POLSKA AKADEMIA NAUK

INSTYTUT MASZYN PRZEPŁYWOWYCH

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

99

GDAŃSK 1995

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

*

THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

Wydanie publikacji dofinansowane zostało przez PAN ze środków DOT uzyskanych z Komitetu Badań Naukowych

REDAKCJA – EDITORIAL OFFICE

Wydawnictwo Instytutu Maszyn Przepływowych
Polskiej Akademii Nauk
ul. Gen. Józefa Fiszera 14, 80-952 Gdańsk, skr. poczt. 621,
☎ (0-58) 46-08-81 wew. 141, fax: (0-58) 41-61-44,
e-mail: tjan@imppan.imp.pg.gda.pl

ISBN 83-01-95102-2
ISSN 0079-3205

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH
1995 Zeszyt 99

ANATOL JAWOREK[1]

Limiting trajectories of dust particles approaching a charged spherical collector

A numerical model for determining the trajectories of the charged dust particles in the vicinity of he oppositely charged droplet (spherical collector) falling freely down is presented. The limiting collision rajectories of the dust particles and the precipitation space are determined theoretically. The model can pe applied in the designing of the charged droplet scrubbers to determine their collection efficiency.

1. Introduction

The difficulty in removal of submicrometer particles by conventional inertial scrubbers lies in small inertial forces acting on dust particles smaller than 1 μm n diameter. Low precipitation efficiency of electrostatic precipitators is caused by the difficulty in charging dust particles of this size, to levels higher than a few elementary charges, what results in low particle mobility. The effective method which allows us to remove the dust particles from exhaust gases in the micrometer and submicrometer size range is the charged droplet scrubbing. In this method of scrubbing, the dust particles and the scrubbing droplets are charged o opposite polarity. The charged droplets act as small spherical collecting electrodes uniformly distributed in the precipitator, and the distances of the dust particles to these collectors are very short. The attractive Coulomb force causes he particles to move toward a charged droplet up to the mechanical contact with it. This type of scrubbers does not have the shortcomings inherent to conventional electrostatic precipitators and inertial scrubbers, which do not allow us to control effectively the dust particles in the submicrometer size range. The charged-droplet-scrubbing, compared to conventional inertial scrubbers, increases manifold the overall collection efficiency. The scrubbers utilizing electrostatic deposition require lower water rate, and lower pressure drop through the equipment,

[1]Department of Plasma Dynamics, Institute of Fluid-Flow Machinery, Fiszera 14, 80-952 Gdańsk

operating at the same over-all collection efficiencies as inertial scrubbers. The equipment utilizing electrostatic forces operates at lower relative velocities than that in which inertial collection is dominant.

The method of cleaning flue gases by means of charged droplets was patented by Penney [1] in 1944. The first theoretical approach to the physical principles governing the removal of charged dust by oppositely charged droplets was given by Kraemer and Johnstone [2] in 1955. They determined the collection efficiency, taking into account the Coulomb image and Stokes forces as well as the space charge effect. Nielsen and Hill [3, 4] have calculated numerically the collection efficiency taking additionally into account the external electric field force and electric dipole interaction force. Wang [5] has stated that Brownian motions affect the collection efficiency for particles smaller than 0.01 μm, and diffusion and thermodiffusion in the 0.1 μm size range. Deshler [6] has presented the results of numerical calculations of the collection efficiency due to diffusiophoresis and thermophoresis.

In the paper by Jaworek [7] the equations of motion of a particle in the vicinity of a charged spherical collector have been solved in 3 dimensional space. In that paper, however, the trajectories of the collector were determined from an approximate equations, whereas in the present paper the differential equations of motion of the collector are incorporated into the model and solved simultaneously with the equation of motion of the particle. A new parameter – 'precipitation space' which characterizes the efficiency of a method of scrubbing has been defined in the paper.

The purpose of this paper is to report on the results of numerical modelling of particle trajectories in the vicinity of a charged spherical collector, and the calculation of the precipitation space for the collector which traverses a flowing cloud of oppositely charged dust. From the numerical value of the volume of the precipitation space, the collection efficiency has been determined. The method of determination of the collection efficiency considerably differs from that given by Kraemer and Johnstone [2], and commonly used in the literature.

In the presented model the collector (scrubbing droplet) is falling freely down, and takes up the velocity of the surrounding gas. Conditions in the case of free fall are quite different from those of the fixed collector (suspended drop), as used in most theoretical models, and for this reason the equations of motion of the collector are incorporated into the model. Simulation of the phenomena of dust particle flow in the vicinity of the collector, in three dimensional space can provide much more information on the complex nature of the scrubbing process and allows us to determine the space from which the particles can be captured.

From the equations of motion the limiting trajectories of the dust particles have been determined. The starting points of the limiting trajectories lie on a semi-infinite pipe which encloses the precipitation space. The precipitation space is determined by means of determination of this pipe. The precipitation space is the measure of the collection efficiency.

2. Equations of motion

The dust particle trajectories in the vicinity of the charged spherical collector (a drop) falling freely down and the precipitation space for this collector are determined numerically by simultaneous solving the equations of motion of the particle and the collector. It is assumed that the Coulomb force, Stokes force and inertial force act upon the dust particle. The image and polarization forces are neglected. The thermo- and diffusiophoresis as well as the gravitational forces on the dust particle are also excluded from the considerations. It is also assumed that both the droplet and dust particle are spherical, and the flow field in the vicinity of the collector is not disturbed by the dust particles. The trajectories are calculated in 3-D space.

The trajectory of a dust particle of mass m_p in the vicinity of the spherical collector charged with the charge Q_c is given by the following differential vector equation:

$$m_p \frac{d\vec{w}}{dt} = \frac{C_d Re_p}{24} \vec{F}_s + \vec{F}_e \tag{1}$$

in which w is the particle velocity, F_s is the Stokes drag force:

$$\vec{F}_s = \frac{6\pi \eta_g R_p (\vec{u} - \vec{w})}{C_c} \tag{2}$$

and F_e the electrostatic force on the particle.

$$F_e = \frac{Q_c Q_p}{4\pi \epsilon_0 r^2} \tag{3}$$

where Q_p, Q_c are the charges on the dust particle and the collector (droplet), R_p the radius of the dust particle, $\vec{u}$ the gas velocity, η_g gas viscosity, r the distance between the particle and the collector centres, ϵ_0 is the permittivity of the free space, C_c is the Cunningham slip correction factor [8, 9], C_d the non-Stokesian drag coefficient [5,10], and Re_p is the Reynolds number for the dust particle. It is assumed that the only electrostatic force is the Coulomb force between two point charges.

The Cunningham slip correction factor is given by the equation [8, 9]:

$$C_c = 1 + AKn \tag{4}$$

with the Knudsen number Kn:

$$Kn = \frac{\lambda}{R_p} \tag{5}$$

where λ is the mean free path of the gas molecules ($\lambda \cong 50$ nm in NTP). The factor A is expressed by the equation:

$$A = \alpha + \beta \exp\left(\frac{-\gamma}{Kn}\right) \tag{6}$$

which is valid for the gas in normal conditions, and the smooth particles or the liquid droplets. Rader [9] has determined values of $\alpha = 1.207$, $\beta = 0.44$, $\gamma = 0.78$, as fitting best the experimental data.

The non-Stokesian drag coefficient for $Re_p < 2 \cdot 10^5$ is given by Brauer and Sucker [5, 10] equation:

$$C_d = 24\, Re_p^{-1} + 3.73\, Re_p^{-1/2} - \frac{4.83 \cdot 10^{-3} \cdot Re_p^{1/2}}{(1 + 3 \cdot 10^{-6} \cdot Re_p^{3/2})} + 0.49 \tag{7}$$

The geometrical dimensions can be normalized to the collector radius R_c. All velocities, i. e. the gas velocity $\vec{u}$, dust particle velocity $\vec{w}$, and collector velocity $\vec{v}$ can be normalized to the gas velocity u_0 in the undisturbed region. Time can be converted into its dimensionless form:

$$\bar{t} = \frac{t u_o}{R_c} \tag{8}$$

The Coulomb force at the characteristic distance R_c normalized to the Stokes drag force for a dust particle moving with the velocity u_o is known as the Coulomb number:

$$Kc = \frac{C_c Q_p Q_c}{24\, \pi^2 \eta_g u_0 \epsilon_0 R_c^2 R_p} \tag{9}$$

The Stokes number is the kinetic energy of the particle divided by the work done by the Stokes drag force at the characteristic distance R_c:

$$St = \frac{2\, C_c R_p^2 \rho_p u_0}{9\, \eta_g R_c} \tag{10}$$

where ρ_p – is the particle density. In its dimensionless form the equation of motion of the particle becomes:

$$\frac{d^2\vec{r}}{d\bar{t}^2} = \frac{1}{St}\frac{C_d Re_p}{24}\left(\vec{u} - \frac{d\vec{r}}{d\bar{t}}\right) + \frac{Kc}{St}\frac{\vec{r}}{\mid \vec{r} \mid^3} \tag{11}$$

where $\vec{r}$ is the radial distance from the collector centre. The flow field around the spherical collector is approximated by the system of two differential equations, as applied by Kraemer and Johnstone [2], which in spherical coordinates r, θ are:

$$\bar{u}_r = \frac{d\bar{r}}{d\bar{t}} = v_c\left(1 - \frac{3}{2\bar{r}} + \frac{1}{2\bar{r}^3}\right)\cos\theta \tag{12}$$

$$\bar{u}_\theta = \bar{r}\frac{d\theta}{d\bar{t}} = -v_c\left(1 - \frac{3}{4\bar{r}} - \frac{1}{4\bar{r}^3}\right)\sin\theta \tag{13}$$

The system (12) and (13) is valid for collector Reynolds numbers $Re_c < 8$. The collector Reynolds number can increase up to about 200 when the drop falls down on the way of 1 m. For $Re_c > 1000$ there is a potential flow around the collector, and well known models of potential flow can be applied in this range of Re_c. However, due to lack of appropriate approximating equations in the range of $8 < Re_c < 1000$, the model given by equations (12) and (13) will be applied also in the transitional region.

The flow field as given by equations (12) and (13) describes the gas velocity in the coordinates in which the flow axis is co-linear with the gas-collector relative velocity, i. e. in the x', y' coordinates (Fig. 1). The angle θ is between the vector

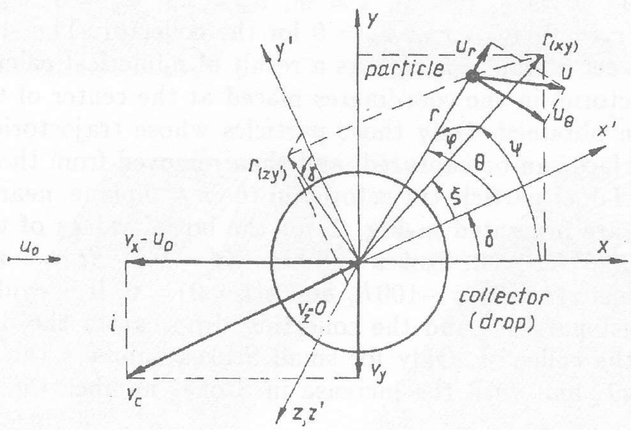

Fig. 1. The spherical collector and the associated system of coordinates

$\vec{r}$ and the vector of the gas - collector relative velocity $\vec{v}_c$ (Fig. 1), which is:

$$\overline{v}_c = \left[(1 + \overline{v}_x)^2 + \overline{v}_y^2\right]^{1/2} \tag{14}$$

The collecting drop enters the scrubbing channel with the initial velocity $v_y = v_{yo}$ and $v_x = 0$. The drop is accelerated due to gravity to the velocity v_y, and is also accelerated by the flowing gas up to the velocity v_x. The velocities are given by the following system of differential equations [11]:

$$\frac{d\overline{v}_x}{d\overline{t}} = 1 - \frac{3}{8}\frac{\rho_g}{\rho_c}\overline{v}_c\overline{v}_x c_x \tag{15}$$

$$\frac{d\overline{v}_y}{d\overline{t}} = g\frac{R_c}{u_o^2} - \frac{3}{8}\frac{\rho_g}{\rho_c}\overline{v}_c\overline{v}_y c_x \tag{16}$$

where c_x is the drag coefficient, which depends on the Reynolds number for the collector:

$$Re_c = \frac{2\rho_g v_c R_c}{\eta_g} \tag{17}$$

For Reynolds numbers $Re \leq 10^5$, c_x is approximated by the Kaskas equation [11]:

$$c_x = 24Re_c^{-1} + 4Re_c^{-1/2} + 0.4 \tag{18}$$

3. Results

The Equation (11), and simultaneously equations (15) and (16), have been solved with the help of the Runge-Kutta method of fourth order with the initial conditions at $t = 0:\ x = x_0,\ y = y_0,\ z = z_0,\ w_x = u_0,\ w_y = 0,\ w_z = 0$ for the dust particle, and $v_x = 0,\ v_y = v_{y0},\ v_z = 0$ for the collector. The starting point of the particle was set at $(x_0,\ y_0,\ z_0)$. As a result of numerical calculations, the dust particle trajectories in the coordinates placed at the center of the spherical collector have been obtained. Only those particles whose trajectories terminate at the collector surface can be captured, and then removed from the gas stream.

The examples of dust particle trajectories in the $z = 0$ plane, near the charged spherical collector are presented in Fig. 2, for the initial values of the Coulomb number $Kc = 10$ and $Kc = 0$, Stokes numbers $St = 0.1$, $St = 1$ and $St = 10$ and initial conditions $x(t = 0) = -100R_c$ and $z(t = 0) = 0$. It is evident that the charging of the dust particles and the collecting drop causes the dust particles to move towards the collector. Only for small Stokes numbers the electrostatic forces are dominant, and with the increase in Stokes number the electrostatic effect diminishes.

The **collision trajectory** of the dust particle is the trajectory which terminates at the surface of the collector. Only these particles which flow along the collision trajectories collide with the collector and can be captured by it.

The **precipitation space** is defined as the geometrical loci of the starting points of the dust particle trajectories which terminate at the surface of the collector (Fig. 3). The precipitation space includes also the collector itself. The boundary of the precipitation space forms a pipe along the collector trajectory which is closed downstream of the collector.

The **limiting trajectory** of the dust particle is the trajectory whose starting point lies at the surface enclosing the precipitation space. The volume of the precipitation space is the measure of the efficiency of the method of scrubbing.

The limiting trajectories are determined numerically by the method of bisection. First, the trajectory of the collector relative to the flowing gas is calculated. Next, each trajectory of a single dust particle is determined assuming its starting point at $(x,\ y,\ z)$, at instant $t = 0$, and its initial velocity $v_x = u_0,\ v_y = v_z = 0$. It is assumed that at the same instant the collector is created at the point $x = y = z = 0$ and starts to fall down with an initial velocity v_{c0}. The equations of motion for the flowing particle and the collector falling down are solved simultaneously up to the collision of both species, or to the distance $r = (4Kc)^{1/2}$ downstream of the collector. It is assumed that the charge and the mass of the collector do not change significantly as the dust particles are captured. The limiting trajectories are determined with the absolute accuracy of 0.01 within the

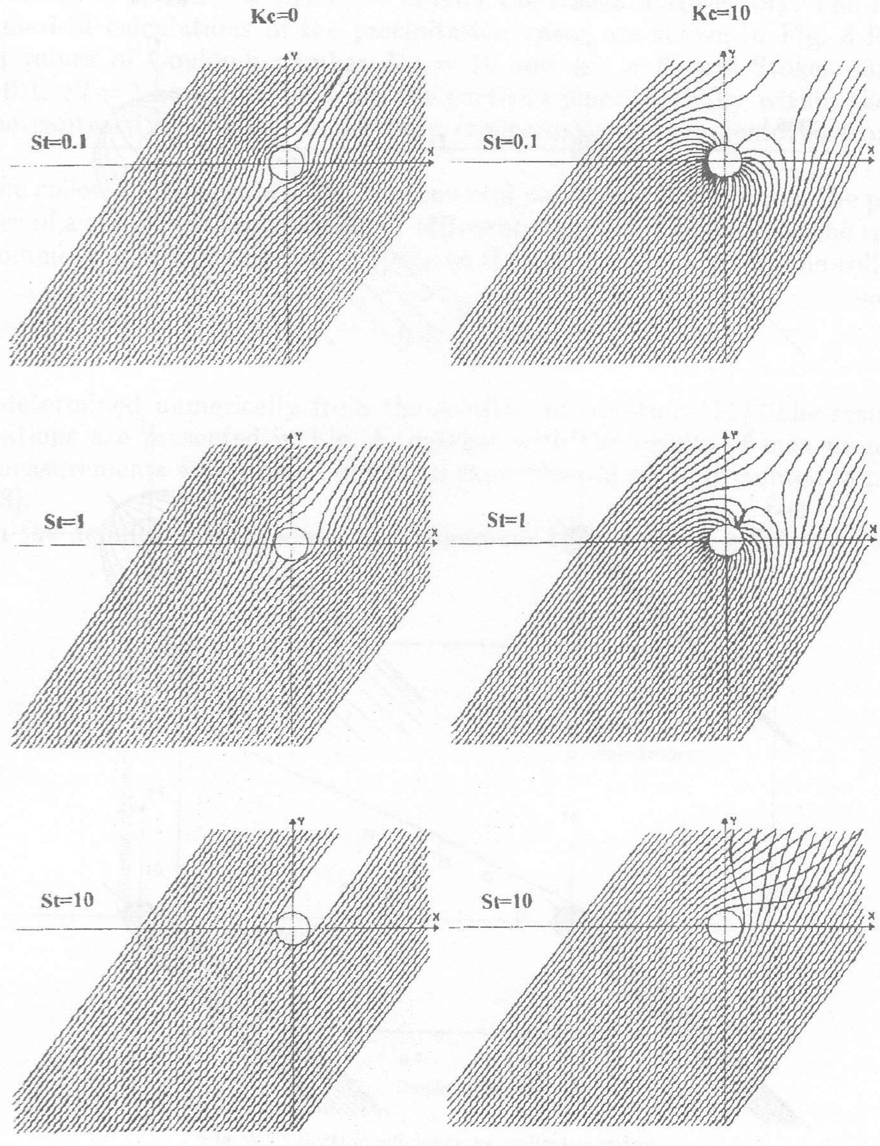

Fig. 2. Dust particle trajectories in the vicinity of the charged spherical collector in the plane $z = 0$, ($R_c = 0.5$ mm, $u_0 = 1$ m/s, $v_{y0} = 1$ m/s)

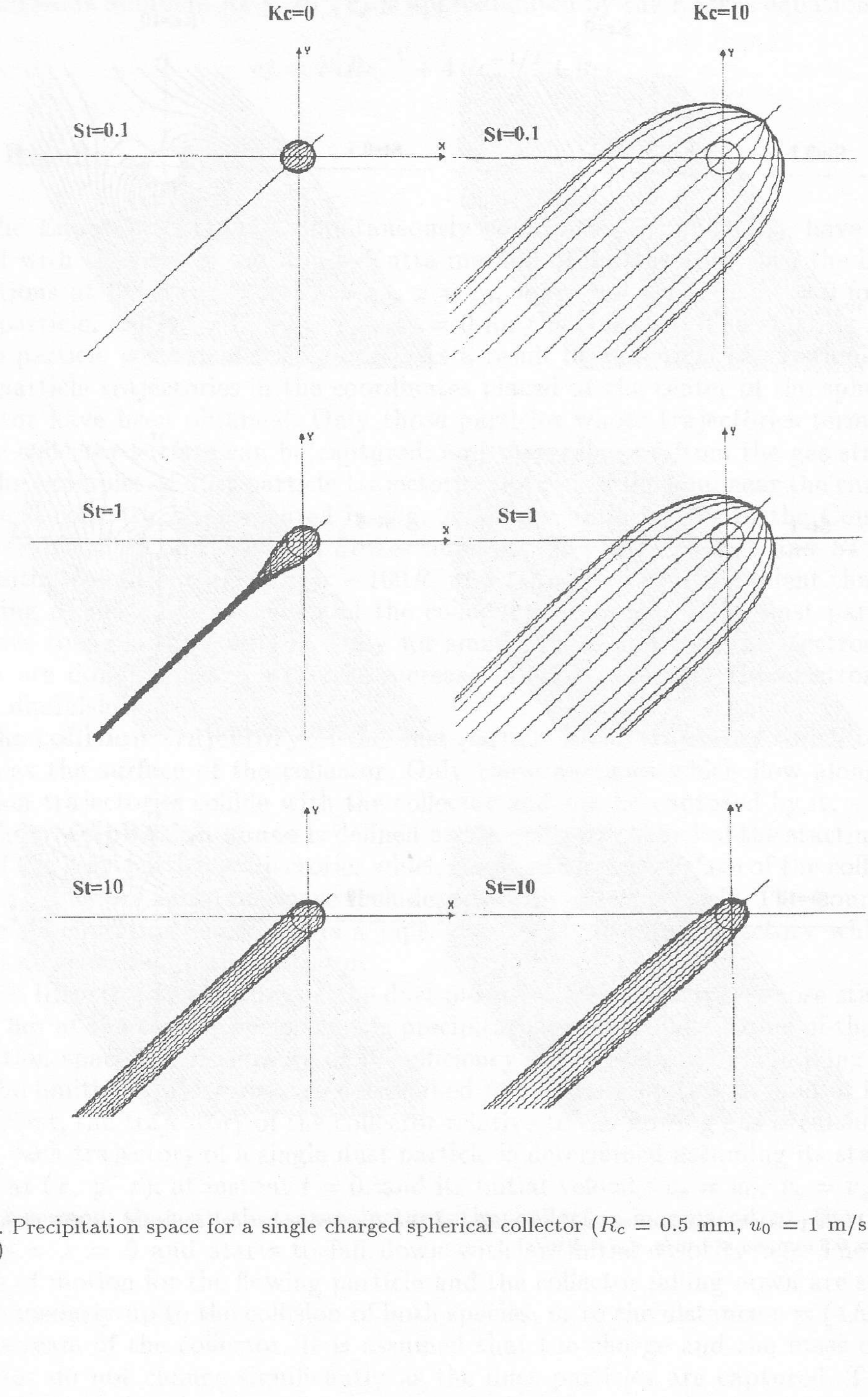

Fig. 3. Precipitation space for a single charged spherical collector ($R_c = 0.5$ mm, $u_0 = 1$ m/s, $v_{y0} = 1$ m/s)

interval of $r = 0$ and $r = (4Kc)^{1/2}$ around the collector trajectory. The results of numerical calculations of the precipitation space are shown in Fig. 3 for the initial values of Coulomb number $Kc = 10$ and $Kc = 0$, and Stokes numbers $St = 0.1$, $St = 1$ and $St = 10$. Only the particles placed initially within the pipe can be captured by the scrubbing drop (collector) and then removed from the gas.

The collection efficiency is the fundamental parameter determining the performances of a scrubber. The **collection efficiency** can be calculated as the ratio of the volume V_c of the precipitation space to the volume V_s swept by the collector:

$$\eta = \frac{V_c}{V_s} \tag{19}$$

both determined numerically from the solution of equation (11). The results of calculations are presented in Fig. 4 together with the results of measurements. The measurements were carried out at an experimental stand presented in papers [12, 13].

In the definition by Kraemer and Johnstone the collection efficiency was de-

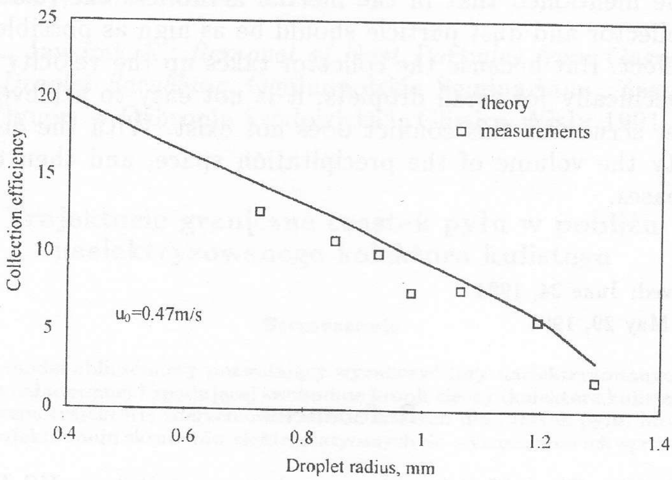

Fig. 4. Collection efficiency vs. collector radius

termined as the quotient of the cross-section for the limiting trajectories starting at $x = -\infty$, to the cross-section of the collector. In practice it is impossible to establish if the starting point of the trajectory is sufficiently distant from the collector. This definition also assumes that the collector is fixed, and the velocity of the particle is constant. These assumptions are however not true in a real precipitator. The definition as proposed by the author of the paper is insensitive to the starting point of the dust particle, and is based on the model which describes real processes that take place in a scrubber.

4. Conclusion

The precipitation space and collection efficiency for a single charged droplet falling freely down in a flowing cloud of oppositely charged dust particles have been determined theoretically. Charging both the dust particles and the collecting droplet causes the increase in volume of the precipitation space and the collection efficiency.

A new method, different from that used in the literature, of determination of the collection efficiency of the charged droplet scrubbing has been presented.

The limiting trajectories of the dust particles and the precipitation space are affected by the Stokes number which is mainly the function of the gas-collector relative velocity, and the Coulomb number which depends principally on the charges on the collector and dust particle.

The deposition of dust particles on a moving spherical collector due to the electrostatic forces are dominant only for small Stokes numbers. With the increase in Stokes number the electrostatic effect diminishes. In order to achieve a high collection efficiency the collecting droplet should be as small as possible, because the Coulomb number increases with the decrease in the collector diameter, at least when the droplets are generated by an electrohydrodynamic method.

It should be mentioned that in the inertial scrubbers the relative velocities between the collector and dust particle should be as high as possible to outweigh the viscosity effect. But because the collector takes up the velocity of the surrounding gas, specifically for small droplets, it is not easy to achieve that. In the charged droplet scrubbers this conflict does not exist. With the decrease in the relative velocity the volume of the precipitation space, and then the collection efficiency increases.

Manuscript received: June 24, 1994
Revised version: May 29, 1995

References

[1] Penney G. W.: *Electrified liquid spray dust-precipitators*, US Patent 2, 357, 354, Sept 5, 1944.

[2] Kraemer H. F.: Johnstone H. F.: *Collection of Aerosol Particles in Presence of Electrostatic Fields*, Ind. Eng. Chem., 47(1955), No 12, 2426-34.

[3] Nielsen K. A., Hill J. C.: *Collection of Inertialess Particles on Spheres with Electrical Forces*, Ind. Eng. Chem. Fundam., 15(1976), No 3, 149-57.

[4] Nielsen K. A., Hill J. C.: *Capture of Particles on Spheres by Inertial and Electrical Forces*, Ind. Eng. Chem. Fundam., 15(1976), No 3, 157-63.

[5] Wang H. C., Stukel J. J., Leong K. H.: *Charged Particle Collection by an Inertial and Electrostatic Forces*, Aerosol Sci. Techn., 5(1985), No 4, 409-21.

[6] Deshler T.: *Measurements of the Rate at which Submicron Particles are Scavenged by Water Drops*, J. Aerosol Sci. 16(1985), No 5, 399-406.

[7] Jaworek A.: *Modelling of Dust Particle Trajectories in the Vicinity of a Charged Spherical Collector in the Charged-Droplet-Scrubbing*, Transactions of the Institute of Fluid-Flow Machinery, 97(1993), 77.

[8] Cunningham E.: *On the velocity of steady fall of spherical particles through fluid medium*, Proc. Royal Soc. 83A(1910), 357-65.

[9] Rader D. J.: *Momentum Slip Correction Factor for Small Particles in Nine Common Gases*, J. Aerosol Sci., 21(1990), No. 2, 161-8.

[10] Brauer H., Sucker D.: Int. Chem. Eng., 18(1978), 367-74.

[11] Orzechowski Z., *Przepływy dwufazowe*, PWN, Warszawa 1990.

[12] Jaworek A., Krupa A.: *Principles of Charged Droplet Scrubbing*, Materials Sci., 16(1990), No. 1-3, 33-8.

[13] Krupa A., Jaworek A.: *Removal of Dust Particles from Gases by Means of Charged Droplet Scrubbing*, Ogólnopolskie Seminarium „Zastosowanie Mechaniki Płynów w Ochronie Srodowiska", Gliwice-Wisła 1991.

Trajektorie graniczne cząstek pyłu w pobliżu naelektryzowanego kolektora kulistego

Streszczenie

Przedstawiono model obliczeniowy pozwalający wyznaczyć tory naelektryzowanych cząstek pyłu w pobliżu przeciwnie naładowanej i spadającej swobodnie kropli cieczy (kolektora kulistego). Wyznaczono teoretycznie graniczne trajektorie zderzeniowe i obszar zderzeń dla cząstek pyłu. Model może znaleźć zastosowanie w projektowaniu skruberów elektrostatycznych do wyznaczania ich sprawności osadzania.