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Prediction of rivulet heat transfer at the liquid film breakdown[3]

An exact analytical solution has been developed to estimate the temperature distribution of the liquid rivulet. The analysis is based on solving the energy equation through the transformation of the rivulet cross-section having the shape of a circular sector into an infinite stripe with the help of the conformal mapping method. When the results are related to those of the continuous liquid film at the instant of its breakdown, an increase in heat transfer coefficient is found for the adjacent rivulets, but if the rivulets are separated by dry distances a drop in heat transfer coefficient is obtained. The present study is also compared with an approximate solution for the flat liquid film.

Nomenclature

- a – parameter defined by Eq. (11a),
- E – energy per unit width,
- h – heat transfer coefficient,
- k – thermal conductivity,
- L – rivulet half width,
- $\dot{m}$ – mass flow rate,
- n – normal,
- q – heat transfer rate,
- q_c – heat flux at the rivulet base,
- R – radius of the rivulet,
- T^+ – dimensionless temperature, $T^+ = (T - T_S)k/q_c''$,
- $\bar{T}$ – average temperature,
- W – film width,
- X – wetting ratio, $X = 2L/W$,
- x – coordinate parallel to the surface,
- y – coordinate normal to the surface,
- z – complex variable ($z = x + iy$),

Greek Symbols

- α, β, γ – parameters defined by Eqs. (11a,11c,11d),
- δ – thickness,
- δ^+ – dimensionless thickness,
- ϕ – function defined by Eq. (A5)
- μ – dynamic viscosity,
- Θ – contact angle,
- ρ – density,
- σ – surface tension,
- τ – shear stress,
- ζ – complex variable, ($\zeta = \zeta + i\eta$).

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Subscripts

e – effective,
f – film,
r – rivulet,
s – liquid free surface.

1. Introduction

Rivulet which is a natural product of breakdown of the continuous liquid film has been a neglected topic for a long time while the continuous liquid film has been studied extensively. The film stability has been studied to prevent the film from reaching the critical thickness thus avoiding its rupture. However, the problem of internal heating of stator blades of steam turbines and other chemical engineering processes, in which the breakdown of the liquid film is observed, necessitate an extensive rivulet study. A limited number of studies concerning hydrodynamics and heat transfer of rivulets have been conducted. In hydrodynamics Towell and Rothfeld [1] developed an approximate solution for the velocity distribution in the flat film. Bentwich et al. [2] obtained a polynominal solution to calculate all the rectilinear flow properties of the rivulet over the complete range of practical interest. Mikielewicz and Moszyński [3, 4] have dealt in their works with the rivulets separated by dry surfaces and their stability. Ryley and Tingxiang [5] studied the free energy loss during the film breakdown theoretically and experimentally. They dealt with different geometries of the rivulet and measured the contact angle. Gowroński [6] studied the rivulet heat transfer experimentally and concluded that the mechanisms of rivulet heat transfer are not considerably different from those of film heat transfer. Fujita and Ueda [7] studied the heat transfer of a falling liquid film before and after the film breakdown on a vertical tube. They concluded that for heating at the low heat flux the heat transfer coefficient is nearly the same as for the continuous liquid film while for a high heat flux a lower value of heat transfer coefficient was noted. Rageb [8] carried out a theoretical and experimental investigation for a rivulet under condition that the heat is transferred through the rivulet free surface according to Newton's cooling law. Hirasawa and Hauptmann [9] experimentally measured the dynamic contact angle and the heat transfer parameters for a vertical heated wall.

The problem of evaporation or condensation of the rivulet requires that the liquid layer interface is at a constant temperature which is equal to the temperature of the solid surface at the edge of the rivulet. This is the most common problem which has been dealt with during the study of the film breakdown. This problem has been tackled only approximately through the application of a one dimensional solution for the flat liquid film.

The aim of the present work is to find an exact analytical solution for the temperature distribution within a rivulet having a constant interface temperature. This solution can be used to find the influence of rivulet formation on the heat transfer coefficient and for comparison with the approximate solution. The analysis is carried out by solving the energy equation using the conformal mapping method.

2. Analysis

The liquid rivulet being a subject of the present analysis is considered to have the cross-section in the shape of a circular sector. It is assumed that all the heat is transferred through the rivulet free surface. Moreover, the rivulet base is subject to a constant heat flux while the free surface is at a uniform temperature.

Under the above assumptions, the temperature distribution $T(x,y)$ of the rivulet cross-section shown in Fig. (1a) is governed by the reduced energy equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0\,, \tag{1}$$

with the boundary conditions:

$$T = T_s \tag{2a}$$

at the liquid free surface ABC, and

$$\frac{\partial T}{\partial n} = -\frac{q_c^{''}}{k} \tag{2b}$$

at the rivulet base (surface DEF).

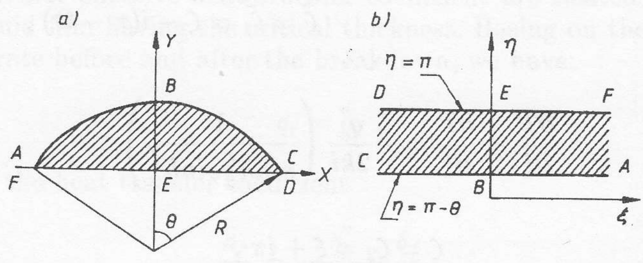

Fig. 1. Rivulet cross-section: a) Z-plane, b) ζ-plane

The equation of the rivulet surface applied for describing the boundary condition in Eq. (2a) is written as;

$$x^2 + y^2 + 2yR\cos\Theta = R^2 \sin\Theta\,. \tag{3a}$$

Eq. (3a) follows from the physical condition of constant temperature and surface tension on the liquid interface. For simplicity it can be assumed that $R\sin\Theta = 1$ so that Eq. (3a) become:

$$x^2 + y^2 + 2y\cot\Theta = 1\,, \tag{3b}$$

The problem can be solved by mapping the z-plane conformally into ζ-plane. Thus the circular sector of Fig. (1a) could be transformated into an infinite stripe Fig. (1b) by using the conformal mapping function

$$\zeta = ln\frac{z-1}{z+1}\,, \tag{4}$$

where

$$\zeta = \xi + i\eta\,, \tag{5a}$$

$$\bar{\zeta} = \xi - i\eta\,, \tag{5b}$$

so that

$$\xi = ln\left(\frac{2y}{[(x+1)^2+y]sin\,\eta}\right)\,, \tag{6a}$$

and

$$\eta = tan^{-1}\frac{2y}{x^2+y^2-1}\,. \tag{6b}$$

In the ζ-plane the transformed problem to be solved becomes

$$\frac{\partial^2 T}{\partial\xi\,\partial\bar{\xi}} = 0\,, \tag{7}$$

with the boundary conditions

$$T = T_s, \qquad \text{at} \qquad \begin{aligned}\zeta = \zeta_1 = \xi + i(\pi - \Theta)\,,\\ \bar{\zeta} = \bar{\zeta}_1 = \zeta - i(\pi - \Theta)\,,\end{aligned} \tag{8a}$$

and

$$\frac{\partial T}{\partial\xi} - \frac{\partial T}{\partial\bar{\xi}} = -\frac{q_c^{"}}{2ki}\left(\frac{1}{sinh\frac{\zeta}{2}sinh\frac{\bar{\zeta}}{2}}\right)$$

at

$$\begin{aligned}\zeta = \zeta_2 = \xi + i\pi\,,\\ \bar{\zeta} = \bar{\zeta}_2 = \xi - i\pi\,.\end{aligned} \tag{8b}$$

The solution of the problem formulated by Eqs. (7) and (8) can be written as

$$T - T_s = \frac{q_c^{"}}{2ki}\left[\frac{1}{sinh\frac{\zeta_2}{2}}ln\left(\frac{coth\frac{\zeta}{4}}{coth\frac{\zeta_1}{4}}\right) - \frac{1}{sinh\frac{\bar{\zeta}_2}{4}}ln\left(\frac{coth\frac{\bar{\zeta}}{4}}{coth\frac{\bar{\zeta}_1}{4}}\right)\right] \tag{9}$$

After rearrangement, the above equation takes a form

$$T - T_s = \frac{q_c^{"}}{2k}\left[a\,ln\left(\frac{\alpha^2+\beta^2}{\beta^2+\gamma^2}\right)\right]\,, \tag{10}$$

where

$$a = \frac{cosh\,\frac{\xi}{2}}{sinh^2\,\frac{\xi}{2} + 1}, \tag{11a}$$

$$\alpha = sinh\,\frac{\xi}{2} cosh\,\frac{\pi - \Theta + \eta}{4}, \tag{11b}$$

$$\beta = cosh\,\frac{\xi}{2} sinh\,\frac{\pi - \Theta + \eta}{4} + sin\,\frac{\pi - \Theta + \eta}{4}, \tag{11c}$$

$$\gamma = cosh\,\frac{\xi}{2} sinh\,\frac{\pi - \Theta + \eta}{4} - sin\,\frac{\pi - \Theta + \eta}{4} \tag{11d}$$

In particular, on the center line of the rivulet cross-section, Eq. (10) is reduced to the following form

$$T - T_s = \frac{q_c''}{k} \ln \frac{tan\,\frac{(\pi - \Theta_o)}{4}}{tan\,\eta/4}. \tag{12}$$

For the purpose of comparison we also present the approximate temperature distribution reffering to the flat film:

$$T - T_s = \frac{q_c''}{k}(\delta - y). \tag{13}$$

In order to estimate the effect of liquid film breakdown on the heat transfer process, the rivulet effective heat transfer coefficient are related to those of the continuous liquid film having the critical thickness. Basing on the equality of the heat transfer rate before and after the breakdown, we have:

$$q_r = q_f, \tag{14}$$

or in terms of the heat transfer coefficient

$$\frac{h_r}{h_f} = \frac{q_c''}{\bar{T}_r - T_s} \frac{\delta_f}{k}, \tag{15}$$

where

$$h_f = \frac{k}{\delta_f}, \tag{16}$$

and

$$h_r = \frac{q_c''}{\bar{T}_r - T_s}. \tag{17}$$

This independent analysis of heat transfer can be now linked together with the hydrodynamic analysis of the broken film done in paper [3]. The analysis based on the equality of masses of the shear driven liquid film before and after its rupture is given in the appendix. On the grounds of this analysis a relation between the rivulet parameters and the film thickness can be written as:

$$\delta_f = \left(\frac{\phi X}{sin\Theta} \right)^{1/2} R \, . \tag{18}$$

Substitution of Eq. (18) into Eq. (14) yields

$$\frac{h_r}{h_f} = \frac{q_c''}{\bar{T}_r - T_s} \left(\frac{\phi X}{sin\Theta} \right)^{1/2} R \, , \tag{19}$$

where the average temperature difference at the liquid-solid interface can be obtained for $\eta = \pi$ as follows:

$$\bar{T}_r - T_s = \frac{q_c''}{2k} \int_0^1 \left[a \, ln \left(\frac{\alpha^2 + \beta^2}{\beta^2 + \gamma^2} \right) \right] dx \, , \tag{20}$$

or

$$\bar{T}_r - T_s = \frac{q_c''}{2k sin\Theta} \left(\frac{\Theta}{sin\Theta} - cos\Theta \right) \, , \tag{21}$$

when linear distribution of the temperature for the flat liquid film is adopted.

Eq. (19) can be applied for the adjacent rivulets with no intervening dry

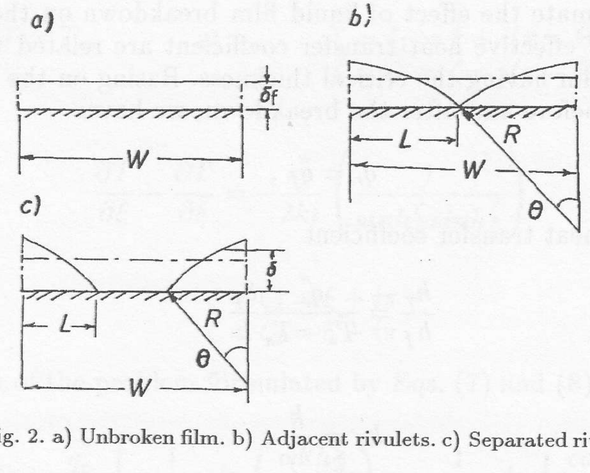

Fig. 2. a) Unbroken film. b) Adjacent rivulets. c) Separated rivulets

surface (Fig. 2b). In this case wetting ratio X is equal to unity and the equation takes the form

$$\frac{h_r}{h_f} = \frac{q_c''}{\bar{T}_r - T_s} \left(\frac{\phi}{sin\Theta} \right)^{1/2} \, . \tag{22}$$

If the rivulets are separated by a dry surface (Fig. 2c) the wetting ratio X is less than unity and its determination is based on the equality of the total energy before and after the breakdown. The relation used to obtain X is given in the appendix.

The effective heat transfer coefficient is an average value of the rivulet heat transfer coefficient and the coefficient reffering to dry areas regarded as adiabatic surfaces. Thus making use of Eqs. (14,19) it is possible to write:

$$\frac{h_e}{h_f} = \frac{q_c''}{T_r - T_f}\left(\frac{\phi X^3}{sin\Theta}\right)^{1/2} . \tag{23}$$

Calculations of the ratios h_r/h_f and h_e/h_f were performed and the results are shown in Figs. 5÷7.

3. Results and discussion

In Figs 3, 4 the dimensionless temperature distribution was drawn at the center line of the rivulet and at the liquid-solid interface respectively using the present analysis and the one-dimensional solution. From these figures it appears

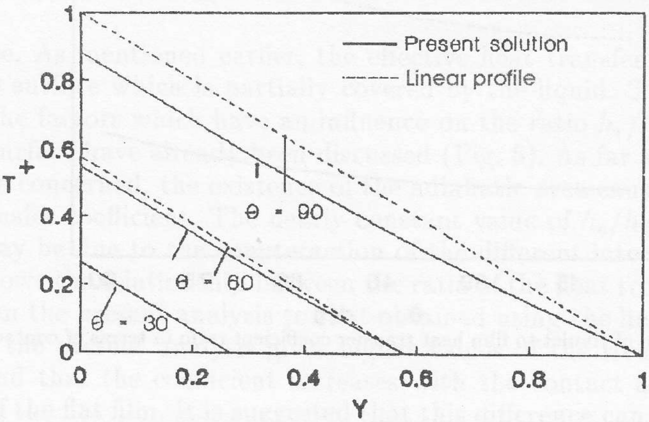

Fig. 3. Comparison of the present solution with the linear profile at the center line

that both solutions are close to each other for small contact angles but for large ones a remarkable difference can be observed. This difference can be explained by the fact that the one dimensional solution gives acceptable results only when the variation of the rivulet thickness is very small and this is true for a slender rivulet. The ratios of mean heat transfer coefficient for the rivulet to that of the flat film having the same liquid-flux as the rivulet are plotted in Fig. 5. The upper curve is drawn for the case when the film rupture creates adjacent rivulets with no intervening dry surface. The observed increase in the heat transfer coefficient may be attributed to the high heat transfer coefficient at the rivulet edge where the thickness approaches zero. The lower curve is drawn for rivulets separated

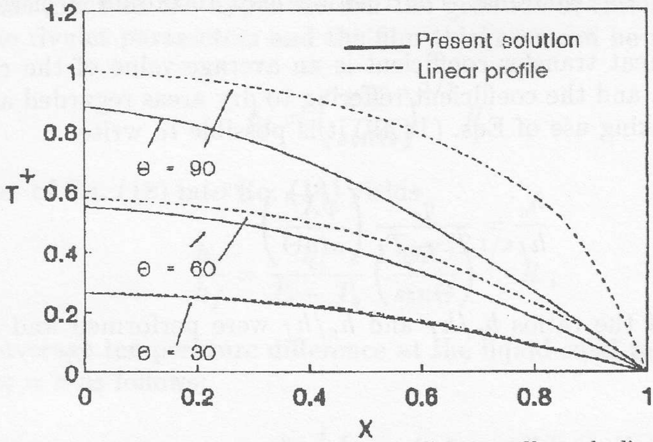

Fig. 4. Comparison of the present solution with the linear profile at the liquid-solid interface.

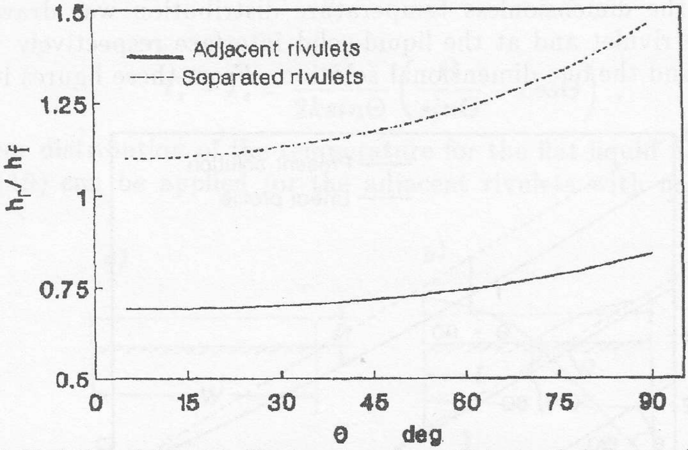

Fig. 5. Variation of rivulet to film heat transfer coefficient ratio in terms of contact angle

by dry areas. Contrary to the previous case, the results show that the ratio of h_r/h_f is less than unity which indicates a significant drop in the heat transfer coefficient after the film breakdown. This is mainly due to the liquid distribution on the surface after the film rupture. The liquid distribution is controlled by the wetting ratio X, described by Eq. (A4), that decreases with the increase in the contact angle. The continuous thin film can produce a rivulet with a relatively large contact angle. Moreover, for the adjacent rivulet since the liquid is concentrated on a smaller area. Therefore, it has a lower heat transfer coefficient and lower h_r/h_f ratio. It can also be seen that h_r/h_f increases with the contact angle. Despite that both the wetting ratio X and the rivulet heat transfer coefficient decrease. This may be explained by taking into consideration the influence of h_f on the ratio h_r/h_f. At the growth of the minimum film thickness with contact angle there is a much faster decrease in the film heat transfer coefficient h_f than in h_r which produces an increase in the ratio h_r/h_f. This should be deemed an explanation for the behaviour of the upper curve.

Fig. 6 shows the variation of the effective heat transfer coefficient with the

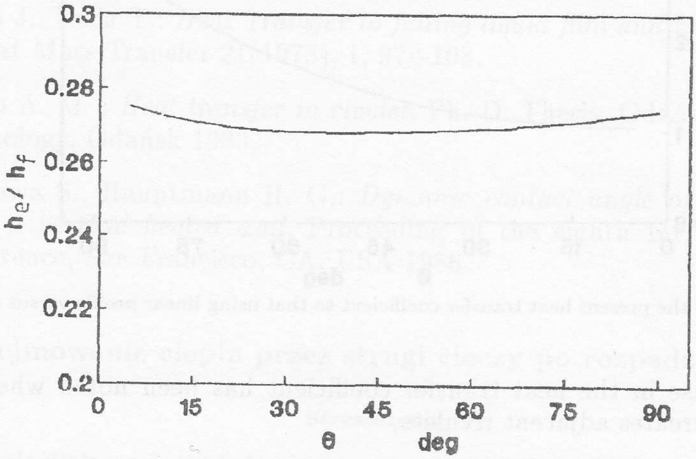

Fig. 6. Variation of effective to film heat transfer coefficient ratio in terms of contact angle

contact angle. As mentioned earlier, the effective heat transfer coefficient is defined for the surface which is partially covered by the liquid. The dry surface is adiabatic. The factors which have an influence on the ratio h_r/h_f for the wetted part of the surface have already been discussed (Fig. 5). As far as the dry part of the surface is concerned, the existence of the adiabatic area causes a reduction of the heat transfer coefficient. The nearly constant value of h_e/h_f that appears in the figure may be due to the counteraction of the different interacting factors.

Fig. 7 shows the relationship between the ratio of the heat transfer coefficients obtained from the present analysis to that obtained using the linear profile. It indicates that the present analysis gives higher values of the rivulet heat transfer coefficient and that the coefficient increases with the contact angle up to 40 % above that of the flat film. It is suggested that this difference can not be neglected and the present analysis is necessary to evalulate the heat transfer parameters for the large contact angle accurately.

4. Conclusions

From the preceding analysis and discussion the following points may be concluded:

- an exact temperature distribution for the rivulet of constant interface temperature has been developed,

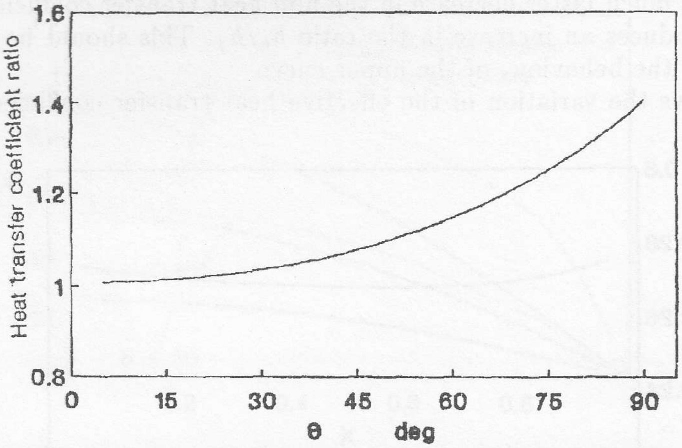

Fig. 7. Variation of the present heat transfer coefficient to that using linear profile versus contact angle

- an increase in the heat transfer coefficient has been noted when the film rupture creates adjacent rivulets,
- a drop in the heat transfer coefficient up to more than 70% of that of the continuous film has been obtained for the case of separated rivulets,
- according to the results of the present study the approximate solution (linear profile) can be used for the case of small contact angle while for large angles it seems to be incorrect to use the approximate solution.

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Przejmowanie ciepła przez strugi cieczy po rozpadzie filmu

Streszczenie

Przejmowanie ciepła przez strugi płynące po powierzchni ciała stałego jest, jak dotychczas, bardzo słabo rozeznane. Niemniej, strugi cieczy wymieniające ciepło występują w wielu zagadnieniach techniki. Celem pracy jest przedstawienie dokładnego rozwiązania analitycznego rozkładu temperatury w strudze, dla przypadku gdy temperatura powierzchni swobodnej strugi jest stała, a na powierzchni styku strugi z ciałem stałym zachowany się stały strumień cieplny.

Dla tych warunków rozwiązano równanie energii (1) przyjmując, że przekrój poprzeczny strugi jest odcinkiem koła. Przy rozwiązywaniu, zastosowano przekształcenie konforemne(4) odcinka koła na pasek na płaszczyźnie zespolonej (rys. 1.). W płaszczyźnie zespolonej, równanie przewodnictwa sprowadza się do postaci (7), a warunki brzegowe do (8). Rozwiązaniem zespolonym jest funkcja analityczna opisana równaniem (9), która w płaszczyźnie rzeczywistej daje rozwiązanie w postaci (10). Pozwoliło to na obliczenie współczynnika przejmowania ciepła od ciała stałego do strug.

W dalszej części pracy, porównano uzyskany rezultat z przejmowaniem ciepła przez film, z którego powstały strugi. Porównanie przeprowadzono dla dwóch przypadków, strug cieczy stykających się i strug oddzielonych suchą powierzchnią. W obliczeniach wykazano, że współczynnik przejmowania ciepła, dla przypadku strug stykających się, jest większy niż dla filmu – natomiast dla strug rozdzielonych suchą powierzchnią – współczynnik przejmowania ciepła jest znacznie niższy, niż dla filmu, z którego postały strugi.

Appendix

Relation Between Liquid Film and Rivulet Dimensions

The relations between the dimensions of the liquid film at the instant of film breakdown and the newly formed rivulet has been investigated using reference [3] and based on the following criteria:

$$\dot{m}_f = \dot{m}_r \ , \tag{A1}$$

$$E_f = E_r \tag{A2}$$

and for stable rivulets

$$\frac{dE_r}{dX} = 0 \ . \tag{A3}$$

The above criteria have been used to obtain the parameter X, which can be written as a function of other rivulet parameters

$$X = \left[\frac{\delta_f \varphi sin^{1/2}\Theta}{2\phi^{3/2}(\frac{\Theta}{cos\Theta} - cos\Theta)} \right]^{2/3} . \tag{A4}$$

where

$$\phi = sin\Theta - \frac{1}{3} sin^3\Theta - \Theta cos\Theta \ , \tag{A5}$$

and

$$\varphi = -\frac{1}{4} cos^3\Theta - \frac{13}{8} cos\Theta + \frac{15\Theta}{8 sin\Theta} - \frac{3}{2}\Theta sin\Theta \, . \tag{A6}$$

δ_f^+ can be found from the following relation obtained as a solution of the system of Eqs. A1, A2, A3

$$3\delta_f^{+2/3} \left(\frac{\varphi^{2/3}}{2^{2/3}\phi} \right) \left(\Theta - \frac{1}{2} sin\Theta \right)^{1/3} - \delta_f^+ = 1 - cos\Theta \ , \tag{A7}$$

where

$$\delta_f^+ = \frac{\rho \tau^2 \delta_f^2}{6\mu\sigma} \, . \tag{A8}$$

The relation between other parameters can be found from the equality of masses as

$$R = \left(\frac{W\delta_f^+}{2\phi} \right)^{1/3} . \tag{A9}$$