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## PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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Wydawnictwo Instytutu Maszyn Przepływowych
Polskiej Akademii Nauk
ul. Gen. Józefa Fiszera 14, 80-952 Gdańsk, skr. poczt. 621,
☎ (0-58) 46-08-81 wew. 141, fax: (0-58) 41-61-44,
e-mail: esli@imppan.imp.pg.gda.pl

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DARIUSZ P. MIKIELEWICZ[1]

# An improved low-Reynolds-number $k \sim \epsilon$ model for flows in pipes[2]

A new low-Reynolds-number $k \sim \epsilon$ turbulence model for calculations of flows in pipes has been suggested. It is a modification of a selection of the $k \sim \epsilon$ models aimed at fine tuning of the model to predict forced convection heat transfer as well as having a capability to cope with the flows where the influences due to buoyancy are very marked. The new model satisfies these objectives and proves to be very reliable model in the case of turbulent flows in pipes.

## Nomenclature

| | | |
|---|---|---|
| $a, b, c$ | | coefficients in Eq. 4, |
| $B$ | – | buoyancy parameter, $Gr/Re^{3.425}/Pr^{0.8}$, |
| $c_p$ | – | specific heat capacity at constant pressure, |
| $C_1, C_2$ | | constants in modelled dissipation equation, |
| $C_{mu}$ | | constant in constitutive equation of eddy viscosity model, |
| $D$ | – | term in low-Reynolds-number $k$-equation, pipe diameter, |
| $E$ | – | term in low-Reynolds-number $\epsilon$-equation, |
| $f_2$ | – | function in dissipation equation, |
| $f_\mu$ | – | function in constitutive equation of $k \sim \epsilon$ model, |
| $g$ | – | acceleration due to gravity, |
| $Gr$ | – | Grashof number, $\beta g D^4 q \bar{\mu}^2/\lambda/\mu^2$, |
| $h$ | – | enthalpy, |
| $k$ | – | turbulent kinetic energy, |
| $k^+$ | – | normalized turbulent kinetic energy, $k/u_{tau}$, |
| $n$ | – | variable property index in Eq. 8, |
| $Nu$ | – | Nusselt number, $q_w D/(T_w - T_b)\lambda$, |

| | | |
|---|---|---|
| $r, z$ | – | cylindrical polar coordinates, |
| $R$ | – | pipe radius, |
| $Re$ | – | Reynolds number, $\rho W_b D/\mu$, |
| $R_k$ | – | non-dimensional distance from the wall, $yk^{1/2}/\nu$, |
| $Re_t$ | – | turbulence Reynolds number, $k^2/\nu\epsilon$, |
| $t$ | – | temperature in degrees $C$, |
| $T$ | – | temperature in Kelvin, |
| $u_\tau$ | – | friction velocity, $(\tau_w/\rho)^{1/2}$, |
| $vw$ | – | Reynolds stress, |
| $V, W$ | – | mean velocity in $r, z$ directions, |
| $y$ | – | distance from the wall, |
| $y^+$ | – | non-dimensional distance from the wall, $\mu_\tau y/\nu$, |
| $y^*$ | – | non-dimensional distance from the wall, $y/\nu(\nu\epsilon)^{1/4}$, |
| $\epsilon$ | – | dissipation of turbulent kinetic energy, |
| $\tilde{\epsilon}$ | – | modified rate of dissipation of turbulence kinetic energy, $\epsilon = \ddot{\epsilon} + D$, |
| $\lambda$ | – | thermal conductivity, |
| $\mu$ | – | dynamic viscosity, |
| $\nu$ | – | kinematic viscosity, |

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[1]Institute of Fluid-Flow Machinery, Department of Thermodynamics and Heat Transfer, Fiszera 14, 80-952 Gdańsk

Present address: Technical University of Gdańsk, Heat Technology Department, Narutowicza 11/12, 80-952 Gdańsk

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| | | | | | |
|---|---|---|---|---|---|
| $p$ | – | pressure, | $\rho$ | – | density, |
| $Pr$ | – | Prandtl number, $C_p\mu/\lambda$, | $\sigma_t$ | – | turbulent Prandtl number, |
| $q_w$ | – | wall heat flux, | $\sigma_k, \sigma_\epsilon$ | – | turbulent Prandl number for diffusion of $k, \epsilon$. |

Subscripts

| | | | | | |
|---|---|---|---|---|---|
| $b$ | – | bulk, | $fc$ | – | forced convection with variable property effects, |
| $cp$ | – | constant property forced convection, | $t$ | – | turbulent, |
| | | | $w$ | – | wall. |

## 1. Introduction

In view of the complexity of the phenomenon of turbulence, its analysis and modelling present great difficulties. At present time, the $k \sim \epsilon$ model is one of the most widely used turbulence models in engineering practice. In one of the reviews of such models, Patel [1] concluded that the damping functions used in $k \sim \epsilon$ turbulence models, especially the one for the eddy viscosity need to be further modified in order to improve the models performance. The author has already embarked on the path of investigating the $k \sim \epsilon$ turbulence models [2-8], available from literature, in the area of their applicability to predict forced convection and buoyancy influenced flows with the account of physical property variation in heated vertical pipes. The main conclusion from these studies has been that only the model due to Launder and Sharma (here after called LS) [9] was capable, to the certain extent, to cope with these rather severe modifications to the shear stress and velocity field. The LS model proved to be quite reliable in predicting buoyancy influenced flows but failed in the cases where a combination of physical properties and buoyancy influences were both strongly present. Other models were generally too slow in their response to the influences of buoyancy, however, were less prone to over-predict the effects of property variation.

In the present paper the near wall asymptotic behaviour of the eddy viscosity model is presented and a modified low-Reynolds-number $k \sim \epsilon$ turbulence model is suggested. The main goal of the study was to devise a $k \sim \epsilon$ model, which would respond to the influences of buoyancy in a similar fashion or better than the $k \sim \epsilon$ turbulence model due to Launder and Sharma. Another objective to be satisfied by the model was not to over-respond to the effects of property variation. Another aspect which focused the author's attention was that the correlating parameters in a new turbulence model should offer possibly wide areas of application, hence could also be used in modelling of flows with separation, flows in corners etc. From the experience, [2-8], it was found that the only parameter which responded satisfactorily to the influences of buoyancy was the turbulence Reynolds number defined as $Re_t = k^2/\nu\epsilon$. This parameter was the sole correlating parameter in the $k \sim \epsilon$ model postulated by Jones and Launder [10-11] and Launder and Sharma [9]. An approach has been made in the present study to find a better fit to the experimental data from [1] of wall damping functions based on $Re_t$. It failed because of very steep gradients in the distributions of turbulence kinetic energy, which makes it very difficult to model. It has therefore been decided that the distance from the wall defined as $R_k = yk^{1/2}/\nu$ will be selected as a parameter for correlating the near wall damping. There were already several turbulence models suggested which utilized the above parameter. They had however certain deficiencies like they were lacking of a correct near wall behaviour or were simply not very well tuned to predict the wall functions if they obeyed the former. The model which showed most promising results was the model due to Yang and Shih [12]. This model has already been tested in earlier studies [3,4] and showed a moderate response to the influences of buoyancy. It had not been too over responsive to the effects of property variation. The author noticed possible areas for improvement of this model and these results are presented in the present work.

The new model proposed here is first compared against the experimental data of Laufer [13] and also against the original Launder and Sharma (LS), Yang and Şhih (YS), Abe, Kondoh and Nagano (AKN) [14] and Sato, Shimada and Nagano (SSN) [15] formulations and validated on the basis of comparison against the empirical correlation for fully developed heat transfer due to Kurganov and Petukhov [16]. In order to show the model capability to predict strongly buoyancy influenced flows the proposed model is also tested against experimental data of Vilemas et al. [17].

## 2. Near-wall turbulence model

In the case of the $k \sim \epsilon$ models the velocity scale is represented by the square root of the turbulence kinetic energy k and the turbulence length scale is the product of its rate of dissipation $\epsilon (= k^{3/2}/\epsilon)$. In the case of the $k \sim \epsilon$ model, the constitutive equation for the turbulent viscosity is written as follows:

$$\mu_t = C_\mu f_\mu \frac{\rho k^2}{\epsilon}. \tag{1}$$

where $C_\mu = 0.09$ and $f_\mu$ is a damping function. As mentioned earlier the form of the damping function is critical in such formulations, since the prediction of the mean velocity field depends primarily on the eddy viscosity model. The modelled equations for the transport of $k$ and $\epsilon$ are as follows:

k-transport

$$\frac{1}{r}\frac{\partial(\rho r V k)}{\partial r} + \frac{\partial(\rho W k)}{\partial z} = \mu_t \left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] - \rho(\tilde{\epsilon} + D), \tag{2}$$

$\epsilon$-transport

$$\frac{1}{r}\frac{\partial(\rho r V \tilde{\epsilon})}{\partial r} + \frac{\partial(\rho W \tilde{\epsilon})}{\partial z} = C_1 \frac{\tilde{\epsilon}}{k}\mu_t \left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu + \frac{\mu_r}{\sigma_\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial r}\right] - C_2 f_2 \frac{\rho \tilde{\epsilon}^2}{k} + E. \tag{3}$$

## 3. Asymptotic analysis

To analyse the near-wall asymptotic behaviour of the eddy viscosity and other turbulent quantities we can expand the turbulent velocity field near the wall after Hanjalic and Launder [18] in the following way:

$$\begin{aligned} u &= a_1 y + a_2 + \ldots, \\ \nu &= b_2 y^2 + \ldots, \\ w &= c_1 c_1 y + c_2 y^2 + \ldots . \end{aligned} \tag{4}$$

where the coefficients $a, b$ and $c$ are generally nonzero. In the boundary layer, the eddy viscosity is usually defined as:

$$-\nu w = \nu_t \left( \frac{\partial W}{\partial r} \right). \tag{5}$$

Using the expansions from equation (4) it follows that the eddy viscosity has a behaviour as $O(y^3)$. Thus of paramount importance is a correct near wall behaviour of the eddy viscosity model. The quantity $k^{3/2}/\epsilon$ is usually considered a characteristic length scale of the energy containing eddies and the near wall analysis shows that $k^{3/2}/\epsilon$ is $O(y^3)$. Near the wall, the shear stress should behave as $O(y^3)$ as concluded from Eq. 1. Since $k$ is $O(y^2)$ we would require the damping function to have a near wall behaviour of $O(y)$. The present wall damping function is a slightly modified version from [12].

$$f_\mu = [1.0 - \exp(-aR_k - bR_k^3 - cR_k^5)]^{0.5} \cdot [1 + 1/Re_t^{0.75} \exp(-\sqrt{Re_t/10})]. \tag{6}$$

where $a = 1.5 \cdot 10^{-4}, b = 5.0 \cdot 10^{-7}$ and $c = 1.0 \cdot 10^{-10}$. Looking at the formulae for $f_\mu$ it becomes apparent that as $y \to 0, R_k \to 0$ as $O(y^2)$ which gives $f_\mu \to 0$ as $O(y)$. Thus, the near-wall asymptotic behaviour for the shear stress is satisfied. The presence of $Re_t$ in the formulation for $f_\mu$ does not change its near-wall behaviour as it approaches zero as $O(y^4)$. Far from the wall, $R_k$ and $Re_t$ are large and $f_\mu \to 1$. The near-wall model then reduces to its counterpart for high Reynolds number.

It should be the case that when $f_\mu$ and $f_2$ are set to unity, and terms $D$ and $E$ are set to zero, the standard high-Reynolds version of the $k \sim \epsilon$ model is retrieved. However, the models use different values of $C_\mu, C_1, C_2, \sigma_k$ and $\sigma_\epsilon$ which results in the fact that the high-Reynolds number version of the $k \sim \epsilon$ model is not achieved. In a widely acknowledged version of high Reynolds number these constants should take the following values: 0.09, 1.44, 1.92, 1.0 and 1.3 respectively. Variation between the values of constants stems chiefly from the fact that the models were tuned for different kinds of flows. Tables 1 and 2 present details of functions and constants incorporated in the $k \sim \epsilon$ models used.
The boundary conditions used in the solution of the $k$ equations are $k = 0$. In the case of LS model $\epsilon_w = 0$ and the other models use $\epsilon_w = 2\nu(\partial k^{1/2}/\partial y)^2$.

Table 1. Damping functions and model terms used in the model

| Model | $f_1$ | $f_2$ | $f_\mu$ |
|---|---|---|---|
| LS | 1.0 | $1-0.3\exp(-Re_t^2)$ | $\exp[-3.4/(1+Re_t/50)^2]$ |
| YS | $\frac{Re_t^{1/2}}{1+Re_t^{1/2}}$ | $\frac{Re_t^{1/2}}{1+Re_t^{1/2}}$ | $[1-\exp(-aR_k-bR_k^3-cR_k^5)]^{0.5}/(1+1/Re_t^{1/2})$ <br> $a=1,5\cdot10^{-4},\ b=5\cdot10^{7},\ c=1\cdot10^{-10}$ |
| AKN | 1.0 | $[1-\exp(-y^*/3.1)]^2$ <br> $[1-0.3\exp[-(Re_t/6.5)^2]]$ | $[1-\exp(-y^*/14)]^2$ <br> $[1+5/Re_t^{0.75}\exp[-(Re_t/200)^2]]$ |
| SSN | 1.0 | $1-0.3\exp[-(Re_t/6.5)^2]$ | $1-\exp(-Re_t/90)$ <br> $\times[1+(7/Re_t)\exp[-\sqrt{Re_t/10}]]$ |
| PRESENT | 1.0 | $[1.0-\exp(-y^*/3)]^2$ <br> $\times\{1.0-0.3\exp[-(Re_t/6.5)^2]\}$ | $[1-\exp(-aR_k-bR_k^3-cR_k^5)]^{0.5}$ <br> $\times[1+1/Re_t^{3/4}\exp(-\sqrt{Re_t/10})]$ <br> $a=1.5\cdot10^{-4},\ b=5\cdot10^{-7},\ c=1\cdot10^{-10}$ |

Table 2. Model constants

| Model | D | E |
|---|---|---|
| LS | $2\mu(\partial\sqrt{k}/\partial y)^2 \quad y^+\geq 2$ <br> $2\mu k/y^2 \quad y^+<2$ | $2\mu\mu_t/\rho(\partial^2W/\partial y^2)^2$ |
| YS | 0 | $\mu\mu_t/\rho(\partial^2W/\partial y^2)^2$ |
| AKN | 0 | 0 |
| SSN | 0 | $\exp[-(y^+/37)^2]\mu\mu_t/\rho(\partial^2W/\partial y^2)^2$ |
| PRESENT | 0 | $\mu\mu_t/\rho(\partial^2W/\partial y^2)^2$ |

Table 3. Model constants in various $k\sim\epsilon$ models

| Model | $C_\mu$ | $C_1$ | $C_2$ | $\sigma_k$ | $\sigma_\epsilon$ |
|---|---|---|---|---|---|
| LS | 0.0 | 1.4 | 1.9 | 1.0 | 1.3 |
| YS | 0.0 | 1.4 | 1.9 | 1.0 | 1.3 |
| AKN | 0.0 | 1.5 | 1.9 | 1.4 | 1.4 |
| SSN | 0.0 | 1.4 | 1.9 | $1.2/(1+3.5\exp(-Re_t/100))$ | $1.2/(1+3.5\exp(-Re_t/100))$ |
| PRESENT | 0.0 | 1.4 | 1.9 | 1.3 | 1.3 |

## 4. Modelling equations

The geometry considered here is the pipe flow and hence the governing equations are written in the 'boundary layer' approximation. The principal flow direction coincides with the axis of the pipe and the main gradients act in the direction normal to the axis. The thermal boundary condition of the second kind has been considered here, i.e. $q_w = const.$ The governing equations read as follows:
continuity equation

$$\frac{1}{r}\frac{\partial(\rho r V)}{\partial r} + \frac{\partial(\rho W)}{\partial z} = 0, \tag{7}$$

momentum equation

$$\frac{1}{r}\frac{\partial(r\rho VW)}{\partial r} + \frac{\partial(\rho W^2)}{\partial z} = \frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}\left[r(\mu+\mu_t)\frac{\partial W}{\partial r}\right] \pm \rho g, \tag{8}$$

energy equation

$$\frac{1}{r}\frac{\partial(\rho r V)}{\partial r} + \frac{\partial(\rho W)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\lambda}{c_p} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial h}{\partial r}\right]. \tag{9}$$

The turbulent Prandtl, $\sigma_t$, number has a value of 0.85 [5].

## 5. Numerical procedures

Discretization of the equation set is performed according to a finite volume/finite difference scheme following Leschziner [19] and the discretised equations are solved using a 'marching' solution procedure. Correction of the pressure gradient and axial velocity profile at each axial station in order to satisfy overall continuity is achieved using the method of Raithby and Schneider [20]. An axial step length of approximately two viscous sublayer thicknesses is employed (the sublayer for this purpose being taken to have a thickness corresponding to $y^+ = 5$). The radial grid consists of 101 nodes which are distributed to give a high concentration of grid lines near the wall (the wall-adjacent node is positioned at $y^+ \approx 0.5$). Physical properties are updated at each stage following the calculation of the local temperature distribution and pressure.

## 6. Results

As an important criterion of the appropriateness of a turbulence model, the examination of the model function fm was emphasized by Patel et al. [1]. Calculated by various models distributions are shown in Fig. 1. The new model function ollows most closely the actual experiment (see Patel et al. [1]). More significant liscrepancies can be observed in the case of other models, mainly in the log region especially in the case of well established LS model and a relatively 'new' AKN nodel. In Fig. 2 the distribution of turbulent kinetic energy in the case of propo- sed model is given. It is compared against the experimental data due to Laufer 13]. In the conventional $k \sim \epsilon$ model, $\sigma_k$ is usually set to 1.0 and $\sigma_\epsilon$ to 1.3 and nence the turbulent diffusion of $k$ from the wall region is much greater than that of $\epsilon$. As a result of it, the distributions of $k$ do not overlap with the experiment n the core region. This problem can be however alleviated by simply putting $\sigma_k$ o a higher number, in the present case 1.3 and such results are presented in Fig. 2. The behaviour of mt is shown in Fig. 3. The fix of setting $\sigma_k$ to 1.3 instead of 1.0 improves the distribution of mt in the core region either.

To assess the performance of the present model for the prediction of internal boundary layer flows, the model has been used first to simulate turbulent forced flow and heat transfer in pipes under conditions of constant properties. In Table 1, the model predictions are compared with the empirical correlation equation of Kurganov-Petukhov [15], which probably provides the most reliable descrip- tion available on constant property developing forced convection in pipes. In the calculations, the hydrodynamically fully developed profiles of velocity, turbulent kinetic energy and dissipation rate were first obtained and the fully developed Nusselt number then calculated. The calculations started with approximate, the- oretical initial profiles and the code ran for 100 diameters in order to ensure that a fully developed fluid flow condition has been reached. Table 4 shows the va- lues of forced convection Nusselt number obtained in the simulations along with those given by the Kurganov-Petukhov correlation for fully developed constant property forced convection. The range of Reynolds number considered is from 5.0 $\cdot 10^3$ to $6 \cdot 10^4$.

There are discrepancies between the values yielded by the various models and the correlation equation but the percentage differences are generally quite small. The majority of models predict Nusselt numbers which agree with the correlation estimates within 5%. The new model predicts the values of Nusselt number wi- thin 4% except for the lowest Reynolds number, where the validity of correlation could be questioned. A largest discrepancy is found in the case of the YS model which overpredicts heat transfer coefficient by as much as 11%. A good agreement between the model calculations and correlation confirms a correct adjustment of the wall functions especially in the wall region. In the case of air the thermal layer is of comparable thickness to the hydrodynamic layer and as a consequence the results are less sensitive to the precise specification of near-wall turbulence, than in the case of liquids such as water for which the thermal layer is much thinner and any discrepancy would be more marked.

Next, attention was focused on establishing the influence of the temperature dependence of physical properties on the model predictions. It is easier and more reliable to perform this sort of analysis for the case of water where the influences are predominantly due to temperature dependence of viscosity, however variation of density also becomes significant in cases where there are buoyancy influen-

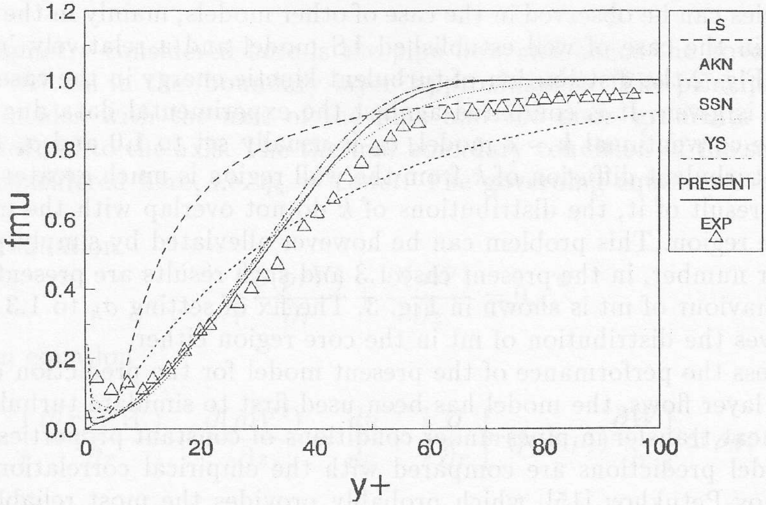


Fig. 1. Distribution of the wall damping function $f_{mu}$.

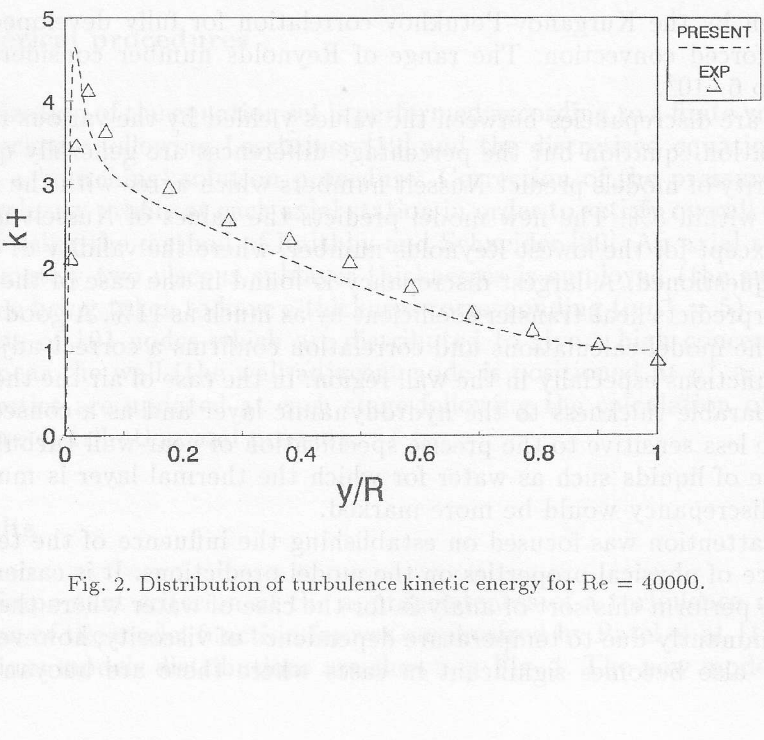


Fig. 2. Distribution of turbulence kinetic energy for Re = 40000.

Table 4. Fully developed constant property Nusselt number for air; $Pr = 0.706, \sigma_t = 0.85$

| Model | Reynolds number | | | | | | |
|---|---|---|---|---|---|---|---|
| | 5000 | 7500 | 10000 | 20000 | 30000 | 40000 | 60000 |
| LS | 17.059 | 23.538 | 29.995 | 51.862 | 71.551 | 89.997 | 124.55 |
| YS | 20.773 | 28.091 | 35.143 | 59.465 | 81.212 | 101.51 | 139.39 |
| AKN | 19.570 | 26.909 | 34.295 | 58.235 | 79.663 | 99.639 | 136.86 |
| SSN | 17.856 | 24.046 | 30.094 | 51.246 | 70.357 | 88.256 | 121.75 |
| PRESENT | 18.688 | 25.300 | 31.449 | 53.459 | 73.246 | 91.769 | 126.44 |
| KURGANOV & PETUKHOV | 17.224 | 24.062 | 30.289 | 52.044 | 71.171 | 88.862 | 121.60 |

ces. The influences of other physical properties are much less significant but also taken into consideration. The influences of variable properties can take two distinct forms, one stemming from buoyancy forces which arise as a consequence of non-uniformity of density and the other from both axial and radial variations of the transport properties viscosity and thermal conductivity. The former depends on both power input and flow rate, becoming less important and eventually negligible as flow rate increases. Under such conditions we are left virtually only with the effect of viscosity variation. The effect of radial viscosity variation under conditions of negligible buoyancy influence is to cause an increase of the heat transfer coefficient. This is due to reduction of viscosity in the near-wall region leading to reduced damping of turbulence and therefore impaired turbulent diffusivity of heat. The effect is usually accounted for using the Sieder and Tate correction [21]. The Sieder and Tate correction involves the application of a viscosity ratio factor to the constant property Nusselt number as follows:

$$Nu = Nu_{cp}\left(\frac{\mu_b}{\mu_w}\right)^n . \qquad (10)$$

The index $n$ is usually assigned the value 0.14. For water at atmospheric pressure the correction is quite small and only results in the enhancement of heat transfer of up to a maximum extent of about 4%. The effects of viscosity variation increase with increase of power input and with decrease of flow rate. In the present case a selection of Buyukalaca's [22] experiments has been simulated and the results are presented in Fig. 4. Calculated index $n$ is plotted as a function of Reynolds

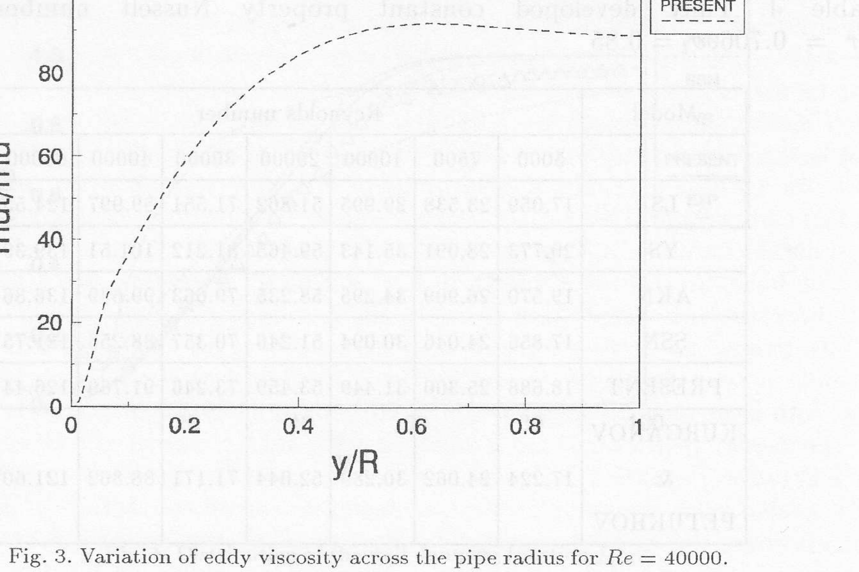


Fig. 3. Variation of eddy viscosity across the pipe radius for $Re = 40000$.

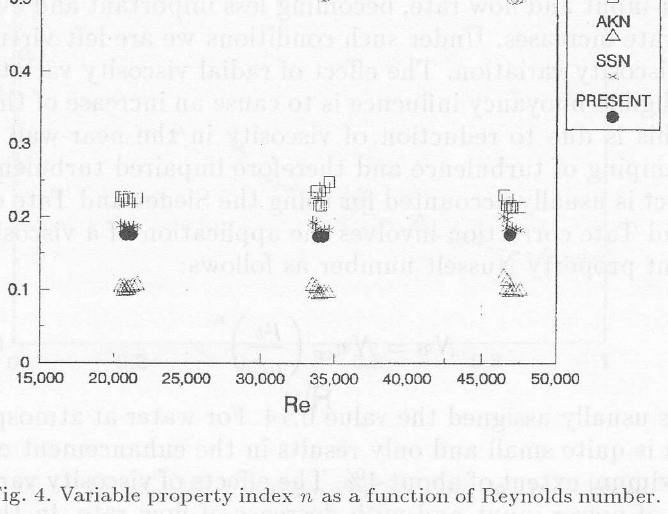


Fig. 4. Variable property index $n$ as a function of Reynolds number.

number. The present model calculates the variable property index closely with experiment at the level of 0.176. In the case of gases it is found that all physical properties can be expressed in the form of absolute temperature ratio correction raised to some power. A typical value quoted for the index n in the case of air is -0.4 (see for example Barnes and Jackson [23]).

Finally, we turn our attention to conditions where buoyancy-influences are present. A buoyancy parameter ($B = Gr/(Re^{3.425}Pr^{0.8})$ of the kind proposed by Jackson and Hall [24,25] serves to quantify the influences of buoyancy. The data base on mixed convection for ascending flow in vertical tubes was recently extended greatly by the publication of a comprehensive set of results by Vilemas, Poskas and Kaupas [17]. Their data encompass a wide range of conditions from what is essentially forced convection, covering the very sensitive region of impaired heat transfer and then extending into the region of enhanced heat transfer. Some cases involve small wall-to-bulk temperature differences ($T_w/T_b \approx 1.05 - 1.1$) and small bulk temperature rise ($\Delta T_b < 30^oC$) but some of the remaining data is strongly influenced by variable properties ($T_w/T_b \approx 1.4 - 1.5, \Delta T_b < 200^oC$).

Eight cases are presented here in two series of four. The first series involves small or moderate influences of variable properties, whereas in the second, the effects of variable properties are quite marked. The inlet Reynolds number varies from 6000 to 20000 and Grashof number is in the range from $4.5 \cdot 10^8$ to $1.4 \cdot 10^9$. The inlet buoyancy parameter varies from $1.19 \cdot 10^{-6}$ to $8.16 \cdot 10^{-4}$. Simulated results are presented in the form of wall temperature development and compared against experimental data from Vilemas et al. In these figures, the development of bulk temperature (same in the case of experiment and all simulations) is also given. Additionally, the ratio of Nusselt number in buoyancy-influenced case normalized by corresponding forced convection value ($Nu_{fc}$) is presented in terms of axial development. Table 5 gives details of the inlet bulk conditions.

Table 5. Conditions (at inlet) for the simulations of Vilemas, Poskas and Kaupas experiments

| SERIES | RUN | Re | $Gr \times 10^{-9}$ | Pr | $B \times 10^6$ | $t_{in}$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 19400 | 0.452 | 0.704 | 1.1875 | 20.19 |
| | 2 | 13300 | 0.239 | 0.706 | 2.3250 | 18.84 |
| | 3 | 8850 | 0.184 | 0.704 | 7.3125 | 20.63 |
| | 4 | 6162 | 0.577 | 0.704 | 76.063 | 21.51 |
| 2 | 5 | 19600 | 1.204 | 0.704 | 3.1625 | 21.12 |
| | 6 | 20700 | 2.389 | 0.704 | 5.0500 | 20.80 |
| | 7 | 11400 | 1.576 | 0.704 | 26.150 | 20.72 |
| | 8 | 7822 | 1.380 | 0.704 | 81.613 | 20.62 |

**Runs 1 to 4** This series is for conditions where the influences of variable properties are quite small and the influences of buoyancy can be observed alone. The buoyancy parameter is in the range from $1.1875 \cdot 10^{-6}$ to $7.606 \cdot 10^{-5}$. These values cover a wide range of buoyancy influences, from conditions where only small modification of heat transfer occurs, through the condition of maximum impairment (where the flow laminarises) and into the region where recovery of heat transfer takes place (recovery of turbulence production and shear stress). With increase of the buoyancy parameter, the measured wall temperatures show the development of the peaks on the distributions (see Figs. 5, 7, 9 and 11). It can be seen that the peak moves upstream with increasing buoyancy parameter. The relative heat transfer development (Figs. 6, 8, 10 and 12) show the trends of heat transfer impairment followed by recovery.

The best simulations of run 1 are revealed by the new model. These are the conditions very close to forced convection and these results reinforce good tuning of the model's functions to predict such flows in the first instance. In the case of run 2, it seems that the LS model is the first model to respond to the modifications of heat transfer due to buoyancy whereas it would seem the new model along with the SSN and AKN models have a slightly delayed response to the above influences. It must be stressed the conditions of complete flow laminarisation (run 2) are very sensitive to the starting parameters in the case of numerical simulations. even a small discrepancy in the input data can lead to completely different model simulations [2]. In authors opinion, the somewhat better response of the LS model can be mainly attributed to the model's over-response to the effects of physical property variation, as we can see that the LS model responds faster than required.

With increased influence of buoyancy (runs 3 and 4) the remaining models start to respond more strongly and all simulations almost overlap. This is a proof that the buoyancy influences have completely damped out the Reynolds stress and in such case where there is no turbulence left in the flow the models are bound to give very similar predictions. The AKN model does not produce results due to numerical difficulties showing its inadequacy to cope with buoyancy-aided flows.

The development of relative heat transfer ratio can be observed in Figs. 6 to 12.

Impairment of heat transfer is calculated by all the models. By far the best general agreement is returned by the new model and the LS model. This is particularly so in the first two runs where these are the only models which are capable of calculating the observed magnitude of heat transfer impairment. In the last run of the series (run 4), enhancement of heat transfer is indicated in the experiment. The present model almost exactly reveals the experimental trends. The LS model calculations differ by some 15% from experiment as a result of the influence of over-prediction of the variable property effect (as do the other predictions). It seems that the new model does not suffer from this effect.

**Runs 5 to 8** In the second series of experiments, the influences of variable properties are stronger and more evident. The wall-to-bulk temperature differences are large. The ratio of the absolute wall to bulk temperatures varies from 1.28 to

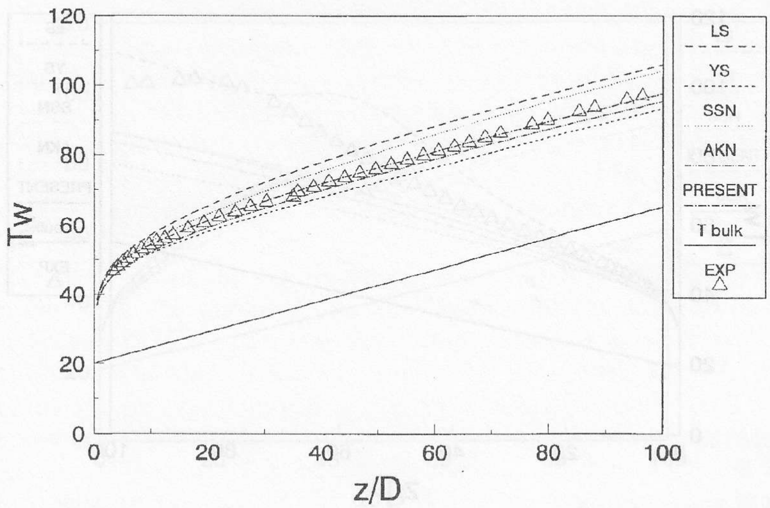


Fig. 5. Wall temperature development – simulation of Vilemas et al. experiment – run 1.

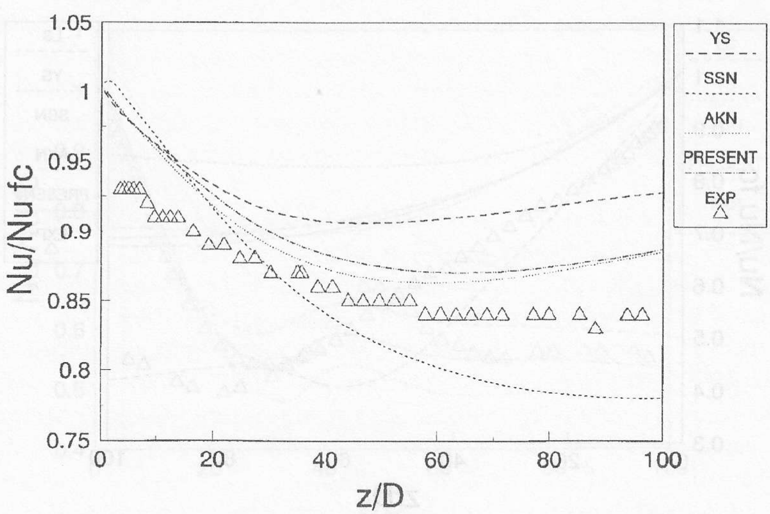


Fig. 6. Relative heat transfer development – run 1.

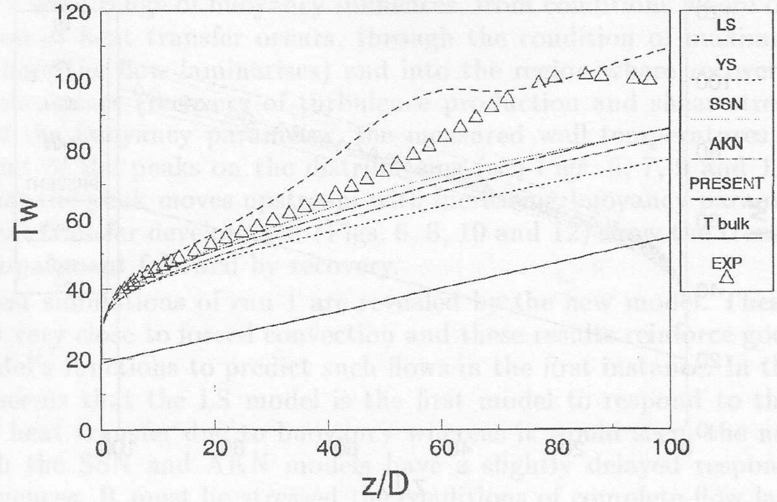


Fig. 7. Wall temperature development – simulation of Vilemas et al. experiment - run 2.

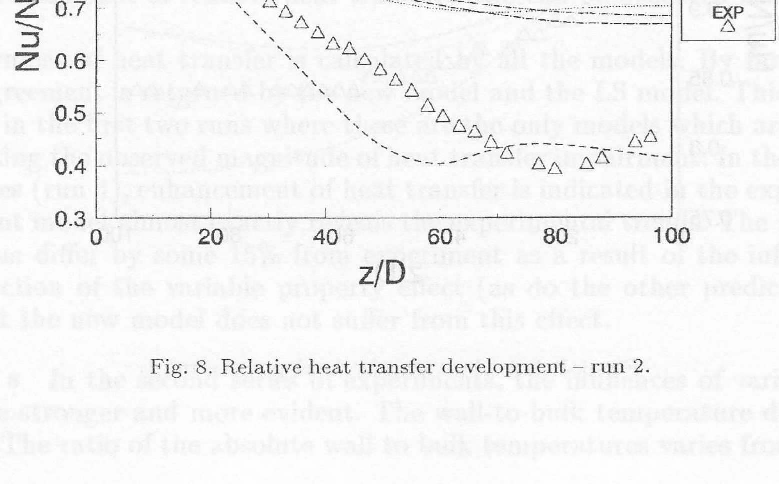


Fig. 8. Relative heat transfer development – run 2.

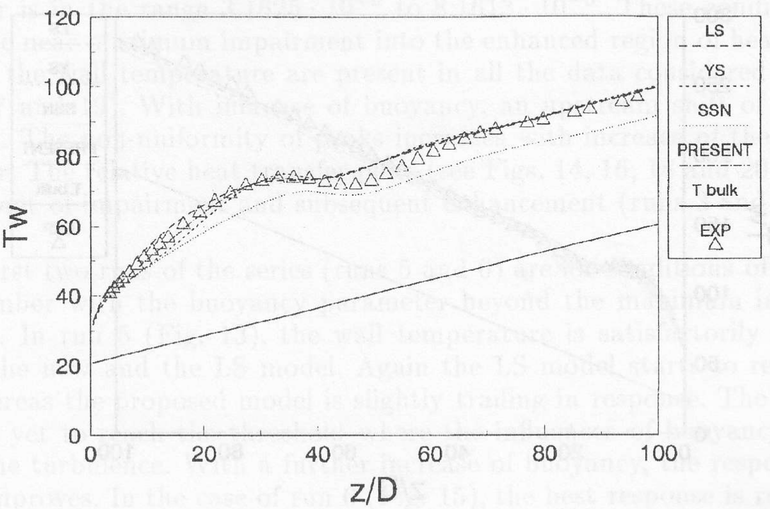


Fig. 9. Wall temperature development – simulation of Vilemas et al. experiment - run 3.

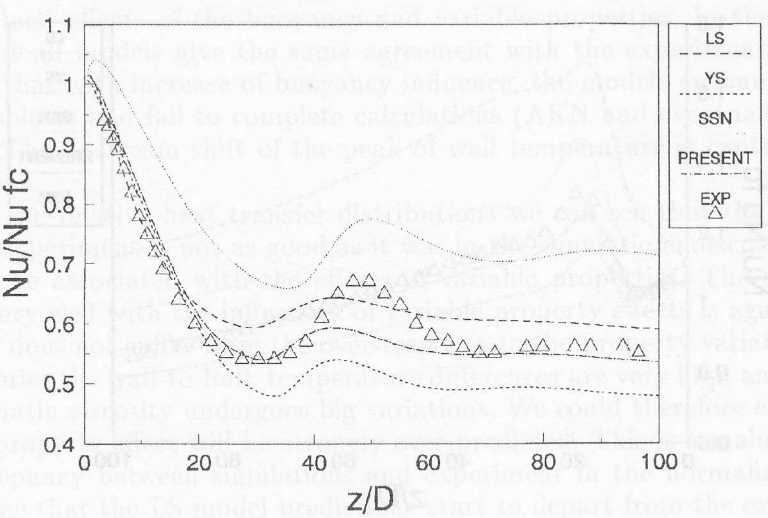


Fig. 10. Relative heat transfer development – run 3.

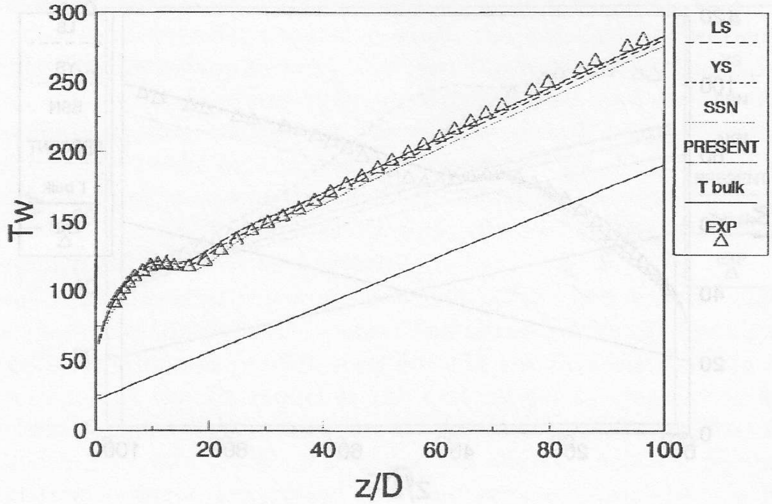


Fig. 11. Wall temperature development – simulation of Vilemas et al. experiment - run 4.

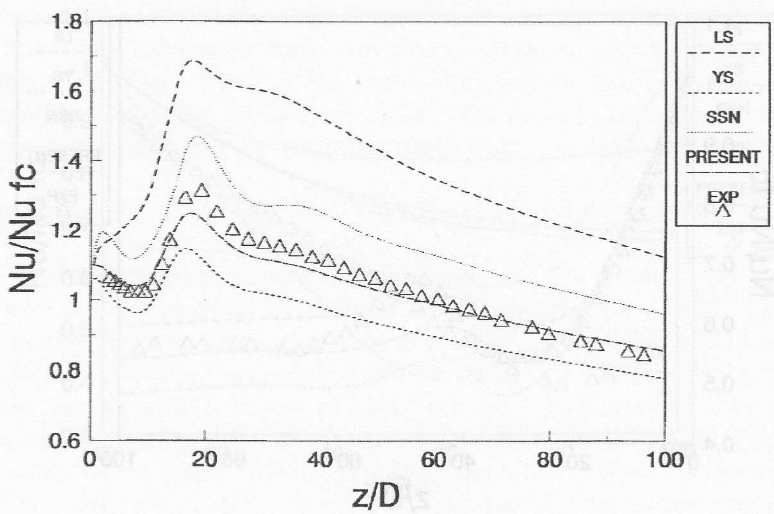


Fig. 12. Relative heat transfer development – run 4.

.35. This would give a temperature ratio correction of 10 to 12%. The buoyancy parameter is in the range $3.1625 \cdot 10^{-6}$ to $8.1613 \cdot 10^{-5}$. These conditions vary from those near maximum impairment into the enhanced region of heat transfer. Peaks on the wall temperature are present in all the data considered (see Figs. 13, 15, 17 and 19). With increase of buoyancy, an upstream shift of the peaks s evident. The non-uniformity of peaks increases with increase of the buoyancy parameter. The relative heat transfer plots (see Figs. 14, 16, 18 and 20) show the development of impairment and subsequent enhancement (runs 3 and 4) of heat transfer.

The first two runs of the series (runs 5 and 6) are for conditions of high Reynolds number with the buoyancy parameter beyond the maximum impairment condition. In run 5 (Fig. 13), the wall temperature is satisfactorily calculated only by the new and the LS model. Again the LS model starts to respond too early whereas the proposed model is slightly trailing in response. The other models have yet to reach the threshold where the influences of buoyancy begin to modify the turbulence. With a further increase of buoyancy, the response of the models improves. In the case of run 6 (Fig. 15), the best response is returned by the new model. Its response is similar to that by the LS models but the postulated model follows the development of the wall temperature up to the first peak on its distribution much better. A failure to predict recovery of heat transfer is evident in the case of all models considered. This is due to the complete laminarization of the flow in the near-wall region. The subsequent recovery of heat transfer cannot be produced as zero values of stress are still being predicted in that region. When the Reynolds number is lower (Figs. 17 and 19) the other models all respond to the combined effects of the buoyancy and variable properties. In these cases it seems that all models give the same agreement with the experiment. It should be noted that with increase of buoyancy influence, the models encounter convergence problems and fail to complete calculations (AKN and eventually the SSN models). The upstream shift of the peak of wall temperature is captured in the calculations.

From the relative heat transfer distributions we can see that the agreement with the experiment is not as good as it was in the simulations discussed earlier. This can be associated with the effects of variable properties. The only model dealing very well with the influences of variable property effects is again the new model. It does not suffer from the over-response to the property variation. In the present series the wall-to-bulk temperature differences are very high and therefore the kinematic viscosity undergoes big variations. We could therefore expect, that variable property effect will be strongly over-predicted. This is a main reason for the discrepancy between simulations and experiment in the normalized results. We can see that the LS model predictions start to depart from the experimental results. In the case of high buoyancy influence and high wall-to-bulk temperature differences this discrepancy increases. The YS and SSN models show similar disagreement with experiment but to a much smaller extent because of the lesser over-response to property variation.

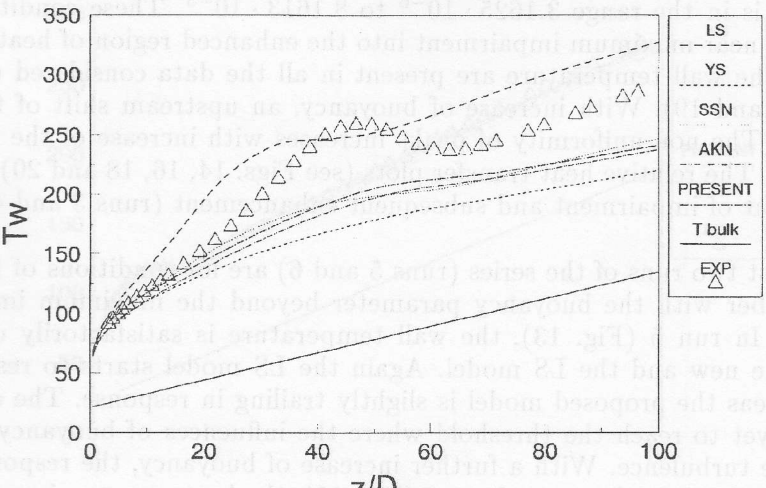


Fig. 13. Wall temperature development – simulation of Vilemas et al. experiment – run 5.

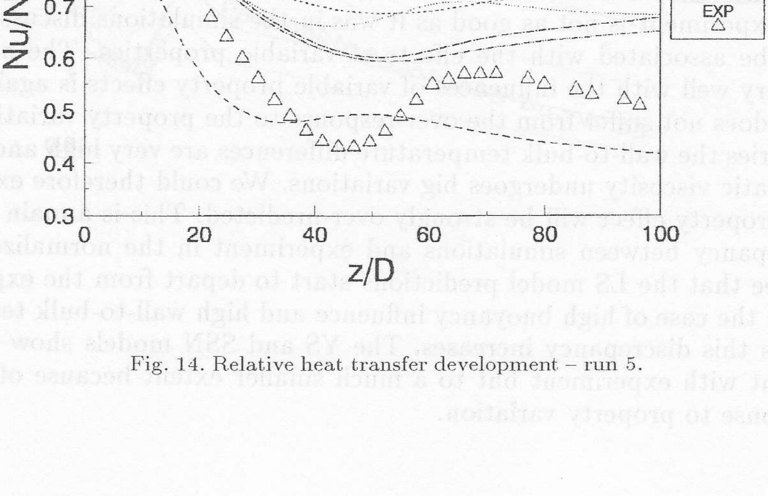


Fig. 14. Relative heat transfer development – run 5.

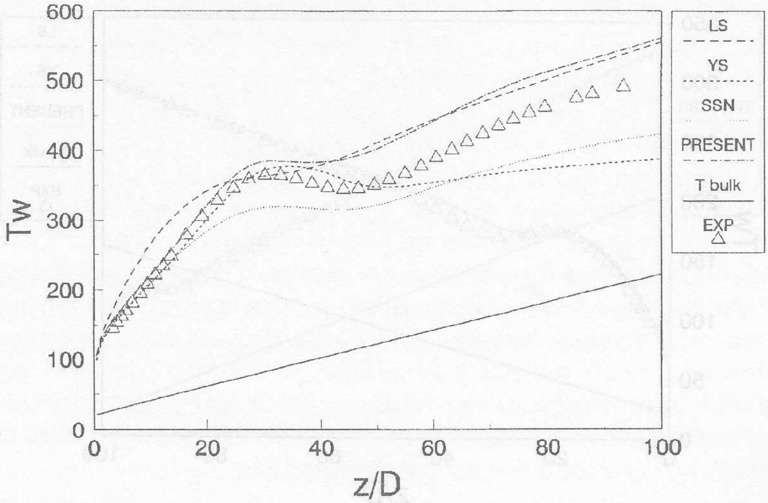


Fig. 15. Wall temperature development – simulation of Vilemas et al. experiment – run 6.

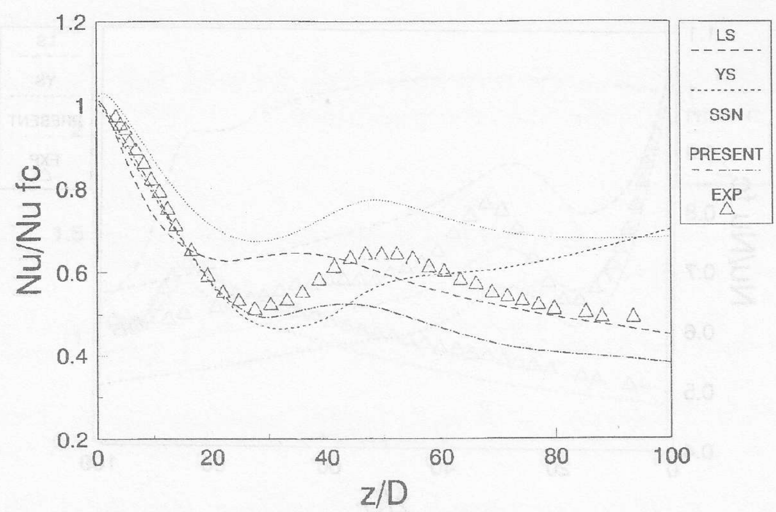


Fig. 16. Relative heat transfer development – run 6.

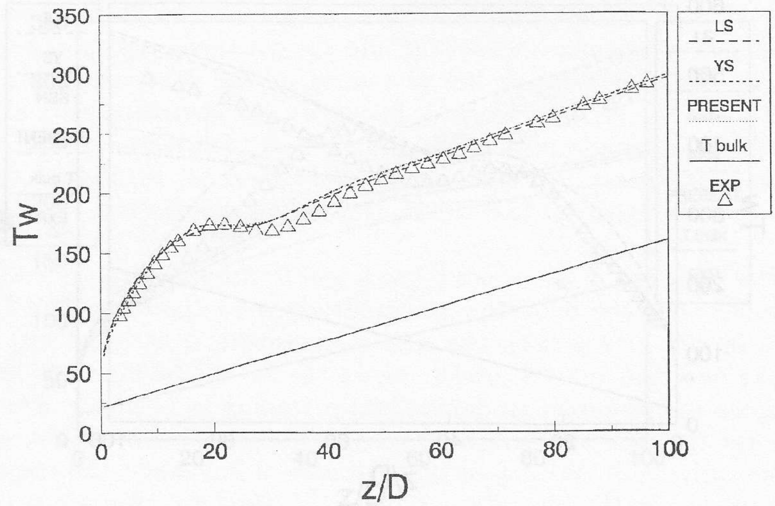


Fig. 17. Wall temperature development – simulation of Vilemas et al. experiment – run 7.

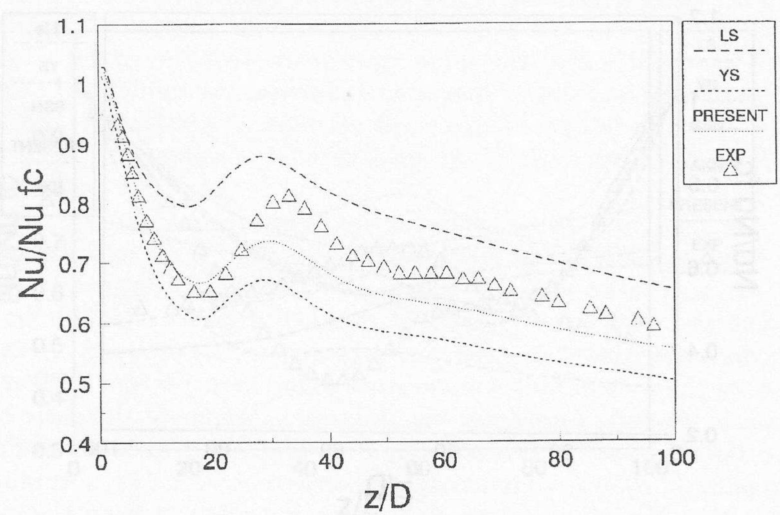


Fig. 18. Relative heat transfer development – run 7.

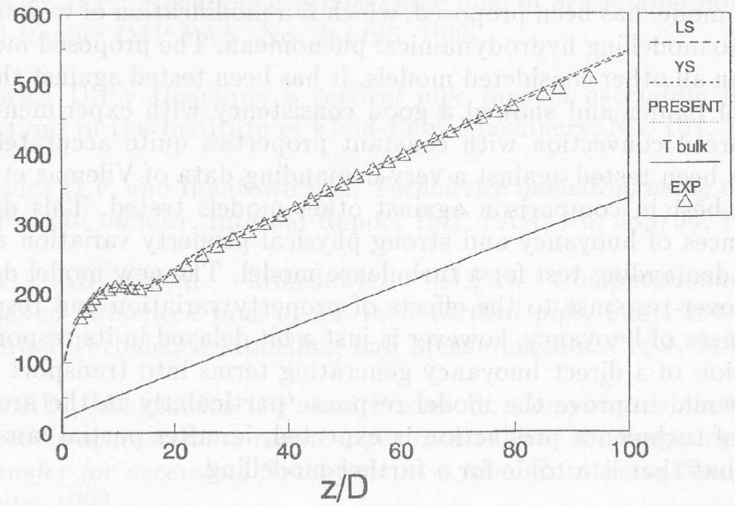


**Fig. 19.** Wall temperature development – simulation of Vilemas et al. experiment – run 8.

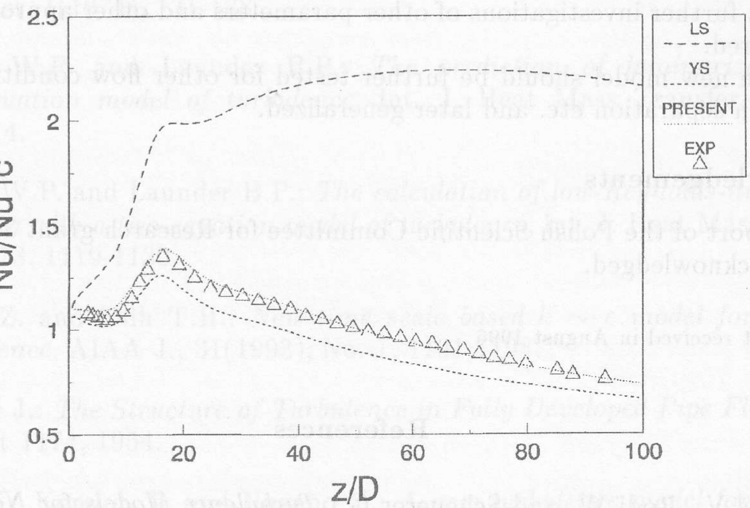


Fig. 20. Relative heat transfer development – run 8.

## 7. Concluding remarks

A new model has been proposed, which is a modification of various authors approaches to modelling hydrodynamical phenomena. The proposed model performs better than all other considered models. It has been tested against the experimental data of Laufer and showed a good consistency with experiment. It can also predict forced convection with constant properties quite accurately. When the model has been tested against a very demanding data of Vilemas et al. it seemed to be the best in comparison against other models tested. This data combines the influences of buoyancy and strong physical property variation and proves to be a very demanding test for a turbulence model. The new model does not suffer from the over-response to the effects of property variation and responds well to the influences of buoyancy, however is just a bit delayed in its response. Probably the inclusion of a direct buoyancy generating terms into transport equations for $k$ and $\epsilon$ would improve the model response particularly in the areas where the recovery of turbulence production is expected, ie. after partial laminarisation of the flow, but that is a topic for a further modelling.

## 8. Conclusions

- For conditions of forced convection with negligible influences of buoyancy the new model is well tuned to predict experimental results.
- The effects of viscosity variation and other physical properties are not over-predicted by the new model.
- It seems that the form of parameter in the damping function of the $k \sim \epsilon$ turbulence models studied here in the form of $R_k = yk^{1/2}/\nu$ is appropriate, but further investigations of other parameters and other approaches are required.
- The new model should be further tested for other flow conditions as flows with separation etc. and later generalized.

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## Zmodyfikowany model $k \sim \epsilon$ do modelowania przepływów w rurach

**Streszczenie**

Zaprezentowano nowy model turbulencji z rodziny $k \sim \epsilon$ do obliczeń przepływów w rurach. Jest to modyfikacja szeregu modeli z rodziny $k \sim \epsilon$, przeanalizowanych wcześniej przez autora. Nowy model dobrze symuluje konwekcję wymuszoną jak i mieszaną, w przypadku stałych i zmiennych własności fizycznych. Nowy model spełnia w zupełności wymagania postawione przed nim w założeniach pracy.