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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

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CFD identification of generalised transmittances for any element of pulsating gas piping system

Pressure pulsation in positive-displacement gas compressor manifolds can significantly affect the quantity of the energy required for gas compression. One of more difficult problems with the analysis of pressure pulsations is proper description of the acoustic effect of gas manifolds. The most used analysis is based on the classical Helmholtz model, where an element of the piping system is substituted by a composition of straight segments of the pipeline of known length and diameter. Quite a large number of elements of a piping system can be described properly in this way. However in many cases this model is insufficient. An attempt to the analysis of other shapes was presented in [2-4], but only simple geometry elements were considered. The aim of the present paper is to show a new method of the computational identification of any element of the piping system, i.e. elements with complex transmittance matrix for such element using the CFD simulation package. On the basis of CFD simulation results the elements of the transmittance matrix are calculated defining in such way the installation element. The results of the developed method have been compared with the results of Helmholtz model showing better accuracy.

Nomenclature

Scalar values

b	– flow damping coefficient,	S	– cross section area,
c	– sound velocity,	$\dot{V}$	– volumetric flow rate,
D	– diameter,	w	– velocity,
l	– length,	ρ	– density,
$m, \dot{m}$	– mass flow rate,	τ	– time,
p	– pressure,	ω	– frequency.

Complex values

$j = \sqrt{-1}$	– imaginary unit,
P	– complex pressure (after FFT),
M	– complex mass flow rate (after FFT),
$\tilde{T}$	– transmittance,

Complex matrices

$\mathbf{A} = \{a_{ij}\}$	– four pole matrix,
$\mathbf{Z} = \{z_{ij}\}$	– impedance matrix,
$\mathbf{T} = \{t_{ij}\}$	– transmittance matrix,

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1. Introduction

In positive-displacement compressor manifolds there are pressure pulsations due to their cyclic operation. This refers mainly to reciprocating compressors, rotary piston and sliding piston compressors rather than to screw or scroll compressors.

The analysis of pressure pulsations in these compressors systems is important for various reasons, some of which are being presented below:

- they directly affect the quantity of energy required for medium compression due to dynamic pressure charging, or inversely, dynamic suppression of suction and discharge processes;
- they cause mechanical vibrations of compressed gas piping systems, which might sometimes lead to their damage;
- they cause aerodynamic and mechanical noise;
- they affect the dynamics of working valves in valve compressors, which leads to their dynamic leakage or early wear;
- they intensify the process of convection in piping systems;
- the flow transient state affects the piping hydraulic resistance;
- the pressure pulsation waves propagating in the piping system carry with them useless acoustic power.

The research and analysis methods of pulsating gas flow used at present can be divided into three groups:

- experimental,
- analogue,
- theoretical, based on computer simulation.

The experimental methods, practically, are used only to confirm theoretical models. The analogue methods based on electro-acoustic analogy used to be important before computers came into common use; now they are rarely employed due to their simplifying assumptions, similar to those used in numerical methods.

The theoretical methods can be divided into two groups:

- methods based on Helmholtz analysis and solution of telegraph equation in complex domain,
- one-dimension (finite difference, method of characteristics) or multi-dimension CFD methods (FEM, BEM, FDM).

An advantage of the CFD method is that the assumption of small disturbance propagation is given up. These methods are useful in vehicle exhaust silencers design. Compressor piping systems, however, are much more complex. In gas compressor stations such systems are several hundred meters long. Incorporating of the whole geometry of such piping into the CFD program would be burdensome even if the computer calculation time, unrealistically long now, were to be shortened in the future. This is why the pure CFD method for vast piping systems seems non-applicable.

The Helmholtz method, which is the basis for commercial companies dealing with pressure pulsation damping (ex. PULSCO), contains many simplifying assumptions. This is because each element of the piping system is substituted by a straight segment of known diameter and length. In many cases, however, this model is insufficient. An attempt of the analysis of other shapes was presented in [2-3], but only simple elements (circle, pipe, cylinder) were considered. The aim of the present paper is to show a new method of identification of any element of the piping system, i.e. elements with complex transmittance matrix for such element by means of CFD simulation.

2. Generalised Helmholtz model

The classical Helmholtz model is based on the solution of wave equation for a straight piping section (2.1). Those equations have several assumptions: the amplitude of the pulsation is small compared to the average value, gas is ideal, only wave motion is considered (no mean flow). The result is a four-pole matrix, as shown in (2.2) and (2.3). Elements of this matrix $\{a_{ij}\}$ are calculated for a straight pipe segment. The complex impedance $\mathbf{Z}$ with elements $\{z_{ij}\}$, defined by (2.3) is used alongside the four-pole matrix. It may be easily transformed both ways **A-Z** and **Z-A**. This approach is limited to rather uncomplicated cases.

$$\begin{cases} -\dfrac{\partial p}{\partial x} = \dfrac{1}{S}\dfrac{\partial \dot{m}}{\partial \tau} + \dfrac{b}{S}\cdot \dot{m} \\ -\dfrac{\partial \dot{m}}{\partial x} = \dfrac{S}{c^2}\dfrac{\partial p}{\partial \tau} \end{cases} \tag{2.1}$$

$$\begin{bmatrix} P_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} ch\gamma L & Z_f sh\gamma L \\ \dfrac{1}{Z_f} sh\gamma L & ch\gamma L \end{bmatrix} \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \tag{2.2}$$

$$\begin{bmatrix} P_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \tag{2.3}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \tag{2.4}$$

where

$$\left. \begin{array}{c} \gamma = \sqrt{(b + j\omega)(\frac{j\omega}{c^2})} \\ Z_f = \frac{1}{S}\sqrt{\frac{(b+j\omega)}{\frac{j\omega}{c^2}}} \end{array} \right\} . \tag{2.5}$$

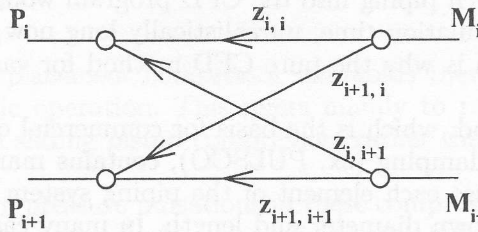

Fig. 1. Graph of the **Z** matrix elements.

The aim of the present paper was to work out a method of identifying the elements of those matrixes. For further investigation it is necessary to define complex transmittance functions. The transmittances are defined below:

$$
\begin{aligned}
&\bullet \text{ pressure transmittance} && \bar{T}_P(i\omega) = \frac{P_{i+1}}{P_i} \\
&\bullet \text{ flow transmittance} && \bar{T}_M(i\omega) = \frac{M_{i+1}}{M_i} \\
&\bullet \text{ pressure-flow transmittance} && \bar{T}_{PM}(i\omega) = \frac{-M_{i+1}}{P_i} \\
&\bullet \text{ flow-pressure transmittance} && \bar{T}_{MP}(i\omega) = \frac{P_{i+1}}{M_i}
\end{aligned}
\tag{2.6}
$$

An interrelation between the **Z** matrix elements and complex transmittance functions can be introduced as (2.7). This system contains two equations with four unknowns with no unique solution.

$$
\left.
\begin{aligned}
1 &= z_{ii} \cdot \bar{T}_{PM} \cdot \frac{1}{\bar{T}_M} - z_{i,i+1} \cdot \bar{T}_{PM} \\
\bar{T}_{MP} &= z_{i+1,i} - z_{i+1,i+1} \cdot \bar{T}_M
\end{aligned}
\right\}
\tag{2.7}
$$

Additional dependencies to solve it can be obtained only when complex impedances Z_0 and Z_k are used (2.8).

$$
Z_{k,0} = \frac{P_i}{M_i},
\tag{2.8}
$$

where 0 – means the pipe beginning, and k – the end.

This means that the manifold transmittance values defined in (2.6) are valid only for an element in a specified place in the defined piping system.

If, however, the transmittance for known values of Z_0 and Z_k could be defined, the interrelation between **Z**, **T** and **A** matrices could be derived. This means that transmittances T could be used to calculate matrices **Z** and **A**. The simplest way to derive them is when $Z_k = 0$ or $Z_0 = \infty$, in case of reverse flow for $Z_0 = 0$ or $Z_k = \infty$. For simulation, see cases shown in Fig. 2.

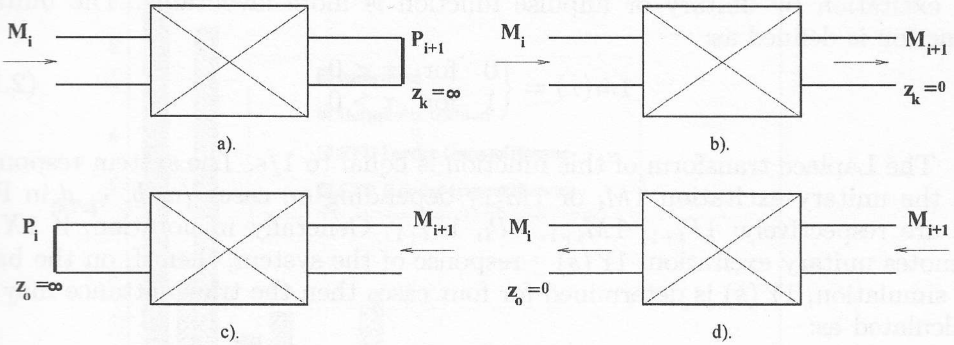

Fig. 2. Four flow cases used for CFD simulation.

The transmittance values described above can be defined by means of CFD simulation because in all cases the flow excitation is determined at the boundary condition. The other four transmittance functions (two straight ones and two reverse) would need pressure inlet conditions, which in case of CFD simulation, with no flow excitation, often gives ambiguous results.

It may be derived that the transmittance values calculated in this way explicitly define the elements of matrices **A** and **Z**. Writing down the impedance dependencies for a, b, c, d cases as shown in Fig. 2 we get:

a)

$$\left.\begin{aligned} P_i &= z_{ii} \cdot M_i \\ P_{i+1} &= z_{i+1,i} \cdot M_i \end{aligned}\right\} \Rightarrow z_{i+1,i} = \bar{T}_a \tag{2.10a}$$

b)

$$\left.\begin{aligned} P_i &= z_{ii} \cdot M_i + z_{i+1,i} \cdot M_{i+1} \\ 0 &= z_{i+1,i} \cdot M_i + z_{i+1,i+1} \cdot M_{i+1} \end{aligned}\right\} \Rightarrow z_{i+1,i+1} = \frac{\bar{T}_a}{\bar{T}_b} \tag{2.10b}$$

c)

$$\left.\begin{aligned} P_i &= -z_{i,i+1} \cdot M_{i+1} \\ P_{i+1} &= -z_{i+1,i} \cdot M_{i+1} \end{aligned}\right\} \Rightarrow z_{i,i+1} = \bar{T}_c \tag{2.10c}$$

d)

$$\left.\begin{aligned} 0 &= -z_{ii} \cdot M_i - z_{i,i+1} \cdot M_{i+1} \\ P_{i+1} &= -z_{i+1,i} \cdot M_i - z_{i+1,i+1} \cdot M_{i+1} \end{aligned}\right\} \Rightarrow z_{i,i} = \frac{\bar{T}_c}{\bar{T}_d} \tag{2.10d}$$

The derived dependencies (2.10) determine uniquely the complex impedance matrix **Z**, which means that the lumped element of the manifold is identified. It was shown that four CFD simulations are necessary, see Fig. 2 *a*, *b*, *c*, *d*. The figures clearly point out the way of defining the closing boundary condition by means of either constant pressure or rigid wall.

The simulation for each transmittance of the four cases and several harmonics is laborious and time consuming, therefore the application of the system response to excitation by unitary or impulse function is more favourable. The unitary function is defined as:

$$1\dot{m}(\tau) = \begin{cases} 0 & \text{for} \quad \tau < 0 \\ 1 & \text{for} \quad \tau > 0 \end{cases} \tag{2.11}$$

The Laplace transform of this function is equal to $1/s$. The system responses to the unitary excitation $1M_i$ or $1M_{i+1}$ depending on cases *(a, b, c, d* in Fig. 2) are respectively: $1P_{i+1}$, $1M_{i+1}$, $1P_i$, $1P_{i+1}$. Generally in notation, if $1X(s)$ denotes unitary excitation, $1Y(s)$ – response of the system, then if, on the basis of simulation, $1Y(s)$ is determined for four cases then the transmittance may be calculated as:

$$\bar{T}(s) = s \cdot 1Y(s). \tag{2.12}$$

This means that for a computed unitary response of the system in each of the four cases *a, b, c, d* the characteristics of the given object can be determined.

A similar analysis can be done using impulse delta function as a flow excitation.

Analytically, the transmittance values can be determined for a section of pipeline of constant cross-section S and length L [1].

3. Application of CFD identification for a pulsation damper of special design

In order to compare the results of the method worked out against the experimental results I have decided to use this method for the analysis of pressure pulsation damper, shown in Fig. 3. For the muffler shown in the Fig. 3, the conventional Helmholtz method was used to calculate its four-pole matrix. Then a CFD model was set up.

A model was made in a cylindrical co-ordinate system, using axial symmetry, which reduced the case to the two-dimensional one. The geometrical model was then implemented in the PHOENICS code on PC. The space grid was divided into 39 parts unequal due to geometry along the radius and 86 parts along the symmetry axis. The analysis was done for an unsteady state flow, at both unitary and impulse function excitation with the velocity amplitude 10 [m/s]. It was decided to use the time range 0÷2[s], divided unequally (more densely at the beginning) into 131 parts.

In order to transfer the CFD calculation results into generalised transmittance functions model the functions of pressure and mass flow rate at inlet and outlet cross-sections were averaged. Examples are shown in Figs. 4 and 5, for the case of flow-pressure transmittance (Fig. 2a).

On the basis of plots (Figs. 4, 5) for response for unitary and impulse excitations the graphic parameters were identified for transmittance of the first and second order, in a general form shown below:

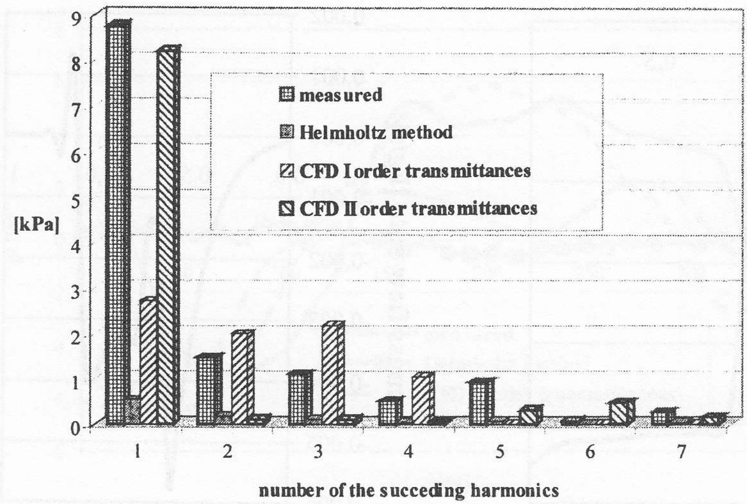

Fig. 3. The gas pulsation muffler of special design used for the method verification.

a) first order transmittance (only damping and time lag):

$$\mathbf{T}(s) = \frac{K}{1 + s \cdot \zeta} \cdot e^{-s\Delta\tau} \tag{3.1}$$

b) second order transmittance:

$$\mathbf{T}(s) = \frac{K \cdot \omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \cdot e^{-s\Delta\tau} \tag{3.2}$$

Values of coefficients for transmittance functions (Fig. 2) are given in Table 1. The experimental verification of the presented identification method was based on the results of measurements of the pressure pulsation before and after the muffler mounted in the compressor manifold. The pressure pulsation registered after the muffler was used as a source, and recalculated in a special way (detailed method is shown in [1]) to obtain moving forward and backward pressure wave. Then it was possible, after some matrix transformations, to calculate pressure before the muffler on the basis of three methods: the Helmholtz model based conventional method, CFD based Ist order transmittances and CFD based IInd order transmittances.

The comparison of pressure pulsation at the measurement point before the muffler, both measured and obtained using transmittances of the first and second order as well as by the Helmholtz method have been shown in Fig. 6.

Significant harmonics calculated from the pressure pulsation run (Fig. 6) have been compared in the Fig. 7.

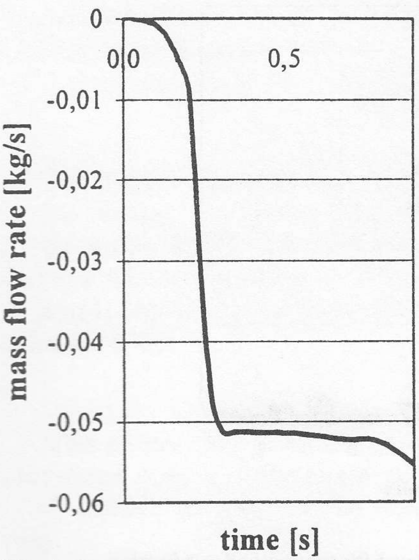

Fig. 4. Response for unitary excitation.

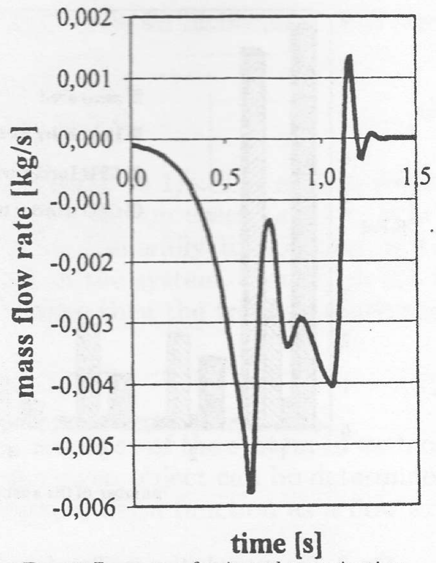

Fig. 5. Response for impulse excitation.

Table 1. Values of coefficients in the 3.1 and 3.2 formulas

Transmittance	K	ω_0	ζ	$\Delta\tau$
T_a	28154	22	0.053	0.187
T_b	−0.8	35	0.095	0.186
T_c	46154	22	0.060	0.190
T_d	−1.12	35	0.130	0.185

The results of calculations have been summed up in Table 2. The agreement with the experiment is definitely improved when using CFD methods, and it is better than in the case of the classical Helmholtz model.

To sum it up, it should be stated that the obtained results of identification using CFD modelling are encouraging. Even a simple model of the first order gives a better solution when compared with the classical Helmholtz model.

4. Conclusions

A direct application of CFD methods in modelling of gas compressor station is useless, not only because of vast computation time, but first of all because of

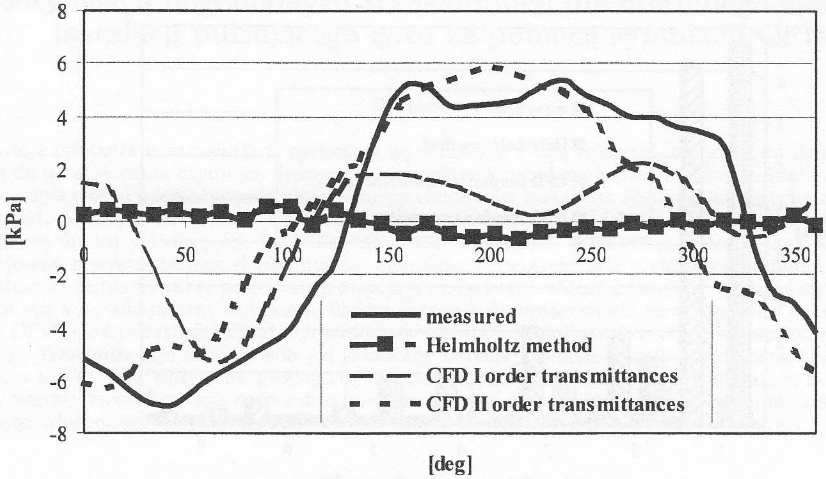

Fig. 6. The comparison of the pressure pulsation: measured and calculated using three methods.

Table 2. Comparison of the peak to peak pressure pulsation amplitudes

[rev/min]	experiment Δp [kPa]	Helmholtz method		CFD simulation I order		CFD simulation II order	
		Δp [kPa]	difference [%]	Δp [kPa]	differenc e[%]	Δp [kPa]	difference [%]
1300	12.5	1.2	90%	7.5	40%	12.2	2.5%

time and labour consumption when feeding the data into computer. That is why it is more convenient to use the presented method to identify the parameters, i.e. complex transmittances. In this method, with the assumption of linearity, the Laplace transform gives complex transmittances of any element of the gas network for identified elements only as a response to impulse or step function excitation using CFD simulation. So the whole network then can be easily modelled using matrices for each element and mathematically assembling them together.

The conclusions from the application of this method are:

- the object identification based on CFD simulation, in the case of strong damping gives better results than the classical Helmholtz model in the aspect of pressure pulsation amplitude, even using transmittance of the first order containing only damping and time lag. However, it should be expected that for simpler geometries (pipe, tank, oil separator), the Helmholtz

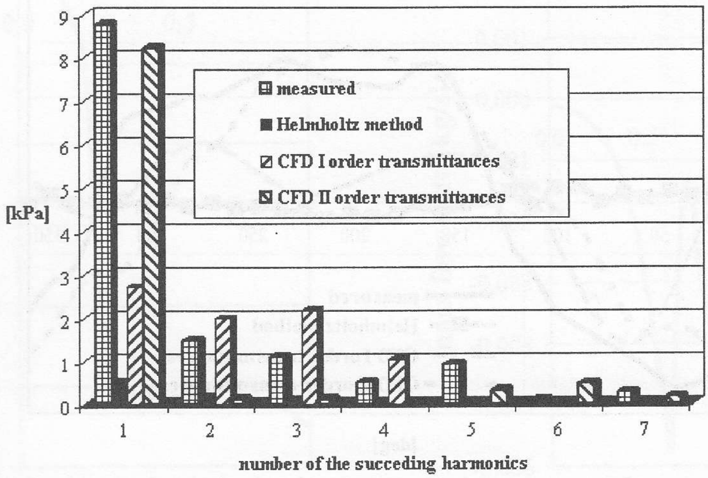

Fig. 7. The comparison of the pressure pulsation harmonics.

model will be still more advantageous than the first order transmittances;

- simulation results are significantly improved when the second order transmittance of four parameters is adopted for the installation element identification. Then the presented out method gives much better agreement with the experiment than the Helmholtz model.

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Identyfikacja uogólnionych transmitancji dla dowolnego elementu instalacji pulsującego gazu za pomocą symulacji CFD

Streszczenie

Pulsacje ciśnienia w instalacjach sprężarek wyporowych mogą znacząco wpływać na ilość energii potrzebną do przetłaczania czynnika. Jednym z trudniejszych problemów przy analizie pulsacji ciśnienia jest właściwy opis efektu oddziaływania akustycznego elementów instalacji. Najczęściej dotychczas stosowaną analizą jest, bazujące na modelu Helmholtza, zastępowanie wszystkich elementów prostymi odcinkami o zastępczej średnicy i długości, które są dobierane arbitralnie. Wiele elementów można w ten sposób zamodelować z wystarczającą dokładnością. Jednakże w przypadkach o złożonej geometrii (np. odolejacze, tłumiki pulsacji) takie podejście prowadzi do znacznych różnic pomiędzy wartościami przebiegów zmierzonych a wynikającymi z obliczeń. Celem autora było opracowanie nowej metody bazującej na symulacji CFD (Computational Fluid Dynamics) określania transmitancji zespolonych dowolnego elementu instalacji. Transmitancje te w sposób jednoznaczny określają oddziaływanie akustyczne elementu w instalacji, a zatem jego wpływ na propagację fali pulsacji ciśnienia. Rezultaty wynikające z opracowanej metody zostały zweryfikowane doświadczalnie i porównane z wynikami uzyskiwanymi metodą klasyczną, wykazując zdecydowanie lepszą zgodność.