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An Analysis of the Unsteady Charging Process in a Cylinder

An analysis of the unsteady process where mass from an external source is used to charge a cylinder is presented in which account is taken of frictional dissipation effects in the nozzle and cylinder with heat crossing the cylinder-nozzle envelope. The relationships are derived for the case where the cylinder volume changes so that they are applicable to charging processes in internal combustion engines and they may also of course be modified to deal with the charging of a constant volume tank. A comparison is made between the charging process and the process in which mass is discharged from a cylinder. The comparison shows the similarity in form of the relationships for charging and discharging processes while emphasising the fundamental differences between the two processes.

Nomenclature

c – velocity,
c_p – specific heat at constant pressure,
h – specific enthalpy,
M – mass,
P – pressure,
P_x – total restraining pressure at the piston,
q_e – external heat per unit mass,
q_f – internal heat of friction per unit mass,
q'_{ec} – heat per unit mass of the charging mass leaving the charging mass and passing to the mass in the cylinder,
q''_{ec} – heat per unit mass of the cylinder mass which is obtained from the charging mass after the charging mass enters the cylinder,
q'_{fc} – internal heat per unit mass of the charging mass due to the dissipation of the kinetic energy of the charging mass once it enters the cylinder,
R – gas constant,
s – specific entropy,
t – time,
T – temperature,
u – specific internal energy,
v – specific volume,
V – cylinder volume,
γ – isentropic index.

Additional suffixes

a – denotes conditions in the source of the charging mass also conditions in cylinder a,
b – denotes conditions in cylinder b,
c – denotes cylinder,
n – denotes nozzle,
o – denotes total head conditions,
s – denotes surroundings in general,
1 – denotes initial conditions in cylinder a,
2 – denotes initial conditions in cylinder b,
3 – denotes final conditions in cylinder a,
4 – denotes final conditions in cylinder b.

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1. Introduction

In this paper an analysis is made of the unsteady flow of fluid from a high pressure source into a cylinder of constant or variable volume. As well as making an analysis of this important practical problem it is also the function of the paper to point out the similarities and at the same time the fundamental differences that exist between this charging process and a process in which mass is discharged from a cylinder. In the analysis account is taken of external heat transfer to and of internal irreversibilities in the mass in the cylinder and the charging mass. To account for heat transfer and internal irreversibility the energy equation, applicable to unit mass of a system in any flow (or non-flow) process between state points 1 and 2, takes the form

$$q_e+q_f=u_2-u_1=\int_1^2 P\,dv \tag{a}$$

and the change in entropy (s) is given by

$$s_2-s_1=\int_1^2 \frac{dq_e+dq_f}{T}. \tag{b}$$

Here u is internal energy, T – temperature, P – pressure and v – specific volume. q_e is the external heat crossing the system boundary and q_f, the heat of friction or internal heat, is due to the dissipation of kinetic energy by friction forces. In the absence of such internal irreversibility it may be shown that

$$\int_1^2 P\,dv=\int_1^2 c\,dc+\sum\int_1^2 P_s\,dv \tag{c}$$

in which c is the velocity of the system, P_s – the pressure of fluid surrounding the system (and of any other restraining pressure at the system boundary) and the summation sign ($\sum$) accounts for the possibility that surrounding fluid may be at different pressures at different points around the system boundary. Where friction occurs, causing the dissipation of kinetic energy as heat, equation (c) becomes

$$\int_1^2 P\,dv-q_f=\int_1^2 c\,dc+\sum\int_1^2 P_s\,dv. \tag{d}$$

The right hand side of this relationship is a summation of work terms (work absorbed by the surroundings) and the kinetic energy change of the system and the sum of such forms of energy is referred to here as high grade energy (HGE). Thus equation (d) may be written

$$\int_1^2 P\,dv-q_f=\int_1^2 d(HGE) \tag{e}$$

giving the net HGE produced in the process.

In the paper the above relationships are applied in turn to (i) the mass in the cylinder and to (ii) the charging mass in deriving the various relationships.

For further information about this approach to thermodynamics the reader is referred to references [1, 2 and 3]. An analysis of the discharging process using a similar approach is given in reference [4].

2. The general case of the charging process

Fig. 1a shows a cylinder fitted with a piston and with a nozzle in the cylinder head. At a given time t a mass M occupies a volume V in the cylinder where the properties of state are P, u, v and h and the velocity is c. Thereafter during time δt a mass δM crosses the nozzle outlet section and the volume increases by δV while the piston is restrained by a force corresponding to the pressure P_x. The mass δM expands across the nozzle from a region where the properties of state are P_a, u_a, v_a and h_a and the velocity is c_a and for the passage through the nozzle the external and internal heat quantities per unit mass are q_{en} and q_{fn} respectively. The external heat quantity per unit mass crossing the cylinder is δq_{ec} and normal dissipation inside the cylinder, which would occur in the cylinder even if no mass enters through the nozzle, causes internal heating equal to δq_{fc} per unit mass.

For a charging process heat quantities other than those mentioned above are involved. When mass is discharged from a cylinder the processes in the cylinder and across the nozzle can in practice approximate closely to isentropic processes. For the charging case frictional dissipation is inevitable. This may be explained as follows.

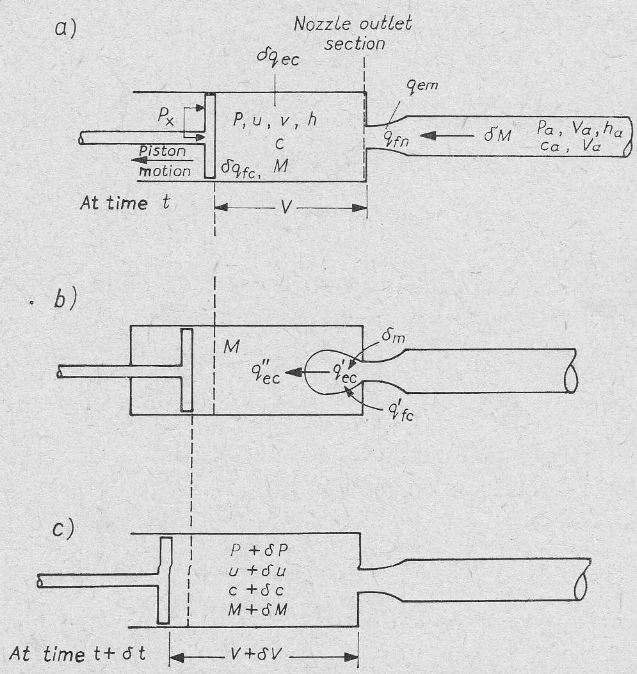

Fig. 1. Arrangement for the charging process; a) conditions before the charging mass enters the cylinder, b) conditions just after mass δM enters the cylinder, c) conditions after the charging mass enters the cylinder

As mass δM crosses the outlet section of the nozzle it does work on the mass in the cylinder and will in general arrive inside the cylinder with a temperature different from that of the mass M which then surrounds it. This state of affairs is indicated in Fig. 1b. Due to the temperature difference there should be an exchange of heat between M and δM. However when the mass δM arrives inside it will have a velocity different from that of mass M. This could be a considerable velocity and the corresponding kinetic energy will be dissipated and will reappear as frictional heat. Let this quantity of frictional heat be $\delta M q'_{fc}$. The effect of this heat will be to increase the temperature of mass δM above the temperature of δM when it first arrived inside the cylinder. Thereafter there will be an exchange of heat between M and δM due to the temperature difference between the two masses. If we consider the mass M as one system and the mass δM as another system then this is an exchange of external heat between the two systems. The direction of this heat flow will depend on the actual circumstances. Suppose that heat quantity $\delta M q'_{ec}$ leaves mass δM so that heat quantity $\delta M q''_{ec}$ enters mass M then

$$\delta M q'_{ec} = M q''_{ec}. \tag{1}$$

The final conditions after time δt are shown in Fig.1c.

For the system of mass M the energy equation is

$$\delta q_{ec} + \delta q_{fc} + q''_{ec} = \delta u + P\delta v + T\delta s \tag{2}$$

so that the specific HGE output of mass M is

$$\delta q_{ec} + q''_{ec} - \delta u = P\delta v - \delta q_{fc}. \tag{3}$$

For the system of mass δM the energy equation is

$$q_{en} + q_{fn} + q'_{fc} - q'_{ec} = u + \delta u - u_a + \int_a P dv \tag{4}$$

so that the specific HGE output for mass δM is

$$q_{en} - q'_{ec} - u + u_a - \delta u = \int_a P\, dv - q_{fn} - q'_{fc}. \tag{5}$$

Pressure-volume diagrams for the two systems are shown in Fig. 2 and from equations (3) and (5) the total HGE output for the two processes is

total HGE output=

$$M\delta q_{ec} + M q''_{ec} - M\delta u + \delta M q_{en} - \delta M q'_{ec} - \delta M u + \delta M u_a \tag{6}$$

in which the products of small quantities are neglected.

Using equation (1), equation (6) reduces to

total HGE output=

$$M\delta q_{ec} - M\delta u + \delta M q_{en} - \delta M u + \delta M u_a. \tag{7}$$

The total HGE output for the two processes is also given by

total HGE output=

$$P_x \delta V - \delta M P_a v_a + (M+\delta M)\frac{(c+\delta c)^2}{2} - M\frac{c^2}{2} - \delta M\frac{c_a^2}{2}$$

or total HGE output=

$$P_x\delta V-\delta MP_a v_a+Mc\delta c-\delta M\frac{c_a^2}{2}+\delta M\frac{c^2}{2}. \tag{8}$$

Hence from equations (7) and (8)
total HGE output=

$$M\delta q_{ec}-M\delta u+\delta Mq_{en}-\delta Mu+\delta Mu_a$$
$$=P_x\delta V-\delta MP_a v_a+Mc\delta c-\delta M\frac{c_a^2}{2}+\delta M\frac{c^2}{2}. \tag{9}$$

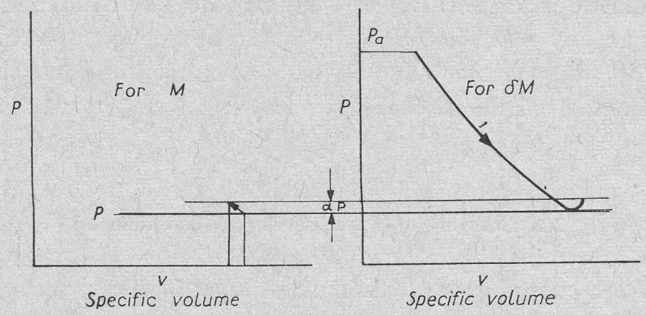

Fig. 2. Process lines for the mass in the cylinder and the charging mass-elemental cylinder process

Equation (9) may be rearranged as

$$\delta M\left(u_a+P_a v_a+\frac{c_a^2}{2}\right)+\delta Mq_{en}=\delta M\left(u+\frac{c^2}{2}\right)+(Mc\delta c+P_x\delta V)$$
$$-M\delta q_{ec}+M\delta u. \tag{10}$$

From equation (2)

$$M\delta u=M\delta q_{ec}+M\delta q_{fc}+Mq''_{ec}-MP\delta v \tag{11}$$

or

$$M\delta u=MT\delta s-MP\delta v \tag{12}$$

and from

$$V=Mv \tag{13}$$
$$P\delta V=MP\delta v+Pv\delta M. \tag{14}$$

Hence substituting for $Mp\delta v$ in equations (11) and (12) from equation (14) gives

$$M\delta u=M\delta q_{ec}+M\delta q_{fc}+Mq''_{ec}+\delta MPv-P\delta V \tag{15}$$

and

$$M\delta u=MT\delta s+Pv\delta M-P\delta V. \tag{16}$$

Substituting for $M\delta u$ from equation (15) in equation (10) gives

$$h_o=h_{oa}+q_{en}-\left[\frac{M(\delta q_{fc}+q''_{cc})}{\delta M}+\frac{Mc\delta c}{\delta M}+\frac{(P_x-P)\delta V}{\delta M}\right] \tag{17}$$

and substituting for $M\delta u$ from equation (16) in equation (10) gives

$$h_o=h_{oa}+q_{en}-\left[-\frac{M\delta q_{ec}}{\delta M}+\frac{MT\delta s}{\delta M}+\frac{Mc\delta c}{\delta M}+\frac{(P_x-P)\delta V}{\delta M}\right]. \tag{18}$$

When mass is discharged from a cylinder (total head condition o) to a constant pressure atmosphere (denoted by a) it has been shown in ref. [5] that

$$h_{oa}=h_o+q_{en}-\left[\frac{M\delta q_{fc}}{\delta M}+\frac{Mc\delta c}{\delta M}+\frac{(P_x-P)\delta V}{\delta M}\right] \tag{17a}$$

and

$$h_{oa}=h_o+q_{en}-\left[-\frac{M\delta q_{ec}}{\delta M}+\frac{MT\delta s}{\delta M}+\frac{Mc\delta c}{\delta M}+\frac{(P_x-P)\delta V}{\delta M}\right]. \tag{18a}$$

It will be noted that equation (18) for the charging case may be obtained from equation (18a) by replacing h_{oa} by $-h_{oa}$ and h_o by $-h_o$. To obtain equation (17) from equation (17a) the term $M\delta q''_{ec}/\delta M=q'_{ec}$ must also be included and it is this which makes the charging process different from the discharging process.

2.1. Charging of a cylinder of constant volume

Consider now the charging case where the volume of the cylinder is constant and where c and δq_{fc} are each zero. Equation (17) then reduces to

$$h=h_{oa}+q_{en}-\frac{Mq''_{ec}}{\delta M} \tag{19}$$

or

$$q''_{ec}=\frac{\delta M}{M}(h_{oa}-h+q_{en}). \tag{20}$$

Hence from equations (20) and (1)

$$q'_{ec}=h_{oa}-h+q_{en} \tag{21}$$

and equation (18) gives

$$h=h_{oa}+q_{en}+\frac{M\delta q_{ec}}{\delta M}-\frac{MT\delta s}{\delta M}, \tag{22}$$

$$\therefore\quad \frac{\delta s}{\delta M}=\frac{h_{oa}-h+q_{en}+\dfrac{M\delta q_{ec}}{\delta M}}{MT}, \tag{23}$$

$\therefore$ from equations (23), (21) and (20)

$$\delta s=\frac{\delta Mq'_{ec}+M\delta q_{ec}}{MT}=\frac{q''_{ec}+q_{ec}}{T}. \tag{24}$$

Equation (21) allows one to evaluate the heat exchanged between M and δM in the mixing process within the cylinder and equation (24) shows that the specific entropy increase of mass M is due to the effects of external heat crossing the solid boundaries of the cylinder and of external heat crossing the fluid boundary of mass M.

Now from equation (22)

$$\delta M(u+Pv)=\delta M\left(u_a+P_a v_a+\frac{c_a^2}{2}\right)+\delta M q_{en}+M\delta q_{ec}-MT\delta s \tag{25}$$

and substituting for $MT\delta s$ in equation (25) from equation (2)

$$\delta M(u+pv)=\delta M\left(u_a+P_a v_a+\frac{c_a^2}{2}\right)+\delta M q_{en}+M\delta q_{ec}-M\delta u-MP\delta v. \tag{26}$$

Since δV is zero we may use equation (14) as

$$-MP\delta v=Pv\delta M. \tag{27}$$

Substituting for $-MP\delta v$ in equation (26) and rearranging gives

$$\delta(Mu)=\delta Mu+M\delta u=\delta M\left(u_a+P_a v_a+\frac{c_a^2}{2}\right)+\delta M q_{en}+M\delta q_{ec}. \tag{28}$$

For a finite mass passing into the cylinder this relationship shows that the difference between the total final internal energy and the total initial internal energy is equal to the sum of the total initial kinetic energy, the total initial flow work and the total external heat passing the solid boundaries of the cylinder and nozzle.

2.2. Adiabatic charging of a constant volume cylinder of a perfect gas

If we now make the additional assumptions (a) that no external heat crosses the solid boundaries of the cylinder or nozzle and (b) that the fluid involved is a perfect gas with constant specific heats then from equation (23)

$$\frac{ds}{dM}=\frac{c_p(T_{oa}-T)}{MT}. \tag{29}$$

Under these circumstances equation (28) gives

$$\delta(MT)=\gamma T_{oa}\delta M \tag{30}$$

which in the limit gives

$$T\,dM+M\,dT=\gamma T_{oa}\,dM \tag{31}$$

or

$$\frac{dT}{dM}=\frac{\gamma T_{oa}-T}{M} \tag{32}$$

which may also be written as

$$\frac{dM}{M}=\frac{dT}{\gamma T_{oa}-T}. \tag{33}$$

Thus for a finite charging process between an initial state point 1 inside the cylinder and a final state point 2

$$\log_e \frac{M_2}{M_1} = \int_{T_1}^{T_2} \frac{dT}{\gamma T_{oa} - T} = \log_e \frac{(\gamma T_{oa} - T_1)}{(\gamma T_{oa} - T_2)}, \tag{34}$$

where $M_2 - M_1$ is the mass which passes across the nozzle and T_{oa} is the constant initial total head temperature of the charging mass.

For the mass inside the cylinder the equation of state gives

$$P = \frac{R}{V}(MT) \tag{35}$$

where R/V is a constant

$$\therefore \; dP = \frac{R}{V}(MdT + TdM) \tag{36}$$

and using equation (31), equation (36) gives

$$\frac{dP}{dM} = \frac{R}{V}\gamma T_{oa}. \tag{37}$$

This shows that the rate of increase of pressure is a constant and is independent of the initial temperature inside the cylinder. For a finite process 1 - 2 equation (37) becomes

$$P_1 - P_2 = \frac{R}{V}\gamma T_{oa}(M_2 - M_1). \tag{38}$$

Using equations (33) and (29) the entropy change in the cylinder may be expressed as

$$ds = \frac{dM}{M}\frac{c_p(T_{oa} - T)}{T} = \frac{c_p(T_{oa} - T)}{T(\gamma T_{oa} - T)}dT \tag{39}$$

and for a finite process 1 - 2 this gives

$$s_2 - s_1 = c_p \log_e \left[\frac{T_2^{\frac{1}{\gamma}} \left(\gamma - \frac{T_2}{T_{oa}}\right)^{\frac{\gamma-1}{\gamma}}}{T_1^{\frac{1}{\gamma}} \left(\gamma - \frac{T_1}{T_{oa}}\right)^{\frac{\gamma-1}{\gamma}}} \right]. \tag{40}$$

2.3. Adiabatic charging of one constant volume cylinder from another constant volume cylinder

The charging of one fixed volume cylinder from another fixed volume cylinder may be approached using the relationships derived above. Consider Fig. 3 and suppose that no external heat crosses the boundaries of the cylinders or nozzle. The volumes of the cylinders are assumed to be large so that instantaneous bulk velocities in the cylinders are negligible. Mass passes from cylinder a to cylinder b and for the charging of cylinder b from cylinder a under these circumstances equation (28) is in the limit.

$$d(Mu)_b = h_a\, dM\,, \tag{41}$$

where h_a is the instantaneous enthalpy in cylinder a.

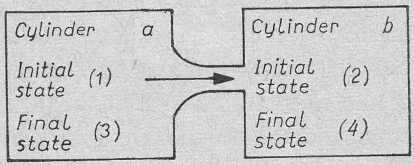

Fig. 3. Charging of cylinder b from cylinder a

Similarly, for the discharge of mass from container a, it may be shown (from equation (24) of reference [4]) that

$$d(Mu)_a = -h_{ob}\, dM \tag{42}$$

and (from equation (25) of ref. [4]) that

$$h_{ob} = h_a\,. \tag{43}$$

Hence by adding equations (41) and (42)

$$d(Mu)_a + d(Mu)_b = 0\,. \tag{44}$$

Consider now a finite process in which the conditions in a and b are designated 1 and 2 respectively and where the final conditions are 3 and 4 respectively.

Then from equation (44)

$$M_3 u_3 - M_1 u_1 + M_4 u_4 - M_2 u_2 = 0 \tag{45}$$

or

$$M_3 u_3 + M_4 u_4 = M_1 u_1 + M_2 u_2\,. \tag{46}$$

As mass is discharged from cylinder a, the pressure P_1 reduces to P_3 while the pressure in cylinder b increases from P_2 to P_4. Equation (46) is applicable even if the process ceases before the pressures equalise in the two vessels. At the instant when the pressures equalise equation (46) will still apply but in general u_3 will not equal u_4. If the passage through the nozzle is left open after the pressures equalise a further interaction between the two masses will ensue until a common state is reached. Application of the energy equation to the single system containing the masses in the two vessels gives

$$M_1 u_1 + M_2 u_2 = (M_1 + M_2)\, u\,, \tag{47}$$

where u is the final common specific internal energy.

3. Concluding remarks

Equations (17) and (18) for the charging process are applicable with heat transfer and internal irreversibilities in the cylinder mass and in the charging mass. They may be applied to the charging process in an engine cylinder where the cylinder volume changes.

The corresponding general relationships for a discharging process are given in equations (17a) and (18a). For the discharging case the sum of the terms inside the square brackets in these equations may be shown to be zero indicating that the "steady" flow energy equation for the nozzle may be applied to this unsteady flow problem provided the appropriate instantaneous values of enthalpy etc. are used. For the charging case however the sum of the terms inside the square brackets in equations (17) and (18) will not in general be equal to zero because of (a) the difference in temperature between the charging mass as it enters the cylinder and the mass in the cylinder which will result in heat transfer between the two masses and because of (b) the dissipation as heat of the kinetic energy of the charging mass after this mass enters the cylinder. One could imagine special circumstances in which no entropy increase would result because of these effects. For example when the velocity leaving the nozzle was the same as the piston velocity and where the total head temperature in the source of the charging mass was such that the temperature after expansion across the nozzle was the same as that in the cylinder. However these two effects are inevitable in most charging processes and it is the result of these effects that makes the charging process differ fundamentally from the discharging process.

Acknowledgements. The form of the relationships for the pressure rise (equation (38)) and entropy change (equation (40)) in the cylinder for the case of adiabatic charging of the cylinder with a perfect gas was first seen by the authors in reference [5].

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Analiza procesu przejściowego napełniania cylindra

Streszczenie

W pracy przedstawiono analizę procesu przejściowego napełniania cylindra masą ze źródła zewnętrznego. Wzięto pod uwagę efekty dyssypacji powodowanej tarciem w dyszy i w cylindrze i przekazywania ciepła przez ścianki cylindra i dyszy. Wyprowadzono zależności dla przypadku cylindra o zmiennej objętości; można je zastosować do procesów napełniania w silnikach spalinowych. Można je oczywiście również zmodyfikować tak by opisywały napełnianie zbiornika o stałej objętości. Dokonano porównania procesu napełniania z procesem wypływu masy z cylindra. Porównanie to pokazuje podobieństwo postaci zależności wyprowadzonych dla procesów napełniania i wypływu, podkreślając równocześnie fundamentalne różnice pomiędzy tymi dwoma procesami.

Анализ переходного процесса наполнения цилиндра

Резюме

В работе представлен анализ переходного процесса наполнения цилиндра массой из внешнего источника. Учитываются эффекты диссипации, вызываемой трением в сопле и в цилиндре, а также эффекты передачи тепла через стенки цилиндра и сопла. Выведены зависимости для случая цилиндра переменного объема; их можно применять относительно процессов наполнения в двигателях внутреннего сгорания. Конечно, можно их также так модифицировать, чтобы можно было применять для описывания наполнения бака постоянного объема. Проводится сравнение процессов наполнения и истечения массы из цилиндра. Результат сравнения показывает на подобие формы зависимостей, полученных для процессов наполнения и истечения и одновременно ярко указывает фундаментальные разницы между этими обоими процессами.