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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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MARIAN TRELA

Gdańsk

Experimental Investigations of the Pressure Drop of Freon 21 During Flow Boiling in Vertical Pipes*

A facility is described, designed for investigation and measurement of the freon 21 boiling pressure drop in vertical pipes. The measurements enabled a comparison between experimental data on the two-phase friction factor and its values calculated after formulas presented in [2]. A good coincidence of the calculated and experimental values has been stated, proving the soundness of the formulas derived in [2].

Symbols

A – channel cross-section area,
C_Φ, C_φ – pressure dependent factors,
d – pipe diameter,
g – acceleration of gravity,
h, L – length,
i – enthalpy,
k, m – pressure dependent factors,
$\dot{m}$ – flow rate,
$\dot{M}$ – mass velocity ($\dot{M}=\dot{m}/A$),
p – pressure,
r – factor, heat of evaporation,
R – two-phase friction factor $R=\dfrac{(dp/dL)_{TPF}}{(dp/dL)_{l_0}}$,
t – temperature,
v – specific volume,
μ – dynamic viscosity,
ρ – density,
φ – void fraction ($\varphi=A_g/A$),
x – quality ($x=\dot{m}_g/\dot{m}_g+\dot{m}_l$),
χ_{tt} – Martinelli parameter $[\chi_{tt}=(\rho_g/\rho_l)^{0,555}\times(\mu_l/\mu_g)^{0,111}(1-x/x)]$,
ζ – single-phase friction factor.

Subscripts refer to:

a – pressure drop due to change of momentum,
F – freon,
g – gas (vapour)
h – hydrostatic pressure drop,
i – i-th measurement section,
l – liquid,
m – mean value,
r – pipe, mercury,
TPF – pressure drop resulting from friction,
u – readings on mercury differential manometer.

1. Introduction

The new concept of the thermal power plant cycle, namely the binary cycle [1], with a vapour of low-boiling fluid in the low-temperature part of the cycle, yields a number of new problems.

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* Praca wykonana w ramach problemu resortowego PAN-19, grupa tematyczna 3.

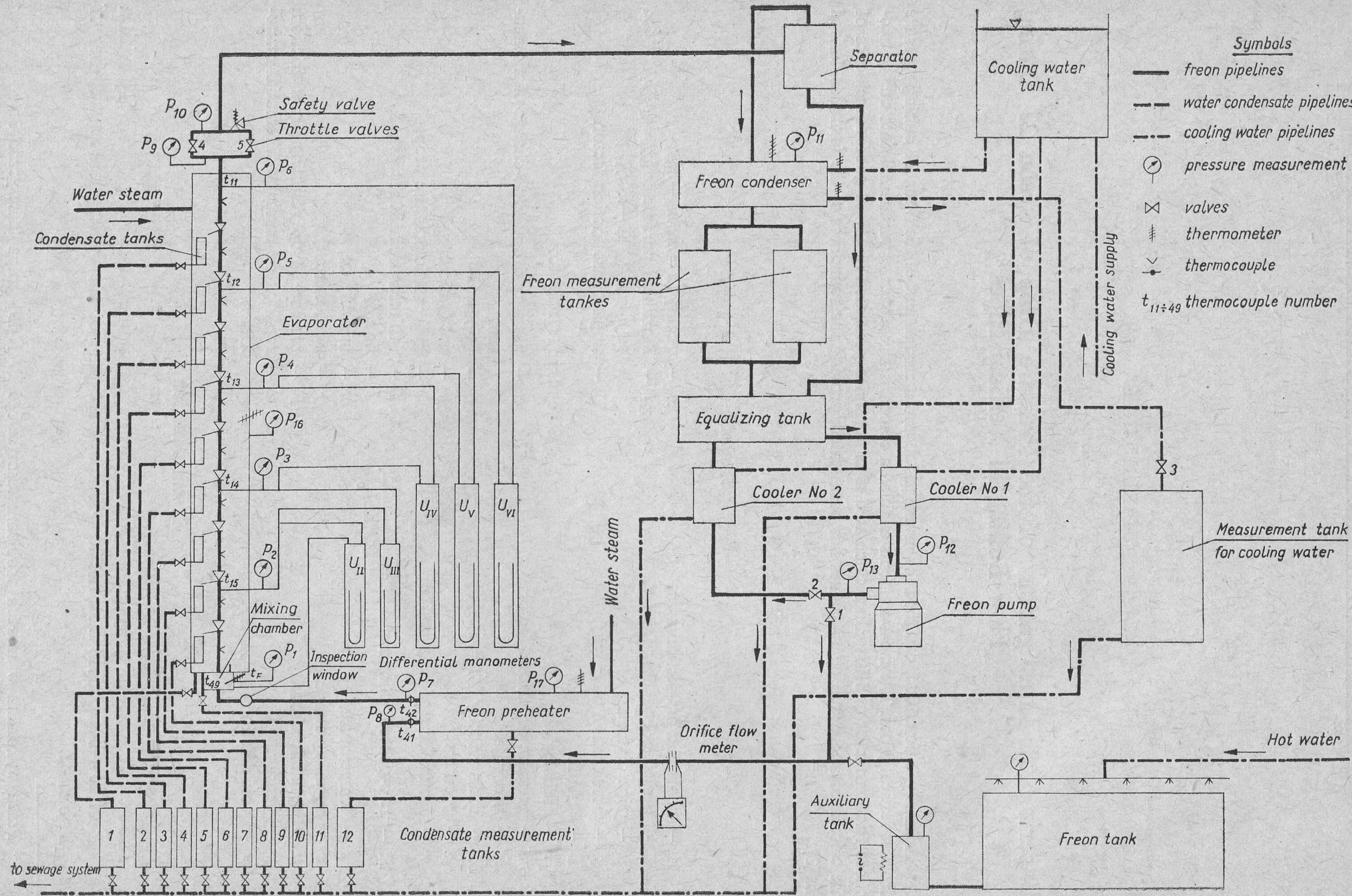

Fig. 1. Schematic diagram of the experimental stand

One of them is the method of calculating the boiling pressure drop for the low-boiling fluid flowing in vertical pipes. Its solution has been put forward in [2]. This paper is aimed at comparing the calculated values of the pressure drop after the method worked out in [2], with those measured in experimental set-up. Freon 21 was used as a working medium. The pressure drop was measured in conditions of bulk boiling only, because the method given in [2] is valid for that type of boiling exclusively.

2. Experimental stand

The experimental stand has been build in the laboratory of the Thermodynamics and Heat Exchange Department of the Institute of Fluid-Flow Machines of the Polish Academy of Sciences in Gdańsk. A schematic diagram of the stand is presented in Fig. 1. The freon 21 circulates in a closed circuit comprising a preheater, evaporator, separator, condenser,

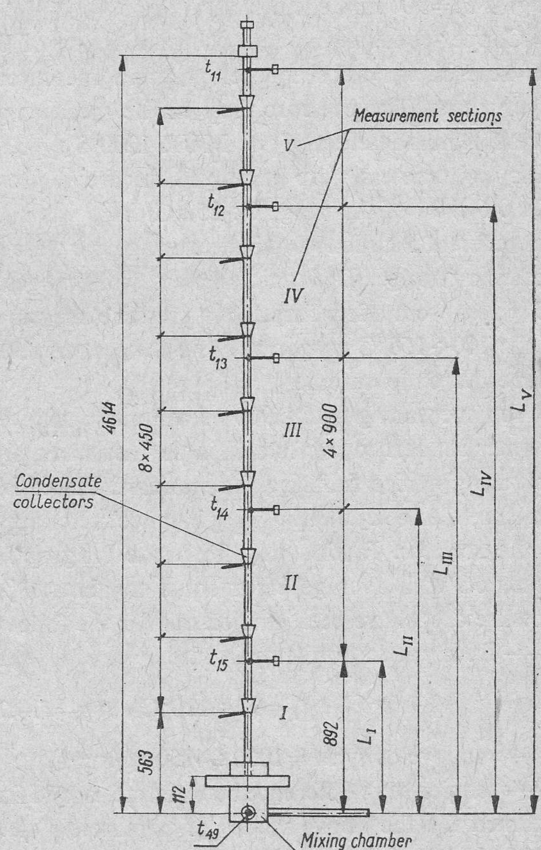

Fig. 2. Measurement pipe

measuring tanks, plenum chamber, coolers and a pump. The main and auxiliary freon tanks are additionally used when filling and draining the system. Freon is heated in the preheater and afterwards it flows through a mixing chamber to the evaporator where partial evaporation takes place. After reducing its pressure outside the evaporator, freon flows to the separator where liquid and vapour phases are separated. The liquid flows to the plenum chamber, whereas the vapour goes to the condenser. The condensed freon flows to the plenum chamber through the measuring tanks. Freon is supplied from the plenum chamber to the pump through the cooler *1*. A part of it comes back from the pump to the plenum chamber through the cooler *2*, whereas the rest goes through a measuring orifice to the heater.

Freon is heated and evaporated by the heat taken from the condensing steam. The steam is supplied in the form of a dry saturated vapour, prepared with the use of a special reducing–cooling station. The evaporator consists of two pipes; steam flows between their walls. The inner pipe forms a measuring element (Fig. 2). Its dimensions are: length $L=4614$ mm and inner diameter $d=11.6$ mm. Mushroom-shaped collectors spaced on the pipe at 450 mm intervals collect the condensed steam which is drained off to the measuring tanks. The measuring pipe shown in Fig. 2 comprises five sections for pressure drop and temperature measurements. The first section is of length $h_r=892$ mm, the other ones — of lengths $h_r=900$ mm each.

Copper-constantan thermocouples made of 0.5 mm diameter wire were used during the experiments for freon temperature measurements. They were calibrated before the experiments. The pressure drop in the measuring pipe was measured with the use of differential manometers of the MUR 1200 S type, with mercury as the manometric liquid. Disk manometers were employed for freon pressure measurements. A disk manometer of a 0.6 class of the range $0\div25$ kG/cm², was installed at the measuring pipe inlet. Other manometers installed on the measuring pipe were of a 0.4 accuracy class and $0\div40$ kG/cm² measuring range. Disk manometers with ranges $0\div4$ kG/cm² and $0\div6$ kG/cm², of the 0.5 class, were used for steam pressure measurements, whereas mercury thermometers of the $100\div150$°C range, with the scale interval equal to 0.2 deg centigrade were used for temperature measurements.

Ten measuring tanks, 900 ml volume each, were employed for measuring the amount of water condensed from steam in the measuring pipe, with accuracy up to 0,5%. A quadrant measuring orifice fulfilling the standard PN-65/M-53950 was used for freon flow measurements, with a „Junkalor" ring balance meter applied for measuring the pressure difference in the reducer. In this way flow rate ranging from $\dot{m}=400$ kg/h to $\dot{m}=1700$ kg/h was measured with average error equal to about 1%. Experiments on the freon 21 boiling pressure drop have been carried out for the following parameter ranges:

— freon saturation temperature

$$t_F=90\div120°\mathrm{C} \qquad (p_F=10.74\div19.67\ \mathrm{bar}),$$

— mass velocity $\dot{M}=1000\div4150$ kg/m²·s,

— temperature difference between steam and freon $\Delta t_{FW}=5\div37$°C. The heat flux density obtained for the above values of parameters t_F, $\dot{M}$ and Δt_{FW} was equal to $q=11\,000\div$ $\div115\,000$ W/m². 114 main measurements i.e. measurements of the two-phase flow pressure drop, and 28 auxiliary measurements aimed at determining the measuring pipe roughness have been made during the experiments. The auxiliary measurements were carried out for the flow of liquid freon at temperature equal to 20°C. The measured values of the pressure drop together with flow rate value and physical parameters of the medium under conside-

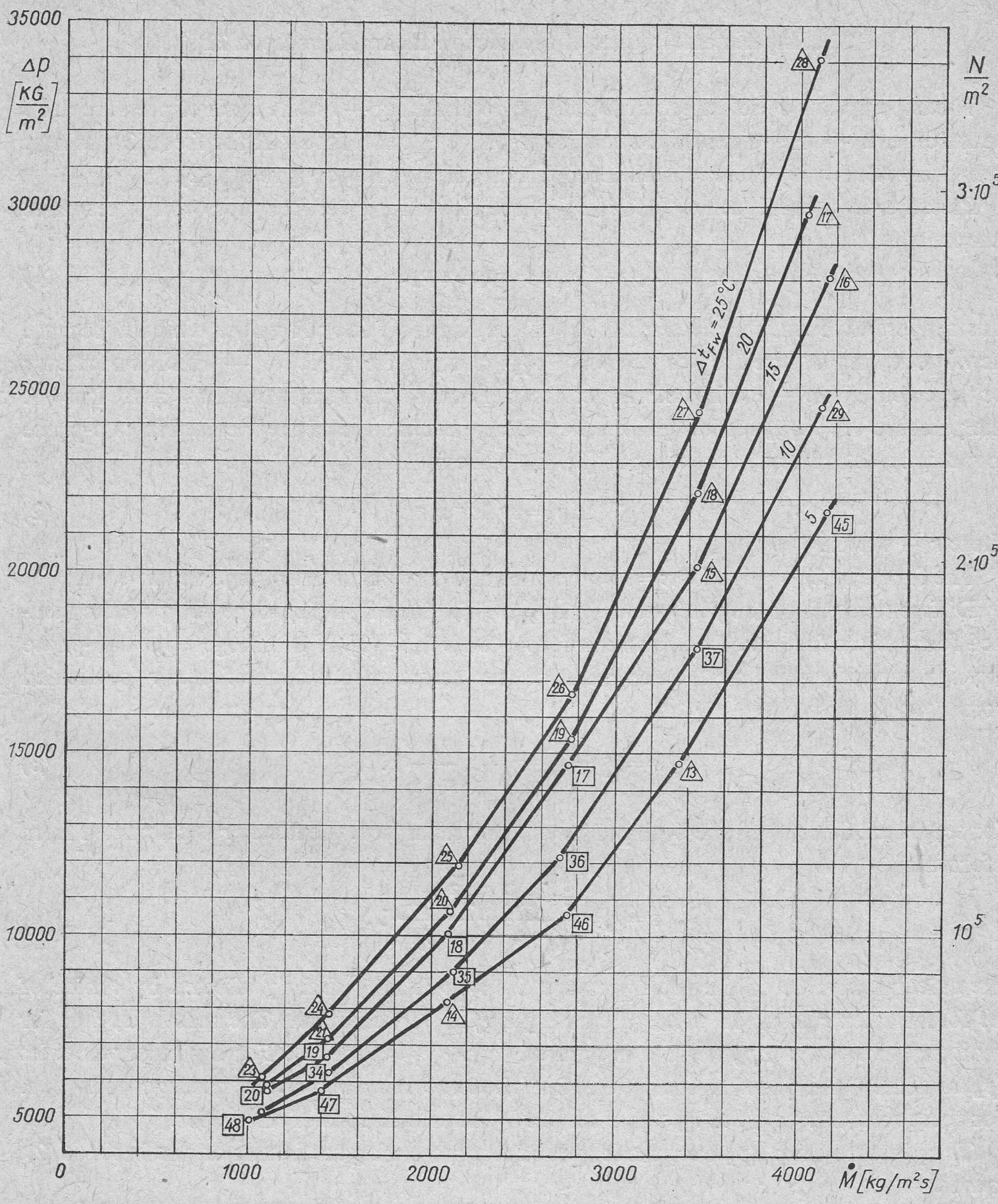

Fig. 3. Some results of measurements of pressure drop along the measurement pipe versus mass velociyt

ration served as the basis for calculating, by means of the Weisbach's formula (17), the friction factor ζ. Afterwards a relative roughness k/d was determined from the calculated Reynolds number and well known relation $\zeta = f(Re, k/d)$ for rough pipes. The absolute roughness, i.e. the height of irregularities obtained therefrom was equal to $k \cong 0.06$ mm. Some results of pressure drop measurements obtained during the experiments are shown in Fig. 3. The measurements were made for the saturation temperature of the freon at the measuring pipe inlet equal to $t_F = 110°C$.

3. Experimental values of the friction factor R for freon 21

The experiments were aimed principally at determining — on the basis of pressure drop measurements — the values of the local two-phase friction factor R for freon 21, and at comparing them with values calculated by means of the relation given in [2].

Pressure drop measurements for two-phase flow along vertical pipes comprises three components

$$\Delta p = \Delta p_{\mathrm{TPF}} + \Delta p_a + \Delta p_h . \tag{1}$$

This renders experimental determination of the friction factor R values difficult, because the value of the pressure drop component Δp_{TPF} resulting from friction — which is necessary for the calculations — can be determined from (1) only if the terms Δp_a and Δp_h are known. The pressure drop Δp_a resulting from the momentum change of the medium in two-phase flow can be determined, according to [3], from the formula

$$\Delta p_a = r v_l \dot{M}^2 , \tag{2}$$

where r – factor.

The formula (2) is valid for $x=0$ at the inlet of a channel with boiling medium.

Different values of the pressure drop Δp_a are obtained from (2), depending on the assumed form of the factor r. Two formulas for r are given in [3]. If a homogeneous two-phase flow is assumed, with liquid and vapour phases perfectly mixed, the factor r is calculed from

$$r = r_H = (1-x) + x \frac{v_g}{v_l} - 1 . \tag{3}$$

For separated flows, with the both phases entirely separated, the factor r is calculated from a formula

$$r = r_s = \frac{(1-x)^2}{1-\varphi} + \frac{x^2}{\varphi} \frac{v_g}{v_l} - 1 . \tag{4}$$

The calculated value of the total pressure drop Δp is only slightly affected by choice of the formula for r assumed when calculating Δp_a in vertical pipes. The reason is, that for small mass velocities Δp_a is small compared with Δp_{TPF} and Δp_h. For big mass velocities Δp_a increases and simultaneously the flow becomes similar to the homogeneous one. For these reasons r equal to r_H was assumed in the calculations of this paper.

The pressure drop Δp_a over the length of any measuring section is, on the basis of (2), equal to

$$\Delta p_{ai} = (r_i - r_{i-1}) v_l \dot{M}^2 , \tag{5}$$

where $i=$I, II, III, IV, V – measuring sections. The above formula is valid for $x>0$ at the inlet of the channel with boiling medium.

The hydrostatic pressure drop for two-phase flow is given by a formula

$$\Delta p_h = \int_0^L [\rho_g \varphi + \rho_l (1-\varphi) g dL] . \tag{6}$$

The dependence of φ on L should be known for a calculation of Δp_h from (6). However, void fraction φ is usually given as a function of quality x rather than of length L. In such case a transformed formula, obtained with the use of relations

$$dL=\frac{dL}{dx}dx, \quad \rho_g \varphi+\rho_l(1-\varphi)=\bar{\rho} \tag{7}$$

will be used in the form

$$\Delta p_h=\int_0^x \bar{\rho} g \frac{dL}{dx} dx. \tag{8}$$

On the basis of this formula the hydrostatic pressure drop in a short measuring section can be determined as

$$\Delta p_{hi}=\frac{h_r}{x_i-x_{i-1}} \int_{x_{i-1}}^{x_i} \bar{\rho} g\, dx=\frac{h_r}{\Delta x}\left[\int_0^{x_i} \bar{\rho} g\, dx-\int_0^{x_{i-1}} \bar{\rho} g\, dx\right]. \tag{9}$$

It is evident, that void fraction φ should be known determine Δp_{hi}. Two formulas were employed in this paper for calculations of void fraction in freon 21. For $x \geqslant 0.05$ calculations were made according to a formula given in [2]

$$1-\varphi=C_\varphi\left(1+\frac{1}{\chi_{tt}}\right)^{-k}. \tag{10}$$

Factors C , k and a number K, determined for freon 21 within a saturation temperature range $t_F=70$ - 120°C are given in a Table.

Values of parameters C_Φ, C_φ, m, k and K for freon 21 at different temperatures

Parameter \ Freon saturation temperature °C	70	80	90	100	110	120
$K=\frac{\rho_l}{\rho_g}\left(\frac{\mu_g}{\mu_l}\right)^{0.25}$	21.65	17.05	13.58	10.81	8.64	6.91
C_Φ	2.07	1.98	1.9	1.8	1.67	1.56
C_φ	0.48	0.50	0.53	0.57	0.6	0.645
m	0.886	0.891	0.897	0.904	0.914	0.924
k	0.858	0.886	0.875	0.885	0.895	0.908

For quality $x<0.05$ the Thom's formula [4] was employed

$$\varphi=\frac{\Theta x}{1+x(\Theta-1)}, \tag{11}$$

where Θ denotes a parameter expressing pressure influence. Values of Θ for water at different pressure values were given in [4]. The last formula can be used, on the grounds of considerations presented in [2], for media other than water, provided the parameter

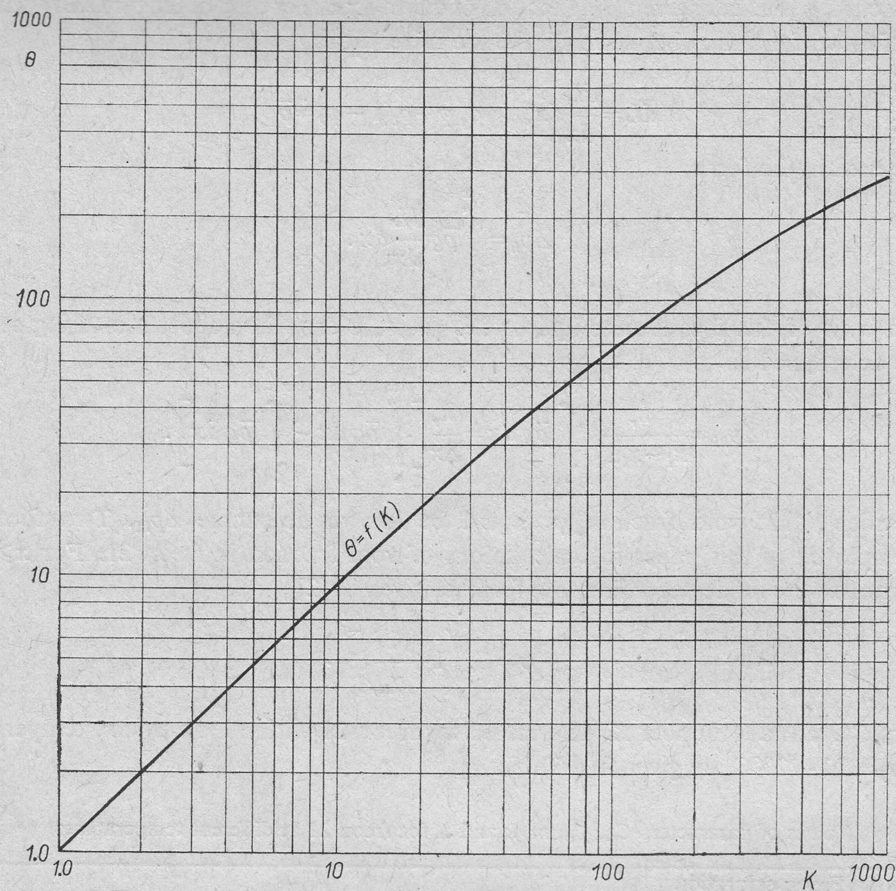

Fig. 4. Θ versus K

Θ will be given as a function of the number K rather than of the pressure value. Θ versus K, as calculated on the basis of [4] and [2], is shown in Fig. 4. Void fraction φ for freon 21 within saturation temperature range $t_F=80$ - 110°C, calculated according to (10) and (11) as a function of x, is shown in Fig. 5. The obtained dependence $\varphi=\varphi(x)$ was employed for calculating an integral

$$\int_0^{x_l} \bar{\rho} g \, dx \tag{12}$$

which was used when calculating Δp_h. Integration was done by means of a Simpson formula

$$\int_{x_0}^{x_0+2k} y \, dx = \tfrac{1}{3} k (y_0 + 4y_1 + y_2) \tag{13}$$

with $k=0.01$ for $0<x\leqslant 0.05$ and $k=0.05$ for $x\geqslant 0.05$.

Calculated values of the integral (12) are shown in Fig. 6.

The above presented formulas were used for calculating a contribution of the pressure drops Δp_a and Δp_h to the total pressure drop Δp which was obtained on the basis of measurements from a formula

$$\Delta p = h_u(\rho_r - \rho_F) g + \rho_F g h_r . \tag{14}$$

From (1), (5), (9) and (14) the friction pressure drop along the i-th measuring section equals to

$$\Delta p_{\mathrm{TPF}i} = h_u(\rho_r - \rho_F) g + \rho_F g h_r - (r_i - r_{i-1}) v_l \dot{M}^2 - \frac{h_r}{\Delta x}\left[\int_0^{x_i} \bar{\rho} g\, dx - \int_0^{x_{i-1}} \bar{\rho} g\, dx\right]. \tag{15}$$

This formula is employed for determination of the average two-phase friction factor $\bar{R}$ for given measuring section, the factor being equal to

$$\bar{R} = \frac{\Delta p_{\mathrm{TPF}i}}{\Delta p_{l_0}}, \tag{16}$$

where Δp_{l_0} – liquid flow friction pressure drop

$$\Delta p_{l_0} = \zeta_l \frac{h_r}{d} \frac{\dot{M}^2}{2\rho_l} . \tag{17}$$

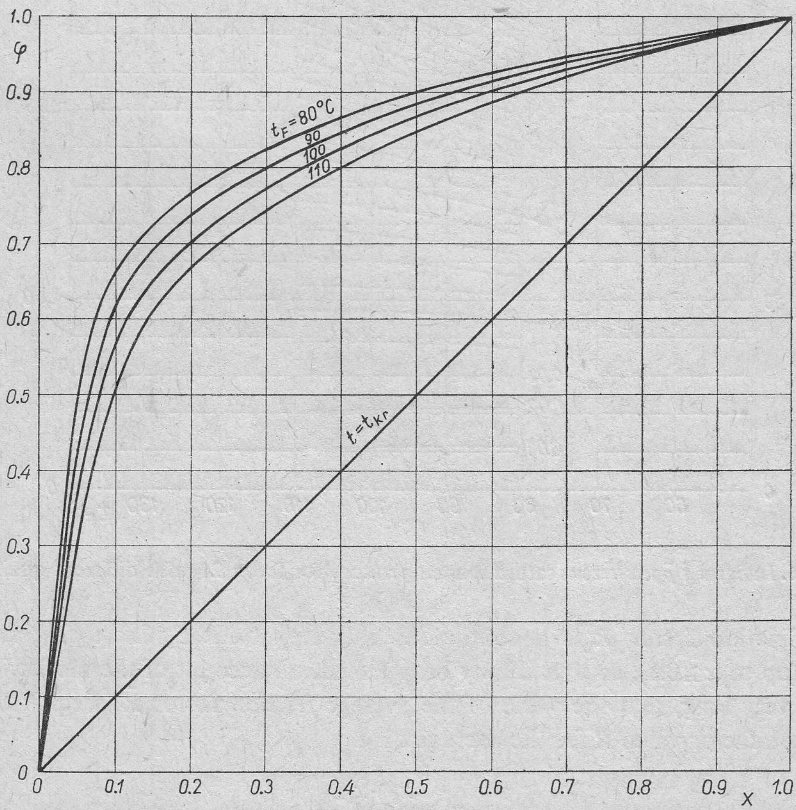

Fig. 5. Void fraction φ versus quality x for freon 21 at different saturation temperatures

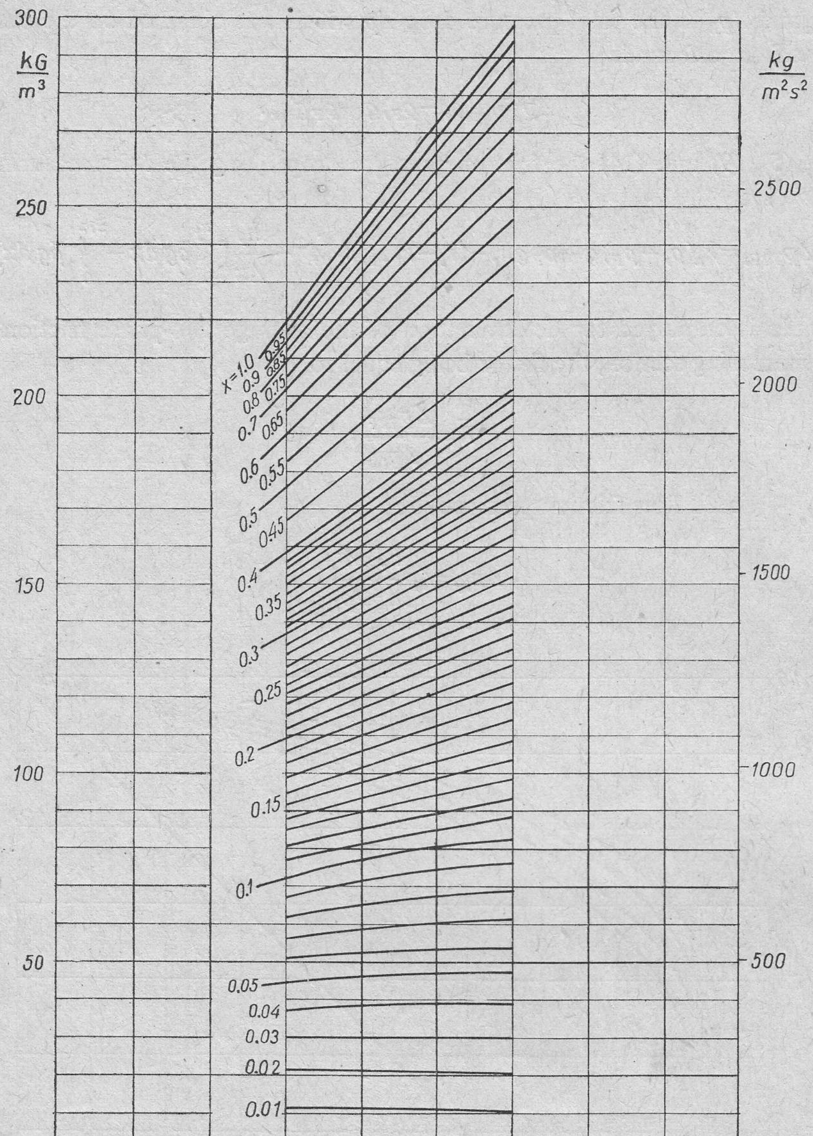

Fig. 6. Integral $\int_0^x \bar{\rho} g dx$ versus saturation temperature for freon 21 with different qualities

Local friction factor R is a continuous and monotonically increasing function of quality x (up to $x \cong 0.8$), so that it may be assumed, on account of the measuring section being not very long, that the value of the average friction factor $\bar{R}$ is equivalent to the local friction factor value R for the average quality

$$x_m = \frac{x_i + x_{i-1}}{2} \tag{18}$$

and the average saturation temperature

$$t_m = \frac{t_{i-1} + t_i}{2}. \tag{19}$$

In the experiments under consideration the quality was determined from the heat balance

$$x = \frac{1}{r_F}\left(\frac{\dot{m}_F i_1 + \dot{m}_k r_k}{\dot{m}_F} - i_2'\right), \tag{20}$$

where i_1 — freon entalpy at the measuring pipe inlet, i_2' — enthalpy of the freon liquid phase at any point along the pipe.

The freon enthalpy was read from tables [5] for freon temperatures measured in the measuring pipe with the use of thermocouples.

4. Comparison of measured and calculated values of friction factor *R*

The measurements of two-phase pressure drop for freon 21 made in the experimental facility under consideration could be divided into 6 groups having different average mass velocities. Their values were equal to:

$$\bar{M}_1 = 1090 \text{ kg/m}^2\text{s}, \quad \bar{M}_2 = 1440 \text{ kg/m}^2\text{s}, \quad \bar{M}_3 = 2115 \text{ kg/m}^2\text{s},$$

$$\bar{M}_4 = 2760 \text{ kg/m}^2\text{s}, \quad \bar{M}_5 = 3450 \text{ kg/m}^2\text{s}, \quad \bar{M}_6 = 4095 \text{ kg/m}^2\text{s}.$$

Mass velocities for individual measurements inside each group differed from the corresponding average values by no more than 7% for the first group and by 1% for the last one. The values of friction factor R obtained from the measurements made at freon tem-

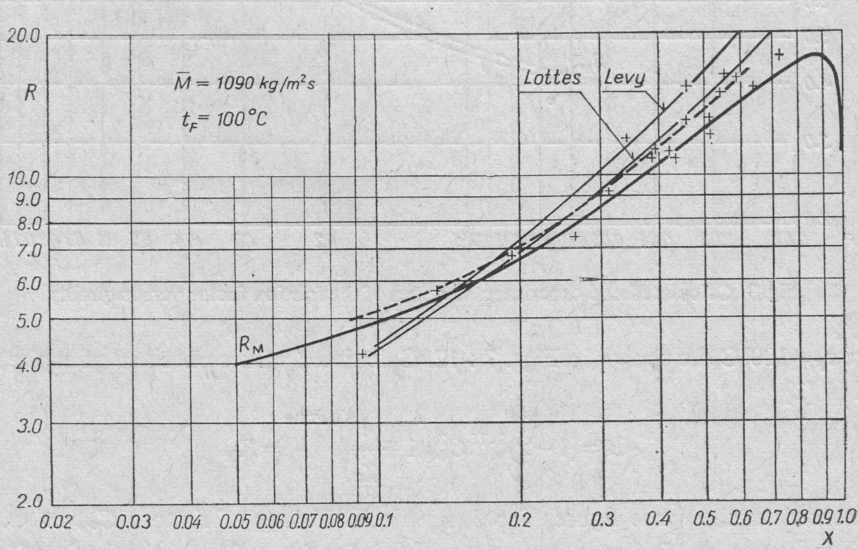

Fig. 7. Calculated and experimental values of the friction factor versus quality

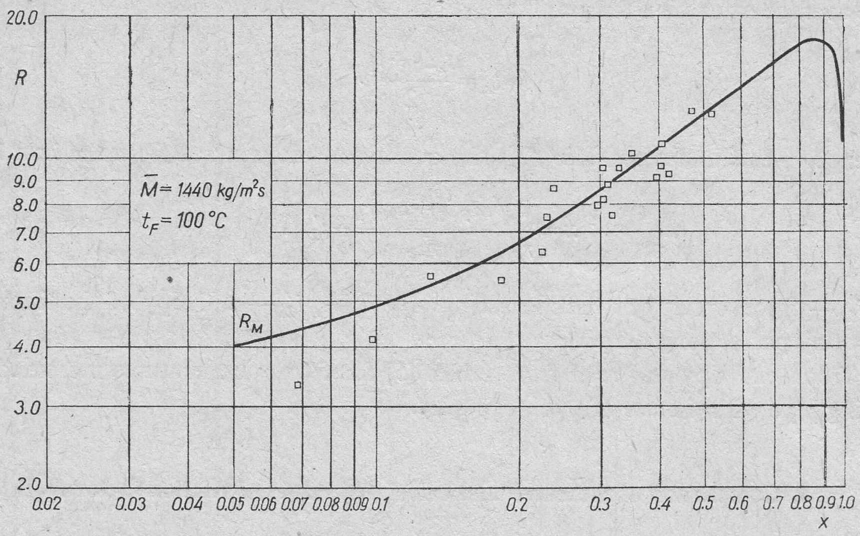

Fig. 8. Calculated and experimental values of the friction factor versus quality

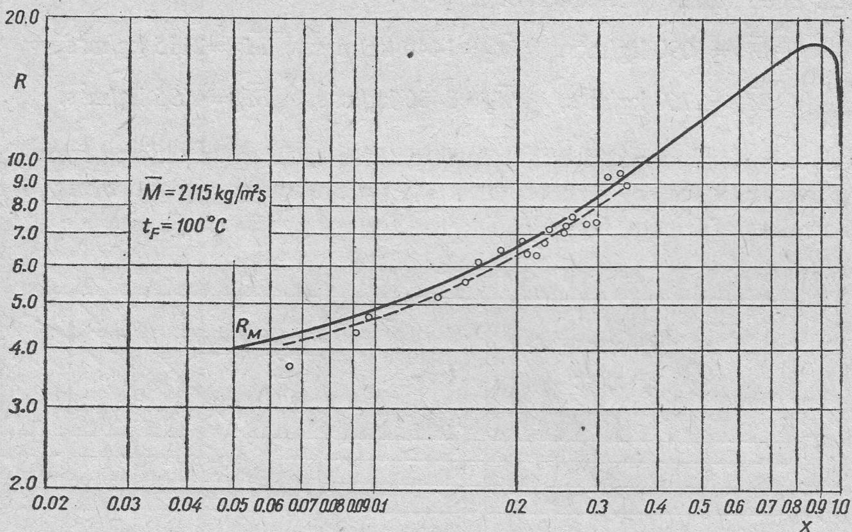

Fig. 9. Calculated and experimental values of the friction factor versus quality

perature $t_F = 100°C$ are shown in Figs. 7÷12. Calculated values obtained after the formula derived in [2]

$$R = (1-x)^{1.75} C_{\Phi}^{2} \left(1 + \frac{1}{\chi_{tt}}\right)^{1.75m} = R_M \tag{21}$$

are presented in the same figures for comparison. Values of factors C_Φ, m expressing pressure influence on R factor were determined for freon 21 on the basis of relations given in [2] for dependence of these factors on the number K. Values of C_Φ, m and K

for freon saturation temperatures ranging from $t_F = 70°C$ to $t_F = 120°C$ are given in the Table.

From the curves presented it is evident that calculated values of the friction factor R_M are sufficiently close to experimental values. This proves that formula (21) is valid, as well as formula (10) for calculation of void fraction φ and criterion of K, which characterizes – as it was shown in [2] – two-phase flow similarities with regard to pressure drops. Experimental values of the friction factor R may be determined from the total pressure drop measurements for two-phase mixture, only if the pressure drops Δp_a and

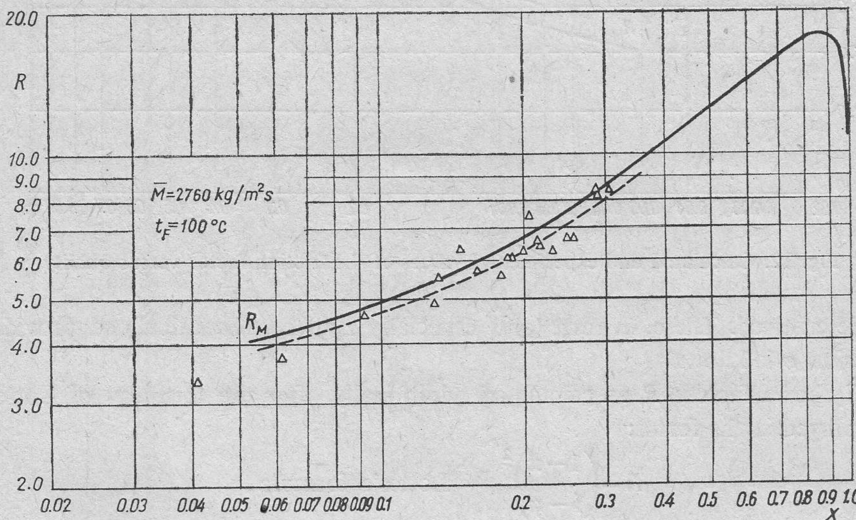

Fig. 10. Calculated and experimental values of the friction factor versus quality

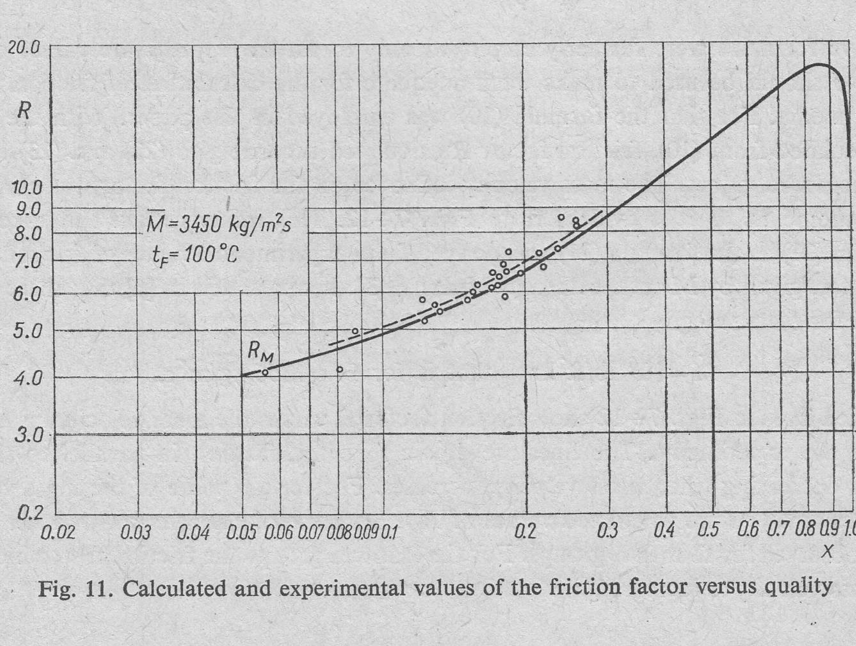

Fig. 11. Calculated and experimental values of the friction factor versus quality

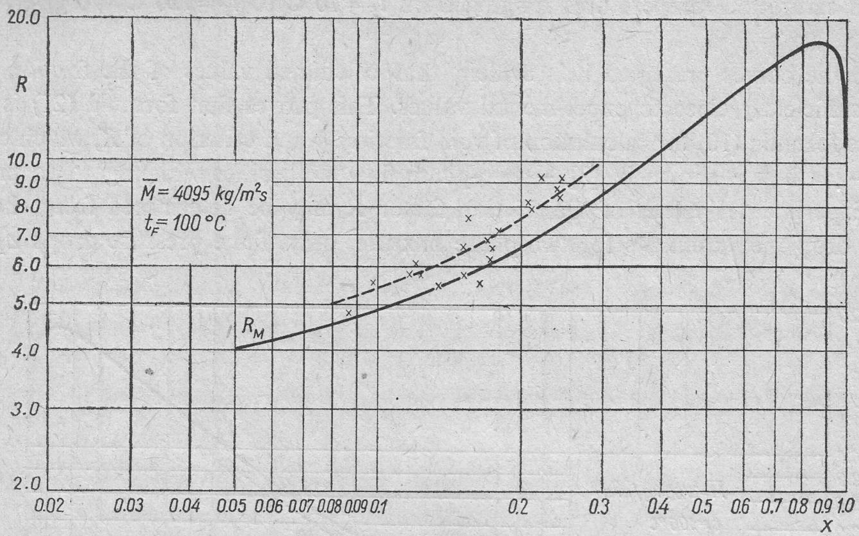

Fig. 12. Calculated and experimental values of the friction factor versus quality

Δp_h are determined. These, in turn (and especially Δp_h) can be calculated provided the void fraction φ is known.

Fig. 7 shows factors R as calculated additionally after the formulas of Lottes and Levy employed in the forms:

$$R=\left(\frac{1-x}{1-\varphi}\right)^2 \quad \text{– Lottes's formula}, \tag{22}$$

$$R=\frac{(1-x)^{1.75}}{(1-\varphi)^2} \quad \text{– Levy's formula.} \tag{23}$$

The above formulas were formerly employed only to water. Appropriate values of void fraction φ should be used to make them adequate for the calculation of friction factors of other media. For that the formula (10) was employed in this paper. With the values of φ calculated from (10), friction factor R calculated according to (22) and (23) is close to the measured values. The mass velocity influence on the friction factor was confirmed by measurements, as it is evident from Figs. 7÷12. This influence was not taken into account in (21) as the formula (21) was derived from experimental values of friction factor R for water, given by Martinelli and Nelson [3]. It was stated in [6] that these values are valid for mass velocity

$$\dot{M}=10^6 \ \text{lb/ft}^2\text{hr} \quad \text{that is for } \dot{M}=1356 \ \text{kg/m}^2\text{s}.$$

The dashed lines in Figs. 7÷12 show friction factor R versus the mass velocity, as evaluated from the experiments. The lines are drawn for simplification in parallel to the R_M line. R is decreasing when mass velocity increases. For certain value of the mass velocity it becomes equal to the calculated values of R_M. Such trend, consistent with other works [6, 7], is observed for four first groups. For the last two groups the friction factor increases when mass velocity is increasing.

A simplified way of drawing the experimental curves of the friction factor values versus mass velocity, namely drawing them parallelly to R_M (dashed lines in Figs. 7÷12) allows analytic formulation of the dependence $R(\dot{M})$ without taking quality into account. Values of the friction factor versus mass velocity, for chosen value of quality ($x=0.3$), were put down in Fig. 13. Then two straight lines a and b were drawn through experimental points, intersecting at the mass velocity $\dot{M}=2400$ kg/m²s. Only the a line, whose course is proved by other papers, was taken for further analysis leading to analytic expression on $R(\dot{M})$ dependence.

As Fig. 13 is drawn in logarithmic scale, the straight line can be described by an exponential formula of the type

$$R=C\dot{M}^{-n}, \tag{24}$$

where n — tangent of the angle α between the straight line a and the abscissa.

The value of n was evaluated from Fig. 13 as equal to about 0.25. The experimental and calculated values of friction factors are equal at $\dot{M}_M \cong 1400$ kg/m²s (see Fig. 13). Obviously, the point $(\dot{M}_M, R_M)$ satisfies the straight line equation (24) i.e.

$$R_M=C\dot{M}_M^{-n}. \tag{25}$$

Dividing (24) by (25) one obtains

$$\frac{R}{R_M}=\left(\frac{\dot{M}}{\dot{M}_M}\right)^{-n}. \tag{26}$$

Hence

$$R=R_M\left(\frac{\dot{M}_M}{\dot{M}}\right)^n=R_M\left(\frac{1400}{\dot{M}}\right)^{0.25}. \tag{27}$$

This expression is valid for $\dot{M}<2400$ kg/m²s, i.e. up to the point of intersection of the lines a and b.

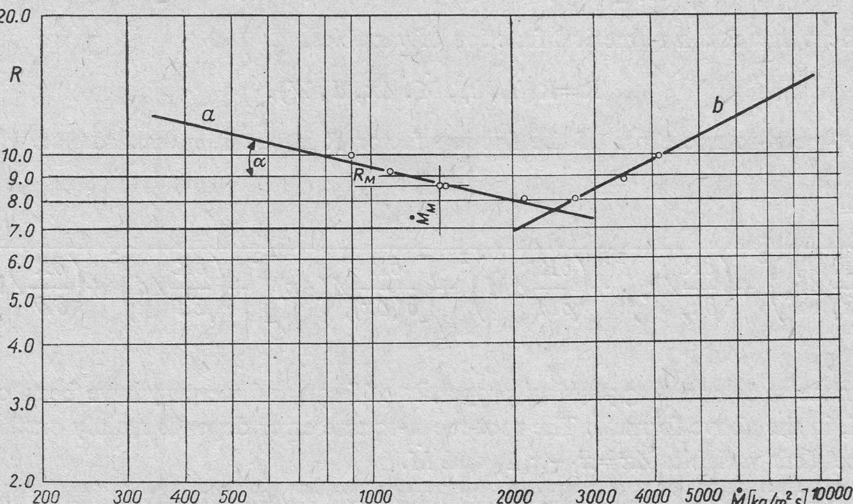

Fig. 13. Dependence of the friction factor on mass velocity, for $x=0.3$ and $t_F=100$°C

5. Error in the determination of friction factor R from the measurements

Measured values of friction factor R are obtained with systematic as well as random errors. Small number of measurements made in identical conditions caused that a random error was not evaluated. A systematic error originating from measurement or calculation method could be evaluated as it did not depend on number of measurements carried on.

It was assumed that the assessment of the total error by simple addition of the errors of independent variables (measured parameters) would not be reasonable. For that reason a more likely error, namely the root-mean-square error [9], was calculated instead of the maximum error.

Generally the root-mean-square error of a dependent variable $y=f(x_1, x_2, \ldots, x_l)$ has a form

$$\Delta y=\sqrt{\left(\sum_{i=1}^{n}\frac{\partial f}{\partial x_i}\Delta x_i\right)^2}\,. \tag{28}$$

The formula (16) can be transformed into the following form

$$R=\frac{h_u(\rho_r-\rho_F)\,g+\rho_F\,g h_r-(r_i-r_{i-1})\,v_L\dot{M}^2-\dfrac{h_r}{\Delta x}W}{\zeta_l\dfrac{h_r}{d}\dfrac{M^2}{2\rho_F}}, \tag{29}$$

where

$$W=\int_0^{x_i}\bar{\rho}g\,dx-\int_0^{x_{i-1}}\bar{\rho}g\,dx\,.$$

It can be rearranged to the form

$$R=\frac{2d}{\zeta_l}\left[\frac{h_u g\rho_F(\rho_r-\rho_F)}{h_r\dot{M}^2}+\frac{\rho_F^2 g}{\dot{M}^2}-\frac{r_i-r_{i-1}}{h_r}-\frac{W\rho_F}{\Delta x\,\dot{M}^2}\right]. \tag{30}$$

It is evident that R is a numerical function of variables

$$R=R(h_u, h_r, \dot{M}, \Delta x, d, W). \tag{31}$$

The root-mean-square error of the friction factor R is equal, in accordance with (28), to

$$\Delta R=$$

$$=\sqrt{\left(\frac{\partial R}{\partial h_u}\Delta h_u\right)^2+\left(\frac{\partial R}{\partial h_r}\Delta h_r\right)^2+\left(\frac{\partial R}{\partial \dot{M}}\Delta\dot{M}\right)^2+\left[\frac{\partial R}{\partial(\Delta x)}\Delta(\Delta x)\right]^2+\left(\frac{\partial R}{\partial d}\Delta d\right)^2+\left(\frac{\partial R}{\partial W}\Delta W\right)^2}\,. \tag{32}$$

Errors made at determining physical properties of freon and mercury have been omitted, as small, in the above formula. The root-mean-square error depends mainly on the errors of independent variables $\Delta x=x_i-x_{i-1}$ and $\dot{M}$.

The four representative measurements have been chosen for calculation of the root-mean-square error. They determine the range of variability for Δx and $\dot{M}$, and, simultaneously, the range of variability of the root-mean-square error. The four variants of error calculations were hence obtained:

Variant A_1 – measurement No. 48
Small values of Δx and $\dot{M}$
Variant A_2 – measurement No. 23
Large value of Δx, small value of $\dot{M}$.
Variant B_1 – measurement No. 46
Small value of Δx, large value of $\dot{M}$.
Variant B_2 – measurement No. 26
Large values of Δx and $\dot{M}$.

Separate partial derivatives in (32) are

$$\frac{\partial R}{\partial h_u}=\frac{2d\rho_F g(\rho_r-\rho_F)}{\zeta_l h_r \dot{M}^2}, \tag{33}$$

$$\frac{\partial R}{\partial h_r}=\frac{2d}{h_r^2\zeta_l}\left[(r_i-r_{i-1})-\frac{h_u(\rho_r-\rho_F)g\rho_F}{\dot{M}^2}\right], \tag{34}$$

$$\frac{\partial R}{\partial \dot{M}}=-\frac{4\rho_F}{\dot{M}^3\zeta_l}\left[\frac{h_u(\rho_r-\rho_F)gd}{h_r}+\rho_F gd-\frac{Wd}{\Delta x}\right] \tag{35}$$

$$\frac{\partial R}{\partial(\Delta x)}=\frac{2d\rho_F W}{\Delta x^2\zeta_l \dot{M}^2}, \tag{36}$$

$$\frac{\partial R}{\partial d}=\frac{2}{\zeta_l}\left[\frac{h_u(\rho_r-\rho_F)g\rho_F}{h_r\dot{M}^2}+\frac{\rho_F^2 g}{\dot{M}^2}-\frac{r_i-r_{i-1}}{h_r}-\frac{W\rho_F}{\Delta x\dot{M}^2}\right], \tag{37}$$

$$\frac{\partial R}{\partial W}=-\frac{2d\rho_F}{\Delta x\zeta_l \dot{M}^2}. \tag{38}$$

The errors of determination of h_u, h_r, $\dot{M}$, Δx, d and W were either calculated or evaluated (for detailed calculations see [10]). Their values are:

$$\Delta h_u=0.003\ \text{m},\quad \Delta h_r=0.003\ \text{m},\quad \Delta\dot{M}=43.4-51.5\ \text{kg/m}^2\text{s},$$

$$\Delta(\Delta x)=0.67\cdot10^{-3}\div36.7\cdot10^{-3},\quad \Delta d=0.0001\ \text{m},\quad \Delta W=0.844.$$

The above data served for calculating, by means of (32), the root-mean-square error of friction factor, together with the relative error. For assumed variants of calculations the corresponding values of errors are:

Variant A_1
$\Delta R=1.395$, $R=4.92$, $\Delta R/R=26\%$.
Variant A_2
$\Delta R=1.322$, $R=12.4$, $\Delta R/R=10.6\%$.
Variant B_1
$\Delta R=0.152$, $R=3.34$, $\Delta R/R=4.55\%$.
Variant B_2
$\Delta R=0.339$, $R=7.31$, $\Delta R/R=4.63\%$.

The measurements used for error calculation determine the range of average error variability. Hence, from the above presented relative error values, the error made for measurements with mean — within limits of assumed range of variability — values of $\dot{M}$ and Δx, can be evaluated as equal to about 10 - 15%.

6. Comparison of measured pressure drop values with the calculated ones

The pressure drop calculated on the basis of (1), (5), (9), (18) together with (10), (21) and (27) served for comparison of the calculated values with those measured. The results of the comparison have been presented in Fig. 14. It is evident from the figure that the calculated and measured values do not differ more than by ± 10 percent. Larger differences exist for some measurements from the fourth group of measurements. However, the mass velocity there, equal to $\dot{M} = 2760\ \mathrm{kg/m^2s}$, is outside the range of $\dot{M}$-values where the formula (27) is valid.

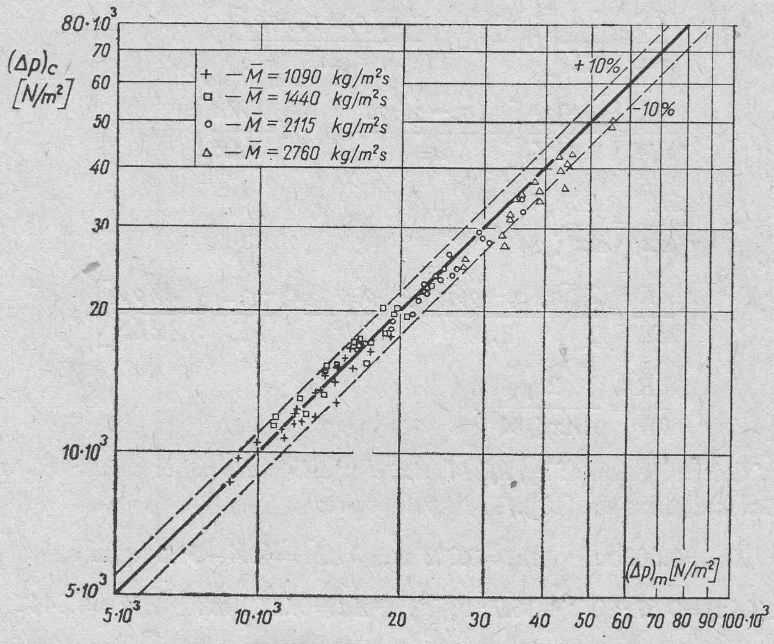

Fig. 14. Calculated values of the pressure drop compared to measured values

In the author's opinion, the differences of about 10 percent can be regarded as permissible from practical standpoint. Thus, the method of calculating the pressure drop for freon 21 from the formulas (10), (21) and (27) may be regarded as sufficiently accurate.

7. Conclusions

Experimental investigations of the boiling pressure drop for freon 21 in a vertical pipe proved the validity of the formulas (10) and (21) derived in [2], as well as the usefulness of number K derived as a similarity parameter for two-phase flows with regard to pressure drops. Additional investigations allowed to take account of the freon 21, mass velocity influence, on the friction factor, for mass velocities not exceeding $\dot{M}=2400$ kg/m²s (formula (27)). For mass velocities larger than $\dot{M}=2400$ kg/m²s the influence appeared different from the picture given by other authors. The lack of thermodynamic equilibrium in the flow, indispensable for the validity of (21), or the influence of heat flux can be the probable reason of that. For the two-phase water flow, the influence of heat flux is not observed until the value of 10^6 W/m² is reached; however, this influence may appear sooner for other media. As to the influence of thermodynamic nonequilibrium on the friction factor R, there is no data up to now. This effect is encountered, as stated in [8], especially at high velocities of two-phase flows in vertical pipes. The thermodynamic nonequilibrium manifests itself by a vapour temperature increase above the saturation temperature at some distance from the pipe inlet.

From the above discussion it is clear that the explanation of the phenomenon of the friction factor R increase for mass velocities exceeding a certain value would require special investigations.

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Eksperymentalne wyniki pomiaru spadku ciśnienia przy wrzeniu freonu R-21 w rurach pionowych

Streszczenie

W pracy opisano stanowisko badawcze, na którym przeprowadzone zostały pomiary spadku ciśnienia przy wrzeniu freonu 21 w rurze pionowej. Pomiary te posłużyły do wyznaczenia wartości doświadczalnych współczynnika tarcia przepływu dwufazowego. Ponieważ dla układu pionowego rur,

spadek ciśnienia w przepływie dwufazowym określa wzór (1), to dla wyznaczenia spadku ciśnienia wynikającego z tarcia Δp_{TPF} — który określa współczynnik tarcia R — należało przedtem wyznaczyć spadek ciśnień Δp_a oraz Δp_h. Spadek ciśnienia Δp_a wyznaczono za pomocą (3) i (5). Natomiast spadek ciśnienia Δp_h obliczono z zależności (9), wykorzystując przy tym (10) oraz (11). Po podstawieniu obliczonych wartości Δp_a oraz Δp_h do (1) przy wykorzystaniu zmierzonych wartości całkowitego spadku ciśnienia, otrzymano spadek ciśnienia Δp_{TPF} (15), a stąd na podstawie (16), doświadczalne wartości współczynnika tarcia R.

Wartości te porównano na rysunkach 7 - 12 z wartościami obliczeniowymi R_M w oparciu o (21), uzyskując dobrą zgodność. Wyniki pomiarów pozwoliły następnie na znalezienie wpływu prędkości masowej na współczynnik tarcia R — zależność (27). Na zakończenie porównano wartości obliczeniowe spadku ciśnienia w przepływie dwufazowym wyznaczone według (1) przy użyciu (10), (21) i (27) (rys. 14) z wartościami doświadczalnymi, uzyskując rozbieżności nie większe jak $\pm 10\%$.

Экспериментальные результаты исследования перепада давления при кипении фреона R-21 в вертикальных трубах

Резюме

В работе описан экспериментальный стенд, на котором проводились измерения перепада давления при кипении фреона R-21 в вертикальной трубе. Эти измерения служили определению экспериментальных значений коэффициента трения двухфазного течения. Так как для вертикальной системы труб перепад давления в двухфазном течении определяется выражением (1), для определения перепада давления, вызванного трением Δp_{TPF}, который определяет коэффициент трения R, заранее следует определить перепады давлений Δp_a и Δp_h. Перепад давления Δp_a определяется при помощи зависимостей (3) и (5), а перепад давления Δp_h подсчитывается по формуле (9) с использованием зависимостей (10) и (11). После подстановки вычисленных значений Δp_a и Δp_h в зависимость (1), с использованием замеренных значений абсолютного перепада давления, получается перепад давления Δp_{TPF} (15), а отсюда на основе зависимости (16) следуют экспериментальные значения коэффициента трения R.

Эти значения сравниваются на рис. 7÷12 с расчетными значениями R_M, полученными из зависимости (21). Достигнута хорошая сходимость. Результаты измерений позволили затем определить влияние массовой скорости на коэффициент трения R (зависимость (27)). В заключении сравниваются расчетные значения перепада давления в двухфазном течении, определенные из зависимости (1), а также (10), (21) и (27) (рис. 14) с экспериментальными значениями, констатируя расходимость не большую, чем $\pm 10\%$.