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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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# Mathematical analysis of compressor unit operation with stepwise capacity control

The mathematical analysis of the compressor unit operation with a stepwise capacity control, with the assumption of under critical air uptake, is incorporated in the paper. The relationships between the parameters characterising the pressure course in the air tank as a function of dimensionless parameters with the air uptake depending on pressure change have been derived. The formulae on the frequency of switching the compressor into idling and on its maximum value is also given in the paper. In the adapted model of the air uptake only a small impact of the pressure changes on the values of time of the pressure rise and full period of pressure change was proved as well as on the mean value of the air pressure uptaken by users. Simple dimensionless relations allowing for the comparison of the energy effectiveness of the compressor units with various stepwise control systems have also been derived. An experimental verification of some of the relationships was carried out as well.

## 1. Introduction

The purpose of this paper is to analyse of the impact of adopting the constant air uptake on the parameters reflecting the pressure course in air tank in the compressor units with a stepwise control of their capacity, such as: the time of pressure rise and its full change, their ratio and the mean pressure value of the air delivered to users. An essential purpose of the paper is also to evaluate the impact of the adapted model of the air uptake on the relations depicting the frequency of switching the compressor into idling (in the compressor units operating with constant rotational speed) or on the frequency of stopping the compressor or switching off an electric engine – in cases where such a capacity control system is applied.

In the light of the contemporary most important problems in the field of power sector systems driven by internal combustion engines, which are: reduction of harmful emissions and fuel consumption, an equally important purpose of this

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paper is introduction of a relation facilitating a comparative analysis of energy conversion in the compressor units with various stepwise capacity control systems.

## 2. Characteristics of the control system

The stepwise compressor capacity control systems are characterised by simplicity and reliability. The structure of such a capacity control system applied most frequently in the compressor units with the displacement compressors is shown in Fig. 1.

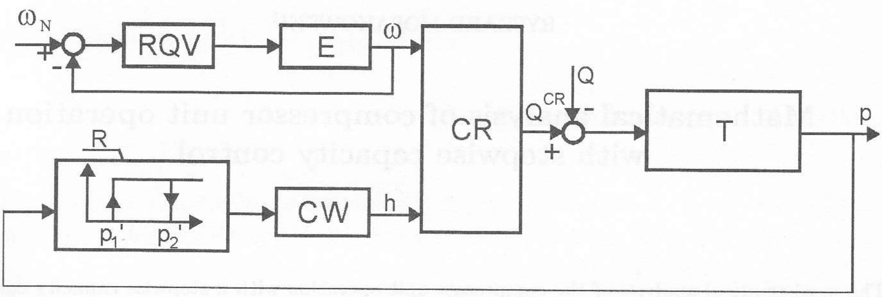


Fig. 1. Structure of the capacity control system in the compressor unit.

The compressor $CR$ is equipped with the controller $R$ of a relay type with hysteresis [1], which maintains pressure $p$ in the air tank $T$ in the range from $p_1 - p_2$. When the pressure increases to value $p_1'$, which is close to $p_2$, the controller $R$ opens the inflow of compressed air from the air tank to the two-state final control unit $CW$, that assumes a position $h = h_{max}$, simultaneously acting on the plates of the compressor's suction valves. Consequently the compressor capacity $Q^{CR}$ drops to zero. In the meantime the pressure in the air tank reaches its maximum value $p_2$. The lasting uptake of the compressed air $Q$ causes a gradual pressure decrease. When the pressure in the air tank drops to value $p_1'$ close to $p_2$ the compressor controller cuts off the connection between the final control unit and the air tank, linking its working space with the ambience. The final control unit returns automatically to its resting state ($h = 0$), not acting on the suction valves' plates, and pressure in the air tank drops to its lowest value $p = p_1$. The compressor starts its operation again with the full capacity. The internal combustion engine runs with a constant rotational speed $\omega$, which is equal to the nominal value $\omega_N$, maintained by the speed governor $RQV$.

Time plots of the compressor mass capacity $Q^{CR}$ and pressure $p$ in the air tank for various values of air mass rate $Q$ uptaken by the air receivers are a graphical pattern of the compressor unit operation. As it follows from Fig. 2, the pressure course in the air tank is of a saw type shape, however, the time periods of the pressure change from $p_1$ to $p_2$ are dependant on $Q$.

Assuming that the time plots shown in Fig. 2 are a graphical model of the

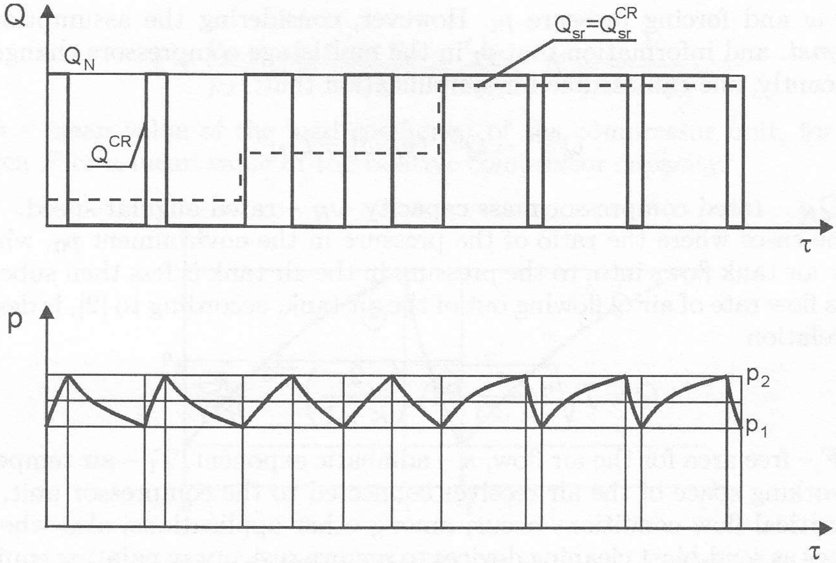


Fig. 2. Time plots of pressure and compressor capacity $Q^{CR}$ for various values of air mean uptake $Q_{sr}$.

compressor unit operation, an attempt of its mathematical description will be presented below.

## 3. Mathematical description

The difference between the mass flow rate of air flowing in $Q^{CR}$ and out of the air tank $Q$ causes a change of the air mass $m$ in the air tank according to equation

$$Q^{CR} - Q = \frac{dm}{d\tau}, \tag{1}$$

where $\tau$ denotes time.

The processes of filling and emptying a tank are accompanied by temperature changes. The range of these changes depends primarily on the mass exchange in the air tank as well as the length of pressure fluctuation period. With small changes of pressure in the air tank (a big volume of the air tank in relation to the rate compressor capacity) the change of a thermodynamic state of air can be depicted by an isothermal process. Based on an assumption, after transformation, equation (1) takes a form

$$dp = \frac{RT}{V}(Q^{CR} - Q)d\tau \tag{2}$$

where: $R$ – gas constant, $T$ – temperature, $V$ – volume of the air tank.

The compressor capacity $Q^{CR}$ in relation (2) is a function of the engine angular velocity $\omega$ and forcing pressure $p_t$. However, considering the assumption $\omega = \omega_N = const.$ and information that $p_t$ in the multistage compressors changes only insignificantly, one can assume for simplification that:

$$Q^{CR} = Q^{CR}(p_t, \omega) = Q_N \tag{3}$$

where: $Q_N$ – rated compressor mass capacity, $\omega_N$ – rated angular speed.

In the cases where the ratio of the pressure in the environment $p_0$, where air from the air tank flows into, to the pressure in the air tank is less then subcritical, the mass flow rate of air $Q$ flowing out of the air tank, according to [2], is described by the relation

$$Q = f\sqrt{2\frac{\kappa}{\kappa-1}RT_1}\left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\frac{p}{RT_1} \tag{4}$$

where: $F$ – free area for the air flow, $\kappa$ – adiabatic exponent, $T_1$ – air temperature in the working space of the air receiver connected to the compressor unit.

Subcritical flow conditions occur, among other applications, also when such appliances as send-blast cleaning devices to remove rust, spray painting equipment etc. are supplied with the air from the compressor unit.

As it follows from relation (4), when disregarding the changes of temperature $T_1$, the air flow rate $Q$ is a function of pressure $p$ and free area $F$ only. In practice the air uptake can be constant, when the air tank is equipped with a control system changing area $F$ in relation to pressure $p$ variations, or variable, when area $F = const.$, irrespective of pressure $p$ changes in the air tank. In the last case, for a given free area $F$ relation (4) may be presented in the following form:

$$Q = C_F \cdot p \tag{5}$$

where

$$C_F = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\sqrt{2\frac{\kappa}{\kappa+1}}\frac{1}{\sqrt{RT}} \cdot F \tag{6}$$

The mean flow rate of air flowing out of the air tank may be determined on the grounds of relation

$$Q_{sr} = Q_{sr}^{CR} = C_F \cdot p_\varphi \tag{7}$$

where

$$p_\varphi = \frac{1}{\tau_2}\int_0^{\tau_2} p(\tau)d\tau \tag{8}$$

and: $\tau_2$ – time period of the full pressure change in the air tank, $p_\varphi$ – mean value of the pressure in the period $\tau_2$.

After substituting relation (7) into (2) and transforming adequately one gets

$$K\frac{dp}{d\tau} = \frac{Q^{CR}}{Q_N} - \frac{\varphi}{x}\frac{p}{p_2} \tag{9}$$

where

$$x=\frac{p_\varphi}{p_2},\quad \varphi=\frac{C_F\cdot p_\varphi}{Q_N}=\frac{Q_{sr}}{Q_N},\quad K=\frac{V}{RTQ_N},$$

and: $\varphi$ – mean value of the load coefficient of the compressor unit, for a given free area $F$ or a mean value of the relative compressor capacity.

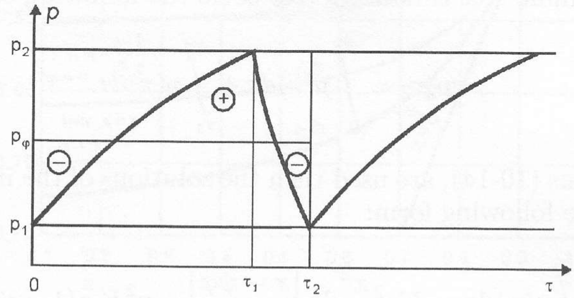


Fig. 3. Diagram of pressure distribution in the air tank for determining $p_\varphi$.

In the period of pressure increase from $p_1$ to $p_2$ the compressor operates with capacity $Q^{CR} = Q_N$. Hence, after integration of equation (9) and taking into account the boundary conditions:

$$p=p_1 \text{ for } \tau=0$$

$$p=p_2 \text{ for } \tau=\tau_1$$

we get

$$p(\tau)=p_2\left[\frac{x}{\varphi}+\left(\psi-\frac{x}{\varphi}\right)\exp\left(-\frac{\varphi}{Kp_2x}\tau\right)\right],\quad \tau\in\langle 0,\tau_1) \tag{10}$$

where $\tau_1$ denotes the time of pressure increase

$$\tau_1=Kp_2\frac{x}{\varphi}\ln\left|\frac{x-\psi\varphi}{x-\varphi}\right|,\quad \psi=\frac{p_1}{p_2}. \tag{11}$$

In the period of pressure decrease from $p_1$ to $p_2$ the compressor capacity $Q=0$. Differential equation (2) depicting the pressure change in the air tank takes the form

$$\frac{dp}{p}=-\frac{\varphi}{Kp_2x}d\tau \tag{12}$$

Solving equation (12) and taking into consideration the boundary conditions:

$$p=p_2 \text{ for } \tau=\tau_1,$$

$$p=p_1 \text{ for } \tau=\tau_2,$$

we get

$$p(\tau) = p_2\left(\frac{x-\psi\varphi}{x-\varphi}\right)\exp\left(-\frac{\varphi}{Kp_2x}\right), \quad \tau \in \langle\tau_1, \tau_2\rangle \tag{13}$$

$$\tau_2 = -Kp_2\frac{x}{\varphi}\ln\left|\frac{x-\varphi}{x-\psi\varphi}\psi\right| \tag{14}$$

In order to determine $x$ it is necessary to solve the following equation:

$$xp_2\tau_2 = \int_0^{\tau_1} p(\tau)d\tau + \int_{\tau_1}^{\tau_2} p(\tau)d\tau. \tag{15}$$

When the relations (10-14), are used then the solutions of the integrals in relation (15) will have the following form:

$$\int_0^{\tau_1} p(\tau)d\tau = Kp_2^2\frac{x^2}{\varphi^2}\ln\left|\frac{x-\psi\varphi}{x-\varphi}\right| - p_2^2K^{\frac{x}{\varphi}}(1-\psi), \tag{16}$$

$$\int_{\tau_1}^{\tau_2} p(\tau)d\tau = Kp_2^2\frac{x}{\varphi}. \tag{17}$$

Substituting relations (14), (16) and (17) to (15) one gets

$$x = \varphi\frac{\psi - \psi^{\frac{\varphi}{\varphi-1}}}{1-\psi^{\frac{\varphi}{\varphi-1}}}; \quad \varphi, \psi \in \langle\psi_0, 1)\ \varphi \in \langle 0, 1\rangle, \quad \frac{p_o}{p} \le \beta \tag{18}$$

where: $\psi_0$ – lowest value of $\psi$ met in practice, $\beta$ – critical pressure ratio.

$$\lim_{\psi\to 0} = \varphi \tag{19}$$

Relation (18) is presented graphically in Fig. 4. As it follows from this figure the mean value of the air pressure delivered to a user drops with decreasing quantity of the air used. For the compressor unit load coefficient $\varphi \le 0,8$ and $\psi \ge 0,85$ this pressure drop is insignificant. It follows from relation (19) that for $\psi \to 0$ expression (18) transforms into equation of a straight line $x = \varphi$, which is tangent to curves $x = x(\varphi, \psi)$ at the point (1,1). In order to illustrate an increasingly strong impact of the decreasing values of $\psi$ on the steepness of function $x = x(\varphi\psi)$, the curves for $\psi < \psi_0$ are also shown in this figure. These curves have therefore theoretical significance only.

Extreme values of $x$ determined on the grounds of de l'Hospital's principle are expressed by relations:

$$x_{min} = \lim_{\varphi\to 0} = \frac{\psi - 1}{\ln\psi} \tag{20}$$

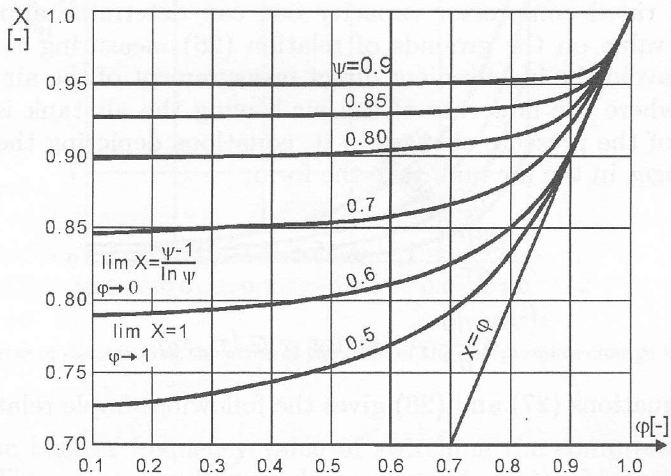


Fig. 4. Dependence of mean relative pressure value in the air tank on compressor unit load coefficient $\varphi$.

$$x_{max} = \lim_{\varphi \to 1} x = 1 \tag{21}$$

It is necessary to note that examination of the limit of $x$ at $\varphi \to 0$, according to relation (20), has a mathematical meaning only.

After substituting relations (18) to (11) and (14) as well as to (10) and (13) we get the following relations:

$$\tau_1 = K p_2 \frac{\varphi}{\varphi - 1} \frac{\psi - \psi^{\frac{\varphi}{\varphi-1}}}{1 - \psi^{\frac{\varphi}{\varphi-1}} \ln \psi} \tag{22}$$

$$\tau_2 = K p_2 \frac{\varphi}{\varphi - 1} \frac{1 - \psi^{\frac{\varphi}{\varphi-1}}}{1 - \psi^{\frac{\varphi}{\varphi-1}} \ln \psi} \tag{23}$$

$$p(\tau) = \frac{p_2}{1 - \psi^{\frac{\varphi}{\varphi-1}}} \Bigg[ \psi - \psi^{\frac{\varphi}{\varphi-1}} + \\ + (1-\psi)\psi^{\frac{\varphi}{\varphi-1}} \exp\left( \frac{\psi^{\frac{\varphi}{\varphi-1}} - 1}{k p_2 (\psi^{\frac{\varphi}{\varphi-1}} - \psi)} \right) \tau \Bigg], \quad \tau \in \langle 0, \tau_1 \rangle \tag{24}$$

$$p(\tau) = p_2 \psi^{\frac{\varphi}{\varphi-1}} \exp\left( \psi^{\frac{\varphi}{\varphi-1}} K p_2 (\psi^{\frac{\varphi}{\varphi-1}} - \psi) \tau \right), \quad \tau \in \langle \tau_1, \tau_2 \rangle \tag{25}$$

Dividing by sides relations (22) and (23) one gets

$$\frac{\tau_1}{\tau_2} = \varphi = \frac{Q_{sr}^{CR}}{Q_N} \tag{26}$$

Knowing the rated compressor capacity one can determine approximately its mean actual value on the grounds of relation (26) measuring $\tau_1$ and $\tau_2$. That way one can avoid the inconvenient direct measurement of the air flow rate.

In cases where the flow rate of the air leaving the air tank is constant, independently of the pressure changes in it, equations depicting the course of the pressure changes in the air tank take the form:

$$k\frac{dp}{d\tau} = 1 - \varphi \quad \text{for } \tau \in \langle 0, \tau_1) \tag{27}$$

$$K\frac{dp}{d\tau} = -\varphi \quad \text{for } \tau \in \langle \tau_1, \tau_2 \rangle \tag{28}$$

Solution of equations (27) and (28) gives the following simple relations for $\tau_1, \tau_2$, and $\frac{\tau_1}{\tau_2}$:

$$\tau_1 = Kp_2(1-\psi)\frac{1}{1-\varphi} \tag{29}$$

$$\tau_2 = Kp_2(1-\psi)\frac{1}{\varphi(1-\varphi)} \tag{30}$$

$$\frac{\tau_1}{\tau_2} = \varphi \tag{31}$$

As it follows from the comparison of relations (22), (23) and (26) with relations (29) to (31), neglecting the impact of the pressure changes in the air tank on $Q$ causes the change of $\tau_1$ and $\tau_2$ but has no effect on the ratio of their values.

In order to estimate the error $B$ in determination of $\tau_2$ on the grounds of relation (30) instead of (23) the following relation was introduced:

$$B = \left[1 + \frac{(1+\psi)(\psi^{\frac{\varphi}{\varphi-1}} - 1)}{\varphi(\psi\psi^{\frac{\varphi}{\varphi-1}} - 1)\ln\psi}\right] 100\% \tag{32}$$

where $\lim\limits_{\varphi \to 1} B = 1 + \frac{1-\psi}{\psi \ln \psi}$; $\lim\limits_{\varphi \to 0} B = 0$.

The results of calculations are presented in Fig. 5. These errors for $\psi \geq \psi_0$ are relatively small and decrease quickly with $\varphi$ drop.

The frequency of switching the compressor into idling $v$ has an important practical significance. This frequency is an adverse value of the full pressure change period in the air tank and can be determined, after some transformation, from relations (23) or (30). Using for that purpose the simpler relation (30) we can get

$$\frac{1}{\tau_2} = v = \frac{\varphi(1-\varphi)}{Kp_2(1-\psi)} \tag{33}$$

After determining the zero of the first order derivative of relation (33) with respect to $\varphi$ one gets a value $\varphi^* = 0.5$, which is in conformity with [3]. It is a value

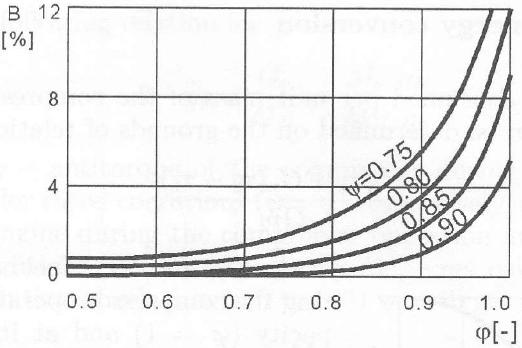


Fig. 5. The course of determining the error of the time of the full pressure change in the air tank.

$\varphi$ to which the biggest frequency value of switching the compressor into idling corresponds. This value simultaneously corresponds to the highest frequency of pressure pulsating in the air tank.

The value of $\varphi^*$ can be also determined by solving the following equation:

$$(\varphi - 1)(1 - psi^{\frac{\varphi}{\varphi-1}})(\psi - \psi^{\frac{\varphi}{\varphi-1}}) - \psi^{\frac{\varphi}{\varphi-1}}(1-\psi)\ln\psi = 0 \qquad (34)$$

The left side of equation (34) constitutes the first order derivative of relation $v = v(\varphi, \psi)$ with respect to $\varphi$ determined on the grounds of relation (23).

The course of $\varphi^*$ values obtained by a numerical solution of relation (34) is presented in Fig. 6. As it follows from this figure, values of $\psi^*$ vary insignificantly and for the values $\psi$ met in practice one can assume $\psi^* = 0.5$. Taking this into consideration and neglecting small errors in determination of $\tau_2$ on the grounds of relation (30) instead of (23) one can accept that relation (33) gives sufficient accuracy of determining the frequency $v$, and especially its maximum value. As it follows up, from Fig. 5 for $\varphi > 0.9$, the errors in determining $v$ on the grounds of relation (33), instead of on the grounds of the relation derived from formula (23), may have some significance for the values $\psi$ met in practice. From the point of view of selecting the design and control parameters of the compressor unit, the value of the maximum frequency of switching the compressor into idling has an essential significance. Taking, therefore, into consideration in relation (33), the fact that $v_{max}$ occurs at $\varphi = 0.5$ and transforming somewhat this relation we get

$$\frac{V \cdot \Delta p \cdot v_{max}}{Q_N} = 4RT \qquad (35)$$

Relation (35) has a universal meaning and may be applied to the selection of the compressor unit parameters with any two-stage step capacity control system, including on/off control systems with respect to electric motor drive.

## 4. Efectiveness of energy conversion

The amount of fuel consumed per unit mass of the compressed air, for any compressor capacity, can be determined on the grounds of relation

$$A = \frac{G_N \tau_1 + G_r(\tau_2 - \tau_1)}{Q \tau_2} \tag{36}$$

where $G_N$ and $Gr$ – fuel consumption during the compressor operation at rated capacity ($\varphi = 1$) and at its idling $\varphi = 0$ respectively, $A$ – specific fuel consumption of the compressor unit.

After transformation of relation (36) and taking into consideration relation (31) one gets

$$A = A_N \left(1 + g_r \frac{1-\varphi}{\varphi}\right); \quad A_N = \frac{G_N}{Q_N}, \quad g_r = \frac{G_r}{G_N} \tag{37}$$

where: $A_N$ – specific fuel consumption of the compressor unit under rated conditions ($\varphi = 1$), $g_r$ – relative fuel consumption of the compressor when the compressor is operated at its idling ($\varphi = 0$).

Dividing the relation (37) by $A_N$ we will get the relation for the relative specific fuel consumption of the compressor unit

$$\lambda = \frac{A}{A_N} = 1 + g_r \frac{1-\varphi}{\varphi} \tag{38}$$

whereas

$$\alpha = \frac{1}{\lambda} \tag{39}$$

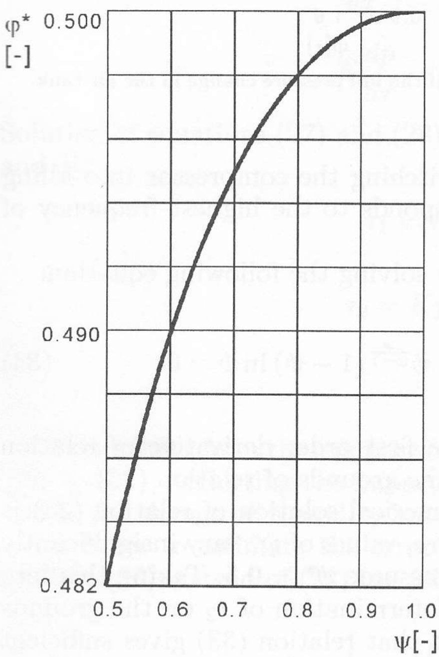


Fig. 6. The course of the compressor unit load coefficient $\varphi^*$ corresponding to maximum value of frequency of switching off the compressor into idling.

It is proposed to name $\alpha$ as the energy effectiveness coefficient of the compressor unit.

As it follows from relation (38), in order to determine $\lambda$ and $\alpha$ for any compressor capacity, it is only necessary to have two values: $G_r$ and $G_N$. Expression (38) can be presented also in a more general form. Making use of the relations on fuel consumption known in the field of internal combustion engines namely:

$$G_r = \frac{M_r \cdot \omega_N}{W_u \cdot \eta_r} \tag{40}$$

$$G_N = \frac{M_N \cdot \omega_N}{W_u \cdot \eta_N}, \tag{41}$$

we will get the following relation for $g_r$

$$g_r = \frac{G_r}{G_N} = \frac{M_r}{M_N}\frac{\eta_N}{\eta_r} \tag{42}$$

where: $M_r$, $M_N$ – antitorque of the compressor during operation at its idling ($\varphi = 0$) and under rated conditions ($\varphi = 1$) respectively, $\eta_r, \eta_N$ general efficiency of the driving engine during the compressor operation at its idling ($\varphi = 0$) and under rated conditions ($\varphi = 1$) respectively, $W_u$ – net calorific value.

After substitution of relation (42) to (38) we will get

$$\lambda = 1 + \frac{M_r \eta_N}{M_N \eta_r}\frac{1-\varphi}{\varphi} \tag{43}$$

Relation (43) is valid either for the compressor units driven by an internal combustion engine or by an electric motor and can be used to compare the energy effectiveness of the compressor units with stepwise capacity control systems. The dimensionless form of the parameters in this relation simplifies the comparative analysis. It follows also from relation (43) that the change of the load coefficient $\varphi$ from 0 to 1 causes $\lambda$ to change from 1 to infinity.

In the compressor units in which the rotational speed variations are significant when switching the compressor into idling, the ratio of the torques in relation (43) should be substituted by the ratio of the respective powers.

## 5. Experimental verification

In order to verify the theoretical relations derived above, tests of the compressor unit WD-53, with the standard equipment, were carried out. The compressor capacity, fuel consumption and temperature in the air tank were measured in the course of these tests. The results of the air temperature and the relative fuel consumption are presented in Fig. 7 and 8.

In Fig. 8 $G_\varphi$ denotes the mass fuel flow rate consumed by the engine during the compressor unit operation at load coefficient $\varphi$. Making use of the curve $g_\varphi = \frac{G_\varphi}{G_N} = f(\varphi)$, the relative specific fuel consumption $g_r$ of the compressor unit operating at its idling was calculated on the grounds of the relation below

$$\left.\frac{G_\varphi}{G_N}\right|_{\varphi=0} = g_r \tag{44}$$

and on the grounds of relation (38) values of $\lambda$ and $\alpha$ in the function of the load coefficient $\varphi$ were calculated and presented in Fig. 9. In the same figure there are also shown $\lambda$ plots obtained from direct measurements in the course of the compressor unit operation. As it follows from this figure the results of calculations show good conformity with the results obtained from direct measurements, for the whole range of the compressor unit capacity change.

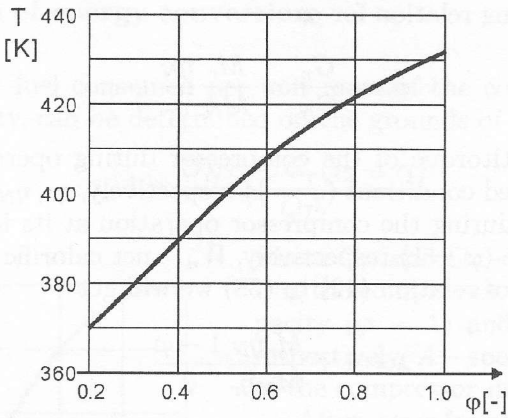


Fig. 7. Air temperature in the air tank in the function of the compressor unit load coefficient $\varphi$.

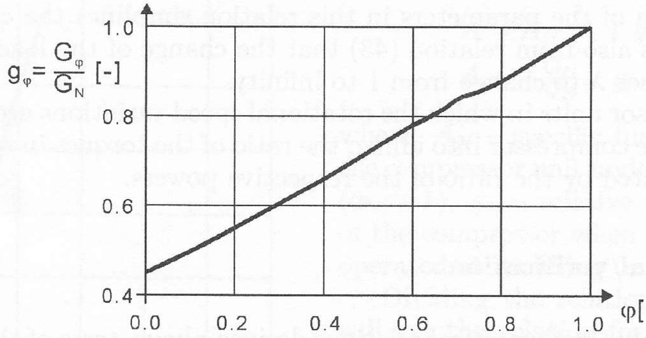


Fig. 8. Dependence of the relative fuel consumption in the function of the load coefficient $\varphi$.

On the grounds of relation (33) the frequency of switching the compressor into idling in the compressor unit WD-53 was calculated in the function of the load coefficient $\varphi$ taking also into account the air temperature changes in the air tank. The results of calculations are presented in Fig. 10. As it follows from this figure, the maximum value of $v$ occurs for $\varphi = 0.53$.

In order to illustrate the impact of the rotational speed reduction on the frequency of switching the compressor into idling in Fig. 10 the values of $v$ at the rotational speed – 800 rev./min are also shown with a broken line. Such a shift in the range of operation of the capacity control system presently used in the compressor units WD-53 is possible in the case when a capacity control system is applied by continuous change of the driving engine rotational speed in the range from 800-1500 rev per min. Then the maximum value of the frequency $v$ would be significantly reduced and the compressor capacity corresponding to $v_{max}$ shifted in the direction of the smaller values. Such a capacity control system ensures greater effectiveness of the energy conversion in the compressor unit.

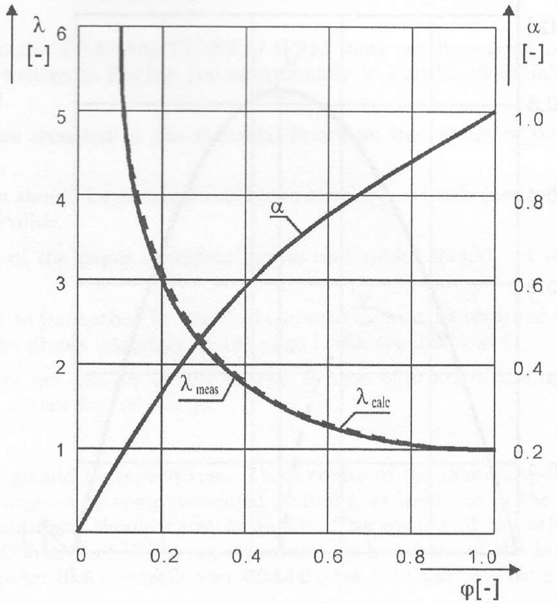


Fig. 9. Relative specific fuel consumption of the compressor unit in the function of the load coefficient $\varphi$.

## 6. Final comments and conclusions

It follows from the analysis of the compressor unit operation, that if the sub-critical pressure ratio is preserved during the outflow of the air from the air tank, the pressure change in this tank in the range met in practice influences the time of pressure increase in it, $\tau_1$, and the time of full pressure change, $\tau_2$, to a small degree only. The pressure change in the air tank, however, has no impact on the ratio of the above mentioned parameters. Also, for the values of pressure ratios $\psi$ in the air tank (minimum to maximum) most often met in practice, the mean value of the air delivered to the users for $\varphi\langle 0.8$ practically does not depend on the values of the load coefficient $\varphi$. Therefore one can use the simpler and more convenient relations for calculating $\tau_1, \tau_2$ and $v$. The relation for calculating the frequency of switching the compressor into idling is valid also for the compressor units equipped with other systems of a stepwise capacity control. The maximum value of $v$ occurs during the compressor operation with mean capacity equal to half of its rated capacity. It is worth mentioning that the air temperature change in the tank has some impact on the value $v_{max}$ causing insignificant shift of $v_{max}$ in the direction of the highest values of the mean compressor capacity. A convenient form of the relations for the relative specific fuel consumption, derived in this paper, allows to compare the energy effectiveness of the compressor units equipped with various systems of stepwise capacity control, with the exception of the capacity control system causing periodical switching off the electric motor (on/off capacity control).

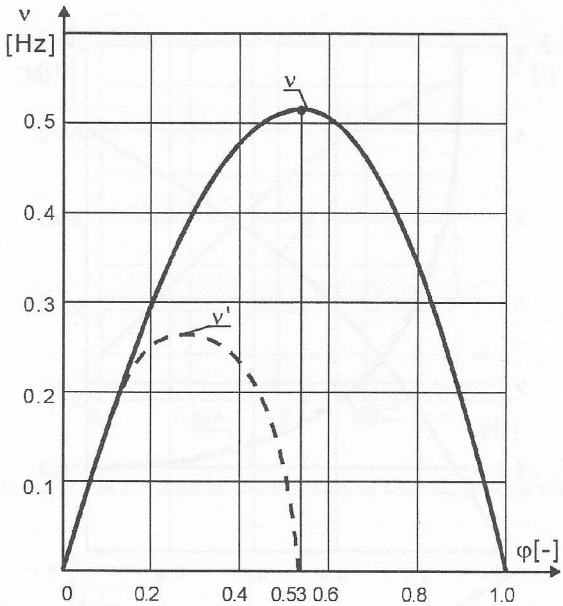


Fig. 10. The frequency of switching the compressor into idling in the function of the load coefficient $\varphi$.

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## Analiza pracy agregatu sprężarkowego ze skokową regulacją wydajności

### Streszczenie

Artukuł zawiera analizę matematyczną pracy agregatu sprężarkowego ze skokową regulacją wydajności, przy założeniu zachowania podkrytycznego stosunku ciśnień podczas poboru powietrza. Wyprowadzono w nim zależności na parametry charakteryzujące przebieg ciśnienia w zbiorniku wyrównawczym w funkcji parametrów bezwymiarowych, przy poborze powietrza zależnym od zmian tego ciśnienia a także zależność przełączania sprężarki na bieg jałowy oraz jej wartość maksymalną. Wykazano mały wpływ zmiany ciśnienia w przyjętym modelu poboru powietrza na wartości czasu narastania i pełnej zmiany ciśnienia w zbiorniku wyrównawczym oraz na średnią wartość ciśnienia powietrza pobieranego przez użytkowników. Wyprowadzono również proste bezwymiarowe zależności umożliwiające porównywanie efektywności energetycznej agregatów sprężarkowych o stałej prędkości obrotowej, wyposażonych w różne układy skokowej regulacji wydajności. Wprowadzono i zdefiniowano nowe pojęcia. Przeprowadzono weryfikację doświadczalną niektórych zależności.