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INSTYTUT MASZYN PRZEPŁYWOWYCH

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

103

WYDAWNICTWO IMP PAN

GDAŃSK 1997

THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

Wydanie publikacji dofinansowane zostało przez PAN ze środków DOT uzyskanych z Komitetu Badań Naukowych

EDITORIAL OFFICE – REDAKCJA

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ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

1997 No. 103

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Influence of plate thermal parameters on non-stationary temperature measurements[3]

In the work presented are the considerations on the non-stationary temperature measurements of the dew point temperature on a flat plate. Presented model enables determination of the dew point temperature by examination of the plate temperature distribution. The model gives very satisfactory results. The influence of selected plate parameters on temperature measurements is also presented.

Nomenclature

a – thermal diffusivity,
A, B, C – constants,
c_p – heat capacity,
g – concentration,
h – enthalpy,
m – humidity, p_s/p,
$\dot{m}$ – mass flow rate,
M – molecular mass,
p – pressure,
r – latent heat,
q – density of heat flux,
Q – heat flux,
Pr – Prandl number,
R_u – gas universal constant,
Sc – Schmidt number,
T – temperature, K
y – vertical co-ordinate,
α – heat transfer coefficient,
δ – plate thickness,
λ – conductivity,
τ – time.

Subscripts

Bp – boiling point,
c – cooling,
e – external,
f – fluid,
g – gas, concentration.
l – liquid,
s – saturation,
w – wall,
0 – initial,

Superscripts

$+$ – non-dimensional quantity,

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[3]The paper was funded by a research project KBN 8 T10B 015 11

1. Introduction

The wet- and dry-bulb psychrometer is widely used to measure the content of moisture in air. In its simplest form, the air is made to flow over a pair of thermometers, one of which has its bulb covered by a wick whose other end is immersed in a small water reservoir. Evaporation of water from the wick causes the wet bulb to cool, and its steady state temperature is a function of the air temperature measured by the dry bulb and the air humidity. Commonly, a psychrometric chart is used to deduce the air humidity from the two temperature readings.

Nowadays the common availability of digital readout thermocouples with built-in cold junctions has led to the increased use of the thermocouple-based psychrometers. Since relatively small thermocouple junctions can be used, their response to changes in temperature and hence humidity is fast, and they are more appropriate than thermometer-based psychrometers in situations where the air state changes rapidly with time, see Trela [1]. The author developed a concept of dew point temperature measurements based on the change of dynamical temperature around the dew point.

There are three possible cases to be considered, which are schematically shown in Fig. 1. First is the one where the cooling process of the plate takes place and finally the dew point temperature is reached. Another case is when the dew point temperature cannot be reached due to insufficient cooling. The last possible case is when we have to heat the plate in order to reach the dew point. In the present work only a first case is considered, i.e. cooling of the plate until the temperature reaches the dew point temperature, as the other possible cases can easily be brought down to the first case.

Consider stagnant air. During the first stage of cooling of wet air in the boundary layer, the mechanism of heat transfer is only by convection to the plate. When the dew point is reached (saturation conditions) then condensation of vapour from wet air begins. This process is controlled by convection and condensation on the surface of the plate. The measurement of the plate temperature usually takes place by a thermocouple embedded from below.

The authors set out the goal to describe the change of regime from that when only convection is present to that where the mass transfer begins. This change of regime is a sign that the dew point temperature has been reached. The thermocouple measurement is usually different from temperature of the plate but we can assume that the plate thickness is small so that a linear variation of temperature takes place across the plate.

2. Theoretical model

Surface energy balances for heat transfer and mass transfer are considered below. An important problem is the condensation of a liquid, for example, con-

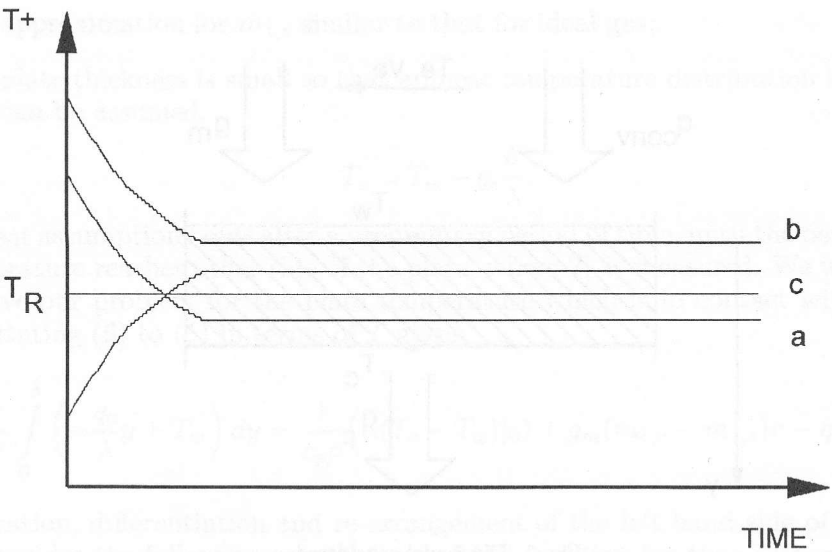

Fig. 1. Possible modes of approaching the dew point temperature, a. cooling curve intersects the dew point, b. cooling curve asymptotes some temperature, which is not a dew point temperature, c. plate needs to be heated in order to reach and intersect the dew point temperature.

densation of water from air, as shown in Fig. 2. The steady-flow energy equation applied to such system requires that

$$\dot{m}\Delta h = \dot{Q}. \tag{1}$$

In the situation considered here, the mass fraction of water vapour in air is relatively small. The highest value is at the surface, due to the cooling process, but even if the air temperature is as high as 50^0C, the corresponding value of m_{H_2O} at 1 bar total pressure is only 0.077. The driving potential for diffusion of water vapour into the interface is $\Delta m = m_{1,s} - m_{1,e}$ and is small compared to unity, even if the free shear air is very dry such that $m_{1,e} = 0$. We can then say that the mass transfer is low and use low mass transfer rate theory. This assumption is valid when we can assume that mass flux and heat flux are independent from each other. For condensation of water vapour on a plate, the error incurred in using low mass transfer rate theory is approximately half of the potential for diffusion of water vapour and a suitable criterion for application of the theory to engineering problems is $\Delta m < 0.2$. If we restrict our attention to conditions, for which the low mass transfer rate theory is valid, we can write $\dot{m}/A = g_m(m_{1,s} - m_{1,e})$. Also, we can then calculate the convective heat transfer as if there were no mass transfer, and write $q_{conv} = \alpha(T_e - T_w)$ .In the present case we consider one-dimensional conduction in the plate. The non-stationary form of the equation for temperature distribution in the plate yields

$$\frac{\partial T}{\partial \tau} = a\frac{\partial^2 T}{\partial y^2} \tag{2}$$

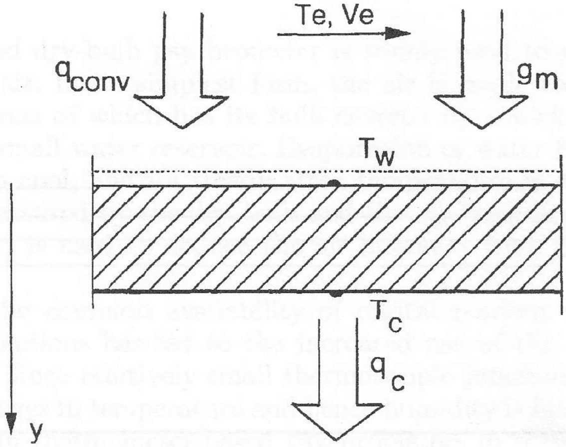

Fig. 2. The model considered.

with the following boundary conditions:

$$\begin{aligned} &\lambda \frac{\partial T}{\partial y}|_0 = \alpha(T_e - T_w)|_0 + g_m(m_{1,e} - m_{1,s})r, && \text{for } y = 0, \\ &\lambda \frac{\partial T}{\partial y}|_\delta = -q_c, && \text{for } y = \delta. \end{aligned} \tag{3}$$

Integration of this equation with respect to y within the limits from 0 to δ gives

$$\frac{\partial}{\partial \tau} \int_0^\delta T\, dy = \frac{1}{c_p \rho}\left(\lambda \frac{\partial T}{\partial y}|_\delta - \lambda \frac{\partial T}{\partial y}|_0\right). \tag{4}$$

According to the boundary conditions we can write that the heat flux at $y = \delta$ is represented by a heat flux due to cooling q_c which is constant and measurable. The heat flux at the distance $y = 0$ is the convective heat flux received from the wet air above the plate. Therefore we can write generally that

$$\frac{\partial}{\partial \tau} \int_0^\delta T\, dy = \frac{1}{c_p \rho}(\alpha(T_e - T_w)|_0) + g_m(m_{1,e} - m_{1,s})r - q_c). \tag{5}$$

In our considerations we have to assume:

- constant material properties,
- constant cooling heat flux,
- constant humidity of air, $m_{1,e}$,

- approximation for $m_{1,s}$ similar to that for ideal gas,
- plate thickness is small so that a linear temperature distribution in a plate can be assumed.

$$T_c = T_w - q_c \frac{\delta}{\lambda}. \tag{6}$$

The last assumption holds after a preliminary period of time, until the penetration temperature reaches other side of the plate, where T_c is measured. We would like to solve our problem for the plate temperature which is in contact with air, so substituting (6) to (5) in terms of T gives

$$\frac{\partial}{\partial \tau} \int_0^\delta \left(-\frac{q_c}{\lambda} y + T_w \right) dy = \frac{1}{c_p \rho} \Big(\alpha (T_e - T_w)|_0) + g_m (m_{1,e} - m_{1,s}) r - q_c \Big). \tag{7}$$

Integration, differentiation and re-arrangement of the left hand side of equation (7) provides the following general form of the equation for time response of the plate

$$\frac{\partial T_w}{\partial \tau} = \frac{1}{c_p \rho \delta} \Big(\alpha (T_e - T_w)|_0) + g_m (m_{1,e} - m_{1,s}) r - q_c \Big). \tag{8}$$

Equation (8) is an ordinary first order equation and hence it presents no difficulties in numerical integration. Before we proceed further we shall cast our equation in a non-dimensional form. Let us introduce the following non-dimensional quantities.

$$T^+ = \frac{T_w}{T_{Bp}} \quad \tau^+ = \frac{\tau \alpha}{c_p \rho \delta} \quad p^+ = \frac{p_{1,s}}{p} \quad q_c^+ = \frac{q_c}{\alpha T_{Bp}}. \tag{9}$$

We can then recast equation (8) into the following form

$$\frac{\partial T^+}{\partial \tau^+} = C + A \left(m_{1,e} - \frac{p^+}{p^+ + 29/18(1 - p^+)} \right) - q_c^+ - T^+ \tag{10}$$

where assuming conditions at $p = 1.01 \times 10^5$ Pa and boiling temperature of 373 K we can read out from tables $r = 2.257 \times 10^6$ J/kg and calculate the constants A, B and C

$$A = \frac{g_m}{\alpha} \frac{r}{T_{Bp}} \approx 5.6 \times 10^6 \quad B = \frac{Mr}{R_u T_{Bp}} \approx 13.1 \quad C = \frac{T_e}{T_{Bp}}. \tag{11}$$

Now we need to resolve the way in which the mass content of water will be calculated. After Mills [2] we write

$$m_{1,s} = \frac{\rho_{H_20}}{\rho} = \frac{p_{1,s}}{p} = \frac{1}{p} \exp \left[-\frac{Mr}{r_u} \left(\frac{1}{T_w} - \frac{1}{T_{Bp}} \right) \right]. \tag{12}$$

For natural convection from a plate, Stanton numbers for heat and mass transfer in gases can be approximated in the following way using the power law relations

$$\frac{g_m}{\alpha} \approx c_p \left(\frac{Pr}{Sc}\right)^{-2/3} . \tag{13}$$

The ratio of Prandtl to Schmidt number is called the Lewis number, a number relevant to simultaneous convective heat and mass transfer. For the case of the air flow past the plate the ratio $g_m/\alpha \approx 925.7$.

It can easily be seen that condensation begins after the system has reached the dew point temperature and up to this point there is no deposition of water droplets, i.e. $A = 0$ as $g_m = 0$. Humidity of external air can be evaluated from the steady-state condition, when we set the time derivative to zero. Then

$$m_{1,e} = \frac{p^+}{p^+ + 29/18(1-p^+)} + \frac{q_c^+ + T^+ - C}{A} . \tag{14}$$

3. Calculation procedure

The problem of cooling without condensation can be solved analytically. The equation (8) in such a case reduces to

$$\frac{\partial T_w}{\partial \tau} = \frac{\alpha}{c_p \rho \delta}\left(T_e - T_w - \frac{q_c}{\alpha}\right) . \tag{15}$$

Integration of equation (15) yields

$$\left(\Theta - \frac{q_c}{\alpha}\right) = C \exp\left(-\frac{\alpha}{c_p \rho \delta}\tau\right) . \tag{16}$$

where $\Theta = T_e - T_w$. The boundary conditions are

$$\text{for} \quad \Theta = 0 \quad \tau = 0. \tag{17}$$

Substitution of boundary conditions to equation (16) results in the following solution of (15)

$$\left(\Theta - \frac{q_c}{\alpha}\right) = -\frac{q_c}{\alpha} \exp\left(-\frac{\alpha}{c_p \rho \delta}\tau\right) . \tag{18}$$

This relation enables us to find time, after which the vapour present in air will start to condensate on a plate. This time is equal to

$$\tau = \ln\left(\frac{C - q_c - T^+}{C - q_c - T_R/T_{Bp}}\right) . \tag{19}$$

This time is used in calculations to determine the point, when the mass transfer starts to come into the model. In the first stage, prior to reaching the dew point temperature by the plate, an analytical solution is used and after the dew point temperature has been reached then the mass transfer term is included and equation (10) is solved in order to obtain the temperature distributions.

Integration of equation (10) is performed using a fifth order Runge-Kutta formula using an adaptive step size [3]. It was found that the results produced using this method are independent of the calculational step assumed in integration procedure.

4. Results

The results of calculations are presented in Figs. 3 to 6. In calculations we have assumed a constant dew point temperature and constant external air temperature. Calculations then proceeded at several cooling rates. Two different external temperatures have been selected.

It is apparent from the figures that the influence of cooling has a significant effect on reaching the dew point temperature. Obviously, it reduces the time required to reach this point. Increasing the cooling rate twice means that the time to reach the dew point temperature will be reduced in a similar fashion.

The influence of the dew point temperature is showing up markedly in the calculations. With the reduction of the dew point temperature we can observe longer times of cooling in order to reach these temperatures. In Figs. 7 and 8 we can observe some deteriorations of plate temperature even after the dew point temperature has been reached. This is probably due to very intensive cooling.

Another very interesting point is that when the dew point temperature is reached then the temperature distributions are almost flat. This finding is very important for assessing the dew point temperature when only looking at the dynamical temperature distributions. Is helps significantly in accurate location of the dew point temperature.

When we consider the non-dimensional time, we can also draw some very important conclusions. When we would like to reduce the real time of measurements we have to enhance the convective heat transfer. On the other hand the stored heat, i.e. the term $\rho c_p \delta$, increases the time of measurements. This means that we cannot sanction too large plate thickness as the response time will be inaccurate.

One would also argue that the plate does not start to respond immediately to the cooling. It is so indeed. We used the penetration theory in order to evaluate the time after which the heat will start to penetrate the plate and the sensor (thermocouple) will start to respond.

Note that the thermocouple temperature will be somewhat delayed compared with the plate temperature. We estimate the temperature lag using the theory of

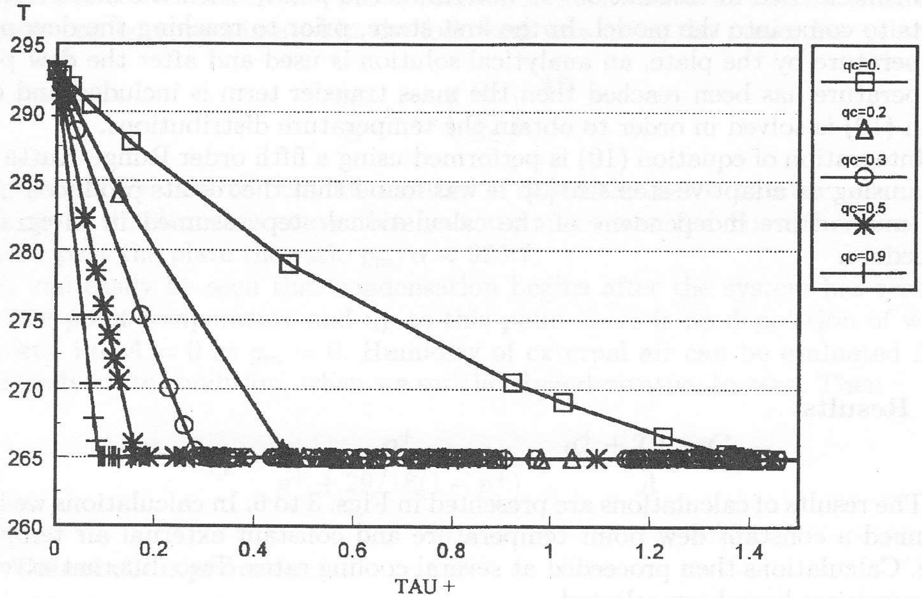

Fig. 3. Dimensional temperature distributions against non-dimensional time for $T_R = 265$ K and $T_e = 293$ K.

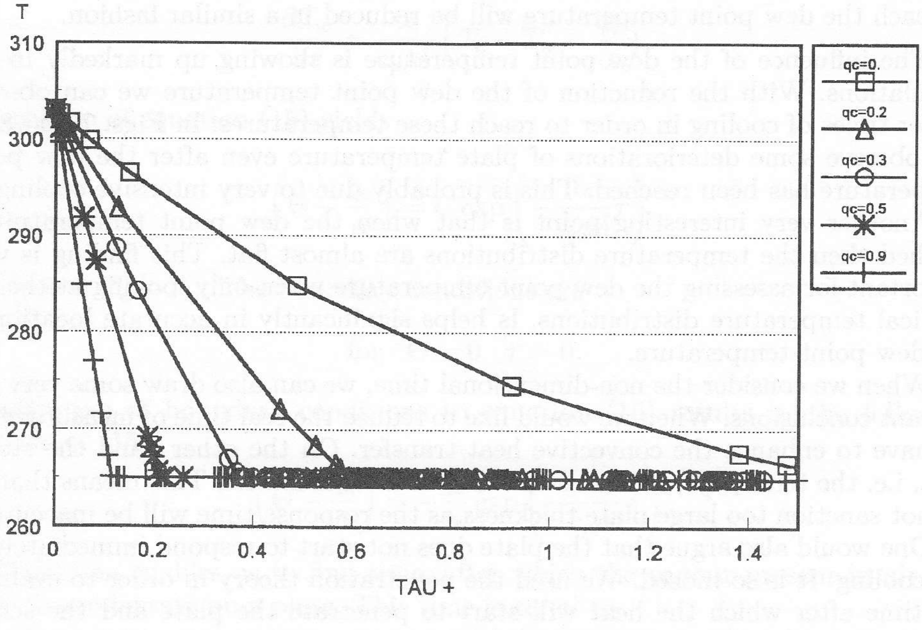

Fig. 4. Dimensional temperature distributions against non-dimensional time for $T_R = 265$ K and $T_e = 303$ K.

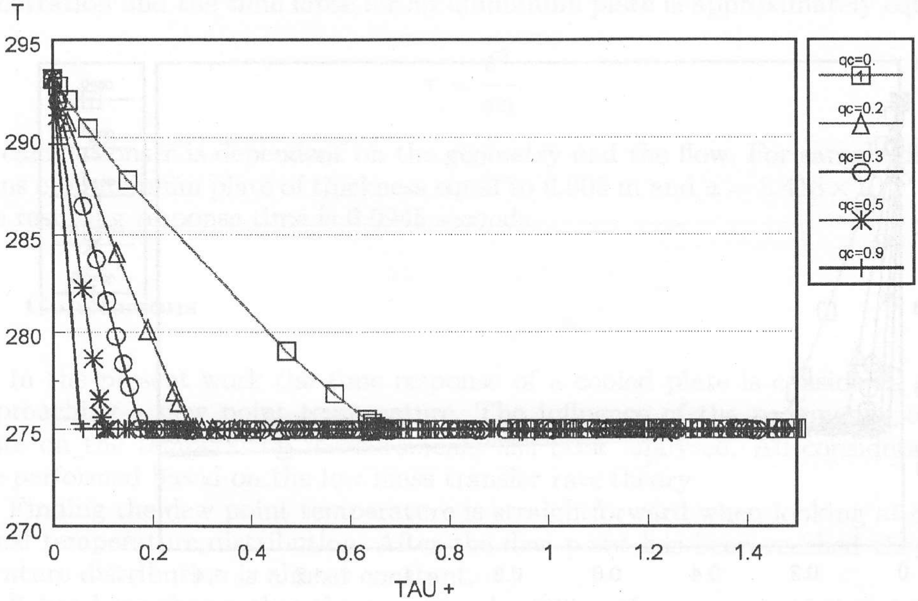

Fig. 5. Dimensional temperature distributions against non-dimensional time for $T_R = 275$ K and $T_e = 293$ K.

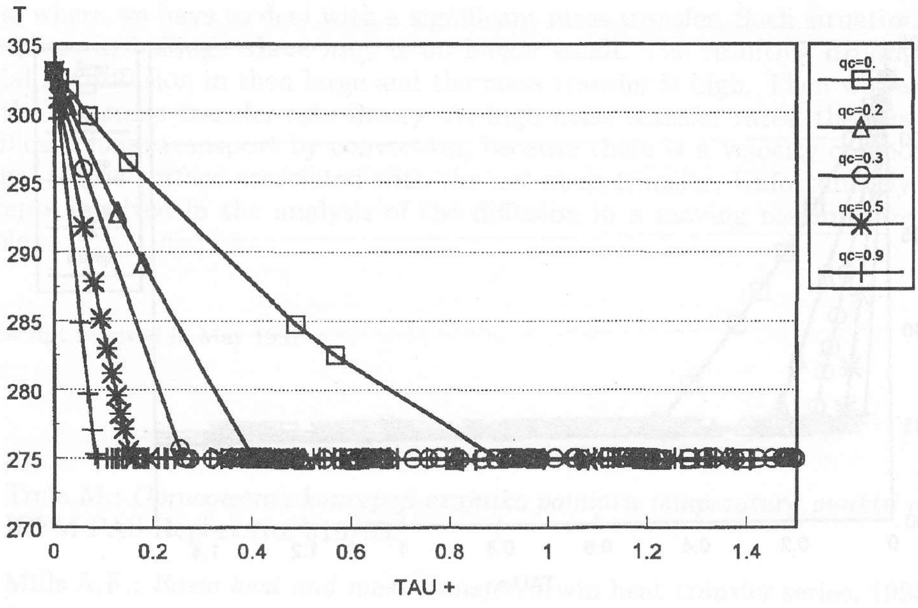

Fig. 6. Dimensional temperature distributions against non-dimensional time for $T_R = 275$ K and $T_e = 303$ K.

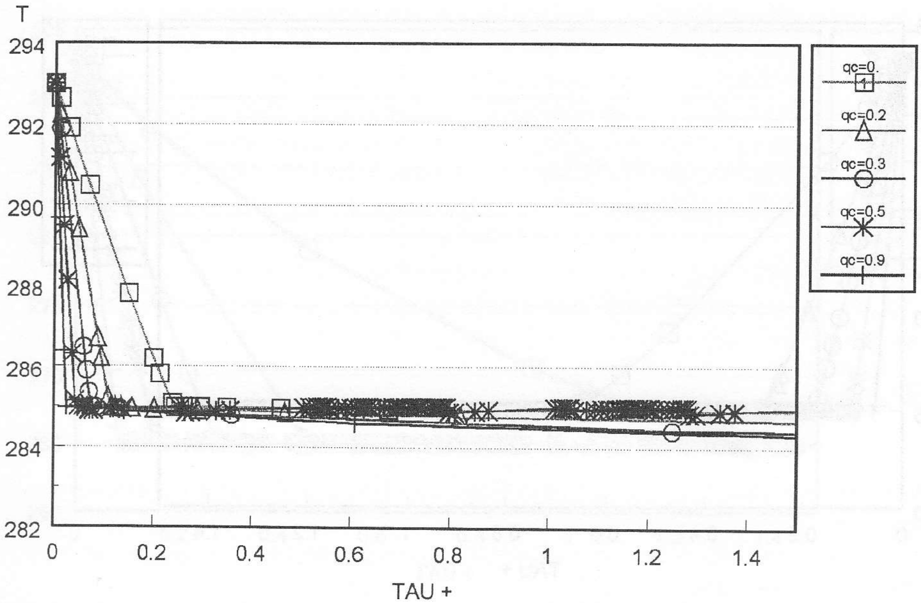

Fig. 7. Dimensional temperature distributions against non-dimensional time for $T_R = 285$ K and $T_e = 293$ K.

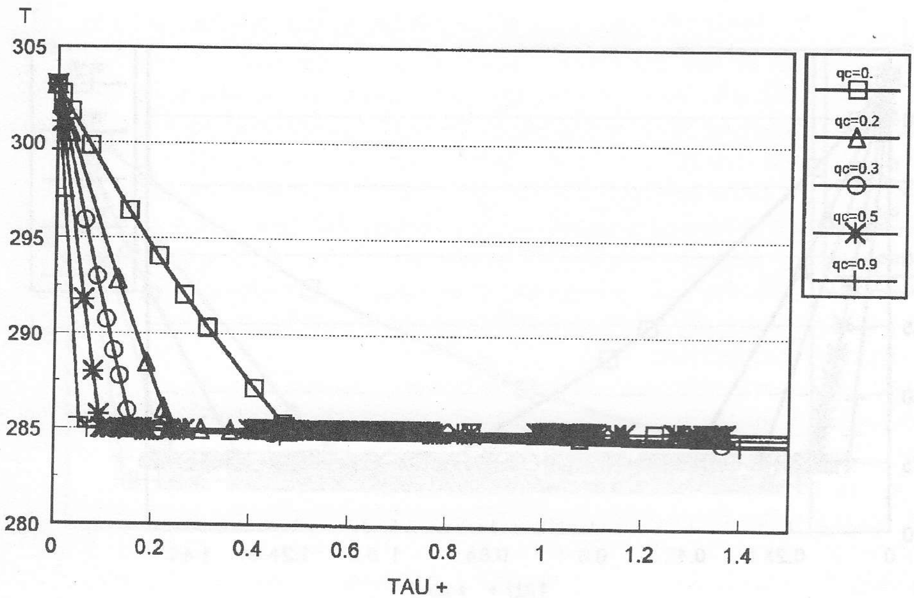

Fig. 8. Dimensional temperature distributions against non-dimensional time for $T_R = 285$ K and $T_e = 303$ K.

penetration and the time lapse for an aluminium plate is approximately equal to

$$\tau = \frac{\delta^2}{\pi a}. \tag{13}$$

In calculations τ is dependent on the geometry and the flow. For sample dimensions of aluminium plate of thickness equal to 0.005 m and $a = 8.418 \times 10^{-5}$ m^2/s the resulting response time is 0.0945 seconds.

5. Conclusions

In the present work the time response of a cooled plate is considered when approaching a dew point temperature. The influence of the parameters of the plate on the temperature measurements has been analysed. All considerations are performed based on the low mass transfer rate theory.

Finding the dew point temperature is straightforward when looking at dynamical temperature distribution. After the dew point has been reached the temperature distribution is almost constant.

It has been shown that the equation describing the response of the plate responds well to the changes of the conditions of air.

Due to the fact that all these considerations are conducted on a basis of the low mass transfer theory, the conclusions reached here cannot be extended to the cases, where we have to deal with a significant mass transfer. Such situation can take place in boiling, where $m_{1,s}$ is no longer small. The resulting driving potential for diffusion in then large and the mass transfer is high. Then we cannot use the low mass transfer rate theory. At high mass transfer rates, there is also significant mass transport by convection, because there is a velocity component normal to the surface associated with the net mass transfer. Unfortunately, the concepts involved in the analysis of the diffusion in a moving medium are not simple.

Manuscript received in May 1997

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Wpływ parametrów termicznych płytki na niestacjonarne pomiary temperatury

Streszczenie

W pracy przedstawiono rozważania dotyczące niestacjonarnych pomiarów temperatury przy pomocy czujnika płytkowego. Analiza oparta jest na rozwiązaniu równania różniczkowego. Przy pomocy tego równania określenie punktu rosy jest bardzo proste. W momencie gdy temperatura punktu rosy zostanie osiągnięta obserwujemy brak zmian w rozkładzie temperatury. Przedstawiona jest dyskusja parametrów płytki na czas odpowiedzi na zmiany temperatury.