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III НАУЧНАЯ КОНФЕРЕНЦИЯ

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ПАРОВЫЕ ТУРБИНЫ БОЛЬШОЙ МОЩНОСТИ

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Thermodynamics of Rapid Non-Equilibrium Expansion of Two-Phase Media

The dynamics of thermal processes in non-equilibrium wet-steam flows has been analysed on the basis of a general model of multiphase media behaviour. The governing dynamic equations have been derived and a method of determining additional exergy losses due to non-equilibrium effects, based on an introduced concept of predicted entropy increments has been suggested. An analysis of flow choking and methods of defining the critical parameters have been presented. Two computational methods have been developed and calculations of non-equilibriun flows around a profile have been carried out.

1. Nomenclature

a — sound velocity, in general,

a_f — "frozen flow" sound velocity,

a_s — sound velocity at zero frequency,

a_∞ — sound velocity at infinite frequency,

$\bar{b}$ — specific exergy per mass flow unit of the mixture,

c, c' — specific heat at const. pressure (for gas and liquid respectively),

D — coefficient in eq. (4.20),

E — flux of thermal energy per volume unit,

e — thermal energy flux density (referred to droplet surface unit),

F — cross-section area per mass flow unit of the mixture,

f — droplet surface area per mass flow unit of the mixture,

g, g' — mass flux content (for gas and liquid respectively),

H — coefficient in eq. (4.20),

h — specific heat of evaporation (condensation),

i, i' — specific enthalpy (for gas and liquid, resp.),

J — coefficient in eq. (4.20),

k — coefficients in eq. (4.17),

l — coefficients in eq. (4.21),

m, m' — mass transfer coefficients in eq. (3.9),

N — droplet concentration

n, n' — heat transfer coefficients in eq. (3.9),

P — mass force per volume unit,

p — static pressure,

$\dot{p}$ — time derivative of the pressure,

Q — heat flux per volume unit,

q — heat flux density (referred to droplet surface unit),

R — gas constant,

r — drag coefficient in eq. (3.13),

s, s' — specific entropy (of gas and liquid resp.),

$\bar{s}$ — specific entropy per mass flow unit of the mixture,

$\bar{s}*$ — "predicted" entropy values per mass flow unit of the mixture,

T, T' — absolute temperature (of gas and liquid resp.),

T_s — saturation temperature,

t — time,

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u — velocity deviation,

v, v' — specific volume (of gas and liguid resp.),

w, w' — absolute velocity (of gas and liquid resp.),

x — path coordinate,

Y — mass flux per volume unit,

y — mass flux density (referred to droplet surface unit),

α — dimensionless critical velocity after eq. (6.20),

β — critical pressure ratio,

$\varDelta$ — finite increment,

δ, δ' — deviation from saturation temperature (for gas and liquid resp.),

ζ — exergy loss coefficient after eq. (5.23),

κ — isentropic exponent

λ — dimensionless critical velocity after eq. (6.21),

μ — root of eq. (4.17),

ρ — droplet radius,

σ — entropy source per volume unit,

Φ — droplet surface area per volume unit,

Ψ — steering functions in eq. (5.19),

ω — entropy source per mass flow unit of the mixture,

ω^* — "predicted" entropy source per mass flow unit of the mixture.

Subscripts

A — active period,

d — dissipation at the wall ("friction"),

e — external forcings (friction included),

ee — external forcings (friction excluded),

K — critical flow conditions of the mixture,

Kg — critical flow conditions of the gas phase,

m — ordinal number of the droplet fraction,

n — number of droplet fractions,

R — relaxational quantities,

S — saturation (equilibrium),

0 — leading values.

2. Introduction

Thermodynamical problems of multiphase media flows are associated with many scientific and technological areas. One of their important application concerns the wet steam flow in LP-steam turbines of both conventional and nuclear power plants.

The properties and the behaviour of wet vapour, which is a heterogeneous medium, differ essentially from those of a homogeneous one-phase medium. Hence there arises a necessity to examine the specific properties of thermal processes in such a medium and to develop appropriate calculation procedures.

The literature on this subject offers several treatments of various aspects of wet vapour flow. However, the generally adopted methods of approach are mostly insufficient for the exact description and calculation of the run of thermal processes in a wet steam flow. This is due to the occurrence of non-equilibrium effects in the medium; a considerable relative rate of processes (e.g. rapid expansion) makes these effects particularly important.

This paper offers a brief presentation of the author's approach based on some of his previous works on the dynamics of thermal processes and multiphase flows ([6, 7, 8, 9, 10, 11, 12, 13)]. The two-phase medium will be considered here as a dynamic system subject to external forcings (disturbances), the action of which causes transient states of internal non--equilibrium in the medium. The resulting follow-up processes will be examined with the purpose to describe their regularities and to develop effective methods of their calculation.

The applications of these methods lead to the following tasks: predicting the time-run of rapid thermal processes in wet vapour flows, determining the choking flow conditions and the critical cross-sections, calculation of the additional thermal losses due to internal non-equilibrium in the medium, dimensioning of the channels and nozzles for two-phase flow.

3. The Non-Equilibrium Model of a Two-Phase Flow

3.1. General Assumptions

The medium in question is wet vapour of an arbitrary substance consisting of a gaseous phase (which is the carrier component of the medium) and drops of various dimensions and physical parameters. Groups of drops having equal diameters and parameters, will be called fractions of the liquid phase. Each of the sub-systems (phases or fractions) is considered as physically homogeneous and its thermal state will be described by means of mass-average parameters.

Drops of liquid are assumed to be spherical and incompressible, their specific volume to be neglected against the specific volume of the gaseous phase

$$v' \ll v. \tag{3.1}$$

The influence of capillary components on the thermal parameters will also be neglected.

A stationary one-dimensional flow of this medium through a duct of varying cross-section area will be considered. In accordance to the one-dimensional character of the flow, the influence of the boundary layer dissipation ("wall friction") will be taken into account as an appropriate combination of the exchange of heat and mechanical work with fictitious outer sources.

3.2. The Method of Process Dynamics Description

The two-phase medium described above will be treated as a dynamic system subject to external forcings working at given time-rates. As for practical reasons the forcings are acting upon the gaseous (carrier) phase, the number of forcing functions which may work independently of each other, is connected with the number of free parameters defining the state of this phase.

A two-phase medium at rest may be subject to the action of two forcings acting simultaneously, a pertinent combination of which can bring about any physically possible process. These two forcings may be chosen to be the pressure variation and the variation of heat supply to the medium, thus defining two important particular cases:
– the adiabatic process (zero heat supply),
– the isobaric process (no pressure variation).

In a one-dimensional channel flow with the state of its gas phase determined by three variables (velocity included) one more independent forcing may act. As this forcing will be chosen the exchange of mechanical work with an external source represented by an external mass force acting upon the gaseous phase.

Another technically important way of defining forcings consists in the choice of the variation of cross-section area of the duct as a forcing function (the other forcings being the external heat supply and the action of the external mass force).

The forcings working at finite time rates upon the gaseous phase cause transient states of internal non-equilibrium which consist in differences of thermal and kinematical parameters *between the gaseous phase and the liquid droplets. This, in turn, leads to the oc-currence* of the irreversible internal mass-, energy- and momentum transfer fluxes between

the phases acting towards equilibrium. The variations of these fluxes with time depend on the structure of the medium (i.e. the size and concentration of the droplets) and on the momentary values of the parameter differences between the phases. Thus the run of the resulting follow-up process will be determined both by the course of interaction between the phases and by the time-run of the forcings. Therefore in the description of the transient processes in a multiphase medium the quantity of time appears as an independent variable (unlike to the "thermostatic" processes where any instantaneous state can be fully determined by means of the leading thermal parameters only, with no relation to time).

In the particular case when none of the forcings is acting the medium undergoes a relaxation process, proceeding asymptotically towards equilibrium.

If the forcings act upon the medium during a finite period of time, at the end of this period the medium will generally be in a non-equilibrium state; then a relaxation process will start. Hence the whole process — the result of the action of forcings on the medium — may be divided into an active period and a relaxation period. Accordingly the resulting changes of parameters at a given moment may be regarded as consisting of an active and a relaxation component.

The deviation from thermal equilibrium occurring in each sub-system (phase or droplet fraction) will be expressed by the difference between its actual temperature and the saturation temperature respective to the actual value of pressure; to describe the kinematical deviations velocity differences between the gas phase and the respective droplet fraction will be used

$$\delta = T - T_s, \qquad \delta'_m = T'_m - T_s, \qquad u_m = w - w'_m \tag{3.2}$$

$$(\text{for } m = 1, 2, ..., n).$$

Thus the state of the flowing medium, containing n fractions of liquid droplets will be defined by the set of parameters:

$$(T_s, w, g'_1, ..., g'_n, \delta, \delta'_1, ..., \delta'_n, u_1, ..., u_n). \tag{3.3}$$

In this set of $(3+3n)$ parameters there are $(1+2n)$ non-equilibrium parameters.

We shall assume that the values of deviations from equilibrium are small compared to their respective basic values

$$\left|\frac{\delta}{T_s}\right| \ll 1, \qquad \left|\frac{\delta'_m}{T_s}\right| \ll 1, \qquad \left|\frac{u_m}{w}\right| \ll 1. \tag{3.4}$$

Balance equations will be written on the basis of mass flux contents of each of the components (phases, fractions); allowing for velocity differences between the components these mass flux contents are not identical with the mass contents of the respective components occupying a fixed volume of the duct.

For the mass flux contents the obvious relation is valid:

$$\sum_m g'_m + g = 1 \tag{3.5}$$

and the total surface area of drops belonging to the m-th fraction — per mass flux unit of the two phase mixture — is:

$$f_m = 3v' \frac{g'_m}{\rho_m}, \tag{3.6}$$

whereas the concentration of these drops i.e. the number of drops per unit volume of the duct is:

$$N_m = \frac{3}{4\pi\rho_m^3} \frac{g'_m}{g} \frac{v'}{v} \frac{1}{1 - \dfrac{u_m}{w}} \tag{3.7}$$

and the total area of drops of the m-th fraction per volume unit of the duct:

$$\Phi_m = \frac{3}{\rho_m} \frac{g'_m}{g} \frac{v'}{v} \frac{1}{1 - \dfrac{u_m}{w}}. \tag{3.8}$$

3.3. Transfer Processes Between the Phases

By virtue of the assumptions (3.4), which are satisfied within a wide range of cases of technical flow processes, the possibility of spontaneous condensation may be excluded, as this kind of condensation practically occurs in strongly subcooled vapour only. Consequently, heat- and mass transfer processes between the phases will take place on the surface of the liquid drops.

The heat- and mass flux densities (fluxes per unit area), directed towards the gas phase, are

$$y_m = m'_m \delta'_m + m_m \delta, \qquad q_m = n'_m \delta'_m + n_m \delta \tag{3.9}$$

on the basis of the author's previous papers ([7, 8]), where also the coefficients used here have been calculated; the density of the thermal energy flux is then:

$$e_m = q_m + y_m i_s. \tag{3.10}$$

Taking into account eq. (3.8) the internal fluxes of mass, heat and thermal energy per unit volume of the duct (connected with droplets of the m-th fraction) may be written:

$$Y_m = \Phi_m y_m, \qquad Q_m = \Phi_m q_m, \qquad E_m = \Phi_m e_m \tag{3.11}$$

and the total internal fluxes per unit volume:

$$Y = \sum_m Y_m, \qquad Q = \sum_m Q_m, \qquad E = \sum_m E_m. \tag{3.12}$$

Similarly, the force (representing internal momentum transfer between the phases) exerted by the droplets of the m-th fraction on the gas phase in a volume unit, will be expressed as a linear function of the respective velocity deviation

$$P_m = -r_m u_m \tag{3.13}$$

and the total internal force

$$P = \sum_m P_m. \tag{3.14}$$

The forms of the transfer equations given above are in accordance with the methods used by the thermodynamics of irreversible processes in assessing the transfer fluxes as linear functions of deviations from equilibrium. The mass- and heat transfer are coupled phenomena, each of them being controlled by both deviations from thermal equilibrium, while the momentum transfer is a separate process. Using appropriate values of transfer coefficients the above equations are valid for both molecular-kinetic and contineous-medium transport situations.

4. Two-Phase Flow Dynamics

4.1. Flow Equations

For a one-dimensional flow of wet steam subject to forcings consisting in pressure variation $p(t)$, external heat supply $Q_e(t)$ and external force action $P_e(t)$ working at arbitrarily given time rates a complete set of equations governing the run of the flow process will be written down. The conservation equations for the gas phase flow are:

$$\frac{d}{dx}\left(\frac{w}{v}\right)=Y, \quad \frac{d}{dx}\left(\frac{w^2}{v}\right)+\frac{dp}{dx}=Yw'+P+P_e, \\ \frac{d}{dx}\left[\frac{w}{v}\left(i+\frac{w^2}{2}\right)\right]=Y\left(i_s+\frac{w'^2}{2}\right)+Q+Q_e+(P+P_e)w, \left.\right\} \tag{4.1}$$

and the conservation equations for a single drop

$$\frac{d}{dt}\left(\frac{4\pi\rho^3}{3v'}\right)_m+4\pi\rho_m^2\, y_m=0, \\ \frac{d}{dt}\left(\frac{4\pi\rho^3}{3v'}\, w'\right)_m+\left(\frac{P}{N}\right)_m-w\frac{d}{dt}\left(\frac{4\pi\rho^3}{3v'}\right)_m=0, \left.\right\} \tag{4.2} \\ \frac{d}{dt}\left(\frac{4\pi\rho^3}{3v'}\, i'\right)_m+4\pi\rho_m^2\, e_m=0,$$

(for $m=1, 2, ..., n$).

In addition, the state equation of the gaseous phase and the Clausius-Clapeyron equation with regard of (3.1) will be needed

$$pv=RT, \tag{4.3}$$

$$dT_s=\frac{v_s T_s}{h}\, dp. \tag{4.4}$$

Enthalpy increases will be treated as:

$$di=cdT, \quad di'=c'dT'. \tag{4.5}$$

As the flow is stationary the relations are valid (for the gas and liquid flow respectively):

$$\frac{d}{dt} = w\frac{d}{dx}, \qquad \frac{d}{dt} = w'_m\frac{d}{dx}. \tag{4.6}$$

Accordingly the action of forcings may be given as functions of time or of the channel path coordinate as well.

Using the above equations and the relations from chapter 3 a set of equations governing the flow of the two phase medium may be written in the form:

$$\frac{dw}{dt} = -v_s\left[\sum_m r_m u_m + \frac{1}{w}\dot{p} - P_e\right], \tag{4.7}$$

$$\frac{dT_s}{dt} = \frac{v_s T_s}{h}\dot{p}, \tag{4.8}$$

$$\frac{d\delta}{dt} = \frac{v_s}{c}\left[\delta\sum_m \Phi_m n_m + \sum_m \Phi_m n'_m \delta'_m + \left(1 - \frac{cT_s}{h}\right)\dot{p} + Q_e\right], \tag{4.9}$$

$$\frac{d\delta'_m}{dt} = \frac{-3v'}{c'\rho_m}\left[(m'h+n')_m\delta'_m + (mh+n)_m\delta\right] - \frac{v_s T_s}{h}\dot{p}, \tag{4.10}$$

$$\frac{du_m}{dt} = -v_s\left[\frac{g}{g'_m}r_m u_m + \sum_{j=1}^{n} r_j u_j + \frac{1}{w}\dot{p} - P_e\right], \tag{4.11}$$

$$\frac{dg'_m}{dt} = -3v'\frac{g'_m}{\rho_m}(m'_m\delta'_m + m_m\delta), \tag{4.12}$$

$$\frac{d\rho_m}{dt} = \frac{3\rho_m}{g'_m}\frac{dg'_m}{dt}, \tag{4.13}$$

(where $m = 1, 2, ..., n$).

The last equation (4.13) follows from the assumption made before, that mass transfer takes place on the surface of existing drops only.

The formulae (4.7)-(4.13) represent a set of $(3+4n)$ equations, governing the variations of as many parameters, $(1+2n)$ of them being non-equilibrium deviations. As can be seen, the variation of each parameter is controlled both by the existing state of non-equilibrium (represented by the deviation values) and by the current values of forcing functions. By setting these values of forcing functions equal to zero a description of an isobaric-adiabatic relaxation process would be obtained.

As a consequence of the linearization of the expressions for the internal fluxes and of the coupling between heat-and mass transfer processes, the variation of any thermal parameter depends on both deviations from the equilibrium temperature whereas the change of the kinematic parameters is controlled separately by the velocity-deviations.

Having determined the run of the flow parameters after the equations (4.7)-(4.13) the value of the cross-section area as a function of the path-coordinate for a given quantity of the total mass flow can be found, thus solving the problem of dimensioning the duct. Using the values of flow parameters calculated as shown above, the required area per unit

mass flow of the two-phase medium at a given section will be:

$$F(x) = R\left(1 - \sum_m g'_m(x)\right) \frac{T_s(x) + \delta(x)}{p(x)w(x)}. \tag{4.14}$$

Obviously, this quantity is the reciprocal of the flow density of the two-phase (i.e. mixture) flow.

The set of equations (4.7)-(4.13) makes also possible taking into account the influence of energy dissipation in the boundary layer at the walls. This influence shall be modeled by with drawing the motion energy (by means of introducing an appropriate mass force) from the flow and supplying to the flow the same amount of heat. The values of external heat- and force actions in eq. (4.7)-(4.13) shall be then treated as:

$$Q_e = Q_{ee} + Q_d, \qquad P_e = P_{ee} + P_d, \tag{4.15}$$

where the wall dissipation ("friction") terms satisfy the condition:

$$Q_d = P_d w \tag{4.16}$$

and the subscript ee denotes action from outside of the channel.

4.2. Dynamic Properties of Two-Phase Flows

The functions determining the run of the deviations from equilibrium values can be written down directly as follow-up functions of the forcings if the respective differential equations are treated as equations with constant coefficients, which in the most cases is a reasonable approximation. For example, the equations (4.9) and (4.10) shall be then written as a set of $(n+1)$ equations, governing the variation of the deviations from the thermal equilibrium:

$$\left. \begin{aligned} \frac{d\delta}{dt} &= \sum_m k_{(n+1)\,m}\delta'_m + k_{(n+1)(n+1)}\delta + \frac{v_s}{c}\left[\left(1 - \frac{cT_s}{h}\right)\dot{p}(t) + Q_e\right], \\ \frac{d\delta'_m}{dt} &= k_{mm}\delta'_m + k_{m\,(n+1)}\delta - \frac{v_s T_s}{h}\dot{p}, \end{aligned} \right\} \tag{4.17}$$

$$(\text{for } m = 1, 2, \ldots, n)$$

where the coefficients are:

$$\left. \begin{aligned} k_{mm} &= -\frac{3v'}{c'}\left(\frac{m'h+n'}{\rho}\right)_m, \qquad k_{m\,(n+1)} = -\frac{3v'}{c'}\left(\frac{mh+n}{\rho}\right)_m, \\ k_{(n+1)\,m} &= \frac{3v'}{gc}\left(\frac{g'n'}{\rho}\right)_m, \qquad k_{(n+1)\,(n+1)} = \frac{3v'}{gc}\sum_m\left(\frac{g'n}{\rho}\right)_m. \end{aligned} \right\} \tag{4.18}$$

Let μ_i $(i = 1, 2, \ldots, n+1)$ be the roots of the characteristic equation:

$$\begin{vmatrix} (k_{11}-\mu) & 0 & \cdots & k_{1\,(n+1)} \\ 0 & (k_{22}-\mu) & \cdots & k_{2\,(n+1)} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ k_{(n+1)\,1} & k_{(n+1)\,2} & \cdots & (k_{(n+1)\,(n+1)}-\mu) \end{vmatrix} = 0. \tag{4.19}$$

The run of each of the temperature deviations is expressed by a function of the form:

$$\delta_j(t) = \sum_{j=1}^{n+1} \sum_{i=1}^{n+1} D_{ij}\delta_j(0)\exp\mu_i t +$$

$$+ \sum_{i=1}^{n+1} \left[H_{ij} \int_0^t \dot{p}(t)\exp(-\mu_i t)\,dt + J_{ij} \int_0^t Q_e(t)\exp(-\mu_i t)\,dt \right]\exp\mu_i t, \qquad (4.20)$$

where D, H and J are coefficients.

From the form of this equation it can be seen that the state of non-equilibrium existing in the medium at a given moment depends not only on the momentary values of appropriate parameters, but also on all the values taken on by these parameters previously during the whole past; hence any present thermal non-equilibrium state is a function of the history of states of the medium. Taking into account the known initial values of the process it may be also said that any present state of the medium depends – apart from the initial conditions – both on the dynamic properties of the medium itself (represented here by the set of values μ_i) and on the history of external forcings (i.e. their time-run during the whole past).

This dependence on the past history has the character of a fading memory, meaning that the influence of states further in the past is smaller than that of more recent states. The said influence disappears in the limit case of infinitely remote states.

In a similar way the equation (4.11) yields a set of equations with constant coefficients governing the variation of deviations from kinematical equilibrium i.e. velocity differences:

$$\frac{du_m}{dt} = \sum_{j=1}^{n} l_j u_j - v_s\left(\frac{\dot{p}}{w} - P_e\right), \qquad (4.21)$$

$$(\text{for } m = 1, 2, ..., n)$$

where the coefficients are:

$$\left. \begin{array}{l} \text{for} \quad j \neq m \quad l_j = -v_s r_j, \\[2mm] \text{for} \quad j = m \quad l_m = -v_s \dfrac{r_m}{g'_m}. \end{array} \right\} \qquad (4.22)$$

The analysis of the solution of this set shows that the state of kinematical non-equilibrium also displays the property of history-dependence and of fading memory of states existing in the past.

4.3. Relaxation Flows

After the active period of the process is finished i.e. after the forcings have ceased to act, in the general case a relaxation process starts, proceeding asymptotically towards equilibrium.

With the choice of forcings assumed above (pressure variation, external heat supply and mass force) the relaxation process in question becomes an adiabatic-isobaric relaxa-

tion. For determining the run of the deviation parameters during the relaxation period equations (4.17) and (4.21), with the respective forcing values set equal to zero are valid. The influence of "wall friction" is to be disregarded (as an external action upon the medium).

The change of parameters of the medium effected during the whole relaxation period, i.e. until full equilibrium is reached, may be determined by writing down the conservation equations for the actual state of the flow (starting point of the relaxation) and for the fully relaxed equilibrium flow. Making use of the assumption that the values of the deviation parameters are small compared to the values of their respective basic parameters and setting for the adiabatic-isobaric relaxation:

$$p = p_R, \qquad T_s = (T_s)_R, \tag{4.23}$$

the values of the relaxation increments of the flow parameters are obtained as:

$$(\Delta g)_R \equiv g_R - g = \frac{1}{h} \left[c' \sum_m g'_m \delta'_m + c g \delta + \tfrac{1}{2} \sum_m g'_m u_m^2 - \tfrac{1}{2} \left(\sum_m g'_m u_m \right)^2 \right], \tag{4.24}$$

$$\Delta w \equiv w_R - w = - \sum_m g'_m u_m \tag{4.25}$$

and the relaxation increment of the cross-section area required for a mass flow unit of the two-phase medium (reciprocal of the flow density) calculated with the use of (4.14) is:

$$(\Delta F)_R \equiv F_R - F = \frac{v_s}{T_s} \left[\frac{c' T_s}{h} \sum_m g'_m \delta'_m + \left(\frac{c T_s}{h} - 1 \right) g \delta + \frac{g T_s}{w} \sum_m g'_m u_m \right]. \tag{4.26}$$

If, instead of the pressure variation, the variation of cross-section area has been chosen as a forcing function, another kind of relaxation process is to be considered: the adiabatic constant-cross-section relaxation, producing respective values of pressure, temperature, velocity and mass contents increments.

5. Entropy Production and Exergy Losses

5.1. The Entropy Sources

In a multiphase medium flow the irreversibilities are caused both by boundary layer dissipation and by non-equilibrium interaction between the phases, the latter cause being specific to heterogeneous media. Basing on the principles of non-equilibrium thermodynamics (e.g. [2]) the problems connected with two-phase flow irreversibilities will be treated here after the methods suggested in ([9, 11]).

The Gibbs equations for the gas phase and each of the drop fractions (written in reference to the mass flux unit of the medium) are:

$$\left. \begin{aligned} d(gi) &= gv\,dp + T\,d(gs) + (i - Ts)\,dg, \\ d(g'i')_m &= (g'i')_m\,dp + T'_m\,d(g's')_m + (i' - T's')_m\,dg'_m, \end{aligned} \right\} \tag{5.1}$$

$$\text{for } m = 1, 2, \dots, n.$$

With the use of the energy conservation equations for each sub-system and of the expressions for inter-phase actions given in chapter 3 the following form is obtained:

$$T\frac{d}{dt}(gs)=\sum_m f_m[e_m-y_m(i-Ts)]-gv\sum_m P_m u_m\left(1-\frac{u_m}{w}\right)+gvQ_e,$$

$$T'_m\frac{d}{dt}(g's')_m=f_m[y_m(i'-T's')_m-e_m] \tag{5.2}$$

$$\text{for } m=1,2,...,n.$$

The rate of entropy increase per mass flux unit of the two-phase medium is:

$$\frac{d\overline{s}}{dt}=\frac{d}{dt}(gs)+\sum_m\frac{d}{dt}(g's')_m. \tag{5.3}$$

Taking into account that after eq. (4.15) the heat supply consists of the wall dissipation heat and of the heat supplied from outside of the channel, the entropy increase rate may be expressed as:

$$\frac{d\overline{s}}{dt}=\omega+\frac{gv}{T}Q_d+\frac{gv}{T}Q_{ee}. \tag{5.4}$$

Here the first term on the right-hand side denotes the entropy source (per mass flux unit of the medium). i.e. the entropy production rate due to non-equilibrium interaction between the phases whilst the second term is caused by the dissipation in the boundary layer ("wall friction"). These two terms are responsible for the entropy production in the flow thus defining its irreversibility. The third term is the entropy supplied to the flow along with the heat supply from outside.

The entropy source caused by the interaction between the gas phase and the droplets of the m-th fraction is:

$$\omega_m=f_m\left[e_m\left(\frac{1}{T}-\frac{1}{T'_m}\right)-y_m\left(\frac{i-Ts}{T}-\frac{i'_m-T'_m s'_m}{T'_m}\right)\right]-\frac{gv}{T}P_m u_m\left(1-\frac{u_m}{w}\right). \tag{5.5}$$

After some rearrangements and rejecting small quantities of higher orders this expression takes the form:

$$\omega_m=\frac{f_m}{T_s^2}[e_m(\delta'_m-\delta)-y_m(i'_s\delta'_m-i_s\delta)]-\frac{gv_s}{T_s}P_m u_m. \tag{5.6}$$

As the quantities e, y and P are linear functions of the deviation values, the entropy source becomes a quadratic function of the deviations from equilibrium; the terms describing the entropy production due to the coupled mass- and heat transfer phenomena are separated from the entropy production term caused by momentum exchange. The minus sign before the last term is due to the assumed notation (eq. (3.13)).

The entropy source caused by the interaction between the gaseous phase and all the fractions of droplets is

$$\omega=\sum_m\omega_m \tag{5.7}$$

whilst the corresponding quantity of entropy source per unit volume (as used in the methods of thermodynamics of irreversible processes) is after linearization

$$\sigma = \frac{1}{gv} \sum_m g'_m \omega_m .$$ (5.8)

With the use of eq. (5.4), the entropy increment due to irreversibilities in the two-phase flow (interaction between the phases and "wall friction") during a limited period of time can be written as:

$$\Delta \bar{s} = \int_0^t \omega(t)\,dt + \int_0^t \frac{g(t)v(t)}{T(t)} Q_d(t)\,dt .$$ (5.9)

5.2. The Adiabatic-Isobaric Relaxation Entropy Increase

Principally the value of the relaxation entropy increment may be calculated by means of determining the run of the deviation parameters during the relaxation process according to the eqs. (4.17) and (4.21), and subsequently applying the defining formula:

$$\Delta \bar{s}_R = \int_{t_R}^\infty \omega(t)\,dt .$$ (5.10)

Taking into account the assumption that the relative values of the deviation parameters are small, the relaxation entropy increment (effected during the whole relaxation period) may be obtained directly. Defining the adiabatic-isobaric entropy increment per mass flux
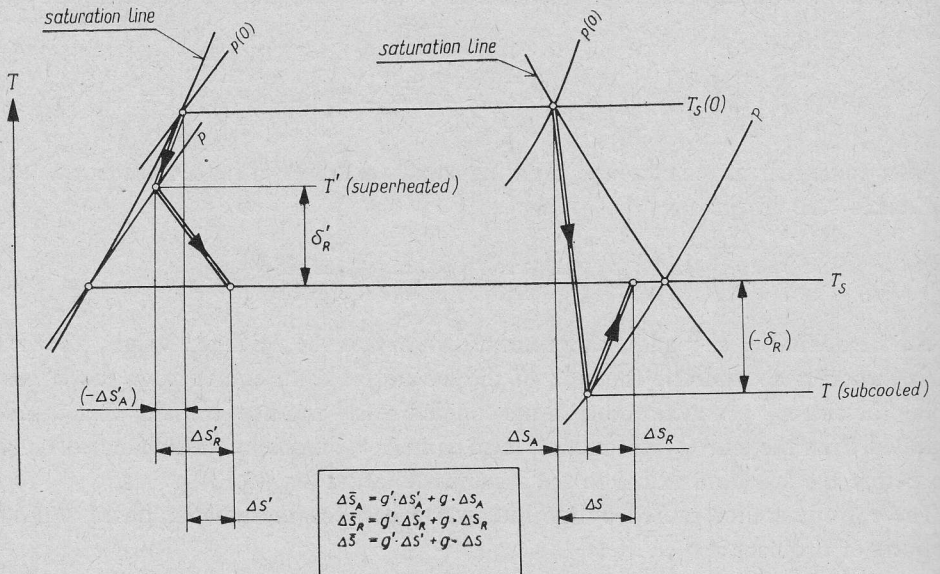

$$\Delta \bar{s}_A = g' \cdot \Delta s'_A + g \cdot \Delta s_A$$
$$\Delta \bar{s}_R = g' \cdot \Delta s'_R + g \cdot \Delta s_R$$
$$\Delta \bar{s} = g' \cdot \Delta s' + g \cdot \Delta s$$

Fig. 1. Adiabatic expansion of wet steam from an initial equilibrium state (scheme)

unit of the medium as:

$$\Delta \bar{s}_R = (g's'_s)_R + (gs_s)_R - \sum_m g'_m s'_m - gs \tag{5.11}$$

and making use of eq. (4.24), the value of the relaxation entropy increment is:

$$\Delta \bar{s}_R = \frac{1}{2T_s^2} \left[cg\delta^2 + c' \sum_m g'_m \delta'^2_m + T_s \sum_m g'_m u^2 - T_s (\sum_m g'_m u_m)^2 \right]_R. \tag{5.12}$$

The total entropy increment of the process consists of the increment arisen during the active period of the process (calculated after eq. (5.9)) and of the relaxation increment

$$\Delta \bar{s} = (\Delta \bar{s})_A + (\Delta \bar{s})_R. \tag{5.13}$$

The basic relations for an elementary adiabatic expansion of wet steam from an initial equilibrium state are depicted schematically in Fig. 1.

5.3. The Concept of the "Predicted Entropy Increments" for the Description of Irreversible Processes

As any thermal process is caused by the action of forcings which evokes non-equilibrium states in the medium, the occurrence of both entropy increments — the active and the relaxational one — should also be considered as a result of the action of the forcings. The relaxation increment remains "latent" at the time when the action of the forcings stops and its value is defined by the then existing values of deviations from equilibrium. In fact, to any non-equilibrium state of the medium, an appropriate value of relaxation entropy increment may be attached (calculated after eq. (5.12)) which will become real if the forcings stop to act at that very moment.

In the author's previous papers ([9, 11]) there has been introduced a concept of a pre-dicted entropy increment representing the total entropy increase which consists of the "active" increment actually occurring within the considered time interval and the associated change of the relaxation increment $\Delta \bar{s}_R$, which is also caused by the action of forcings over the said time interval but which will become physically effective afterwards. Hence an infinitesimal change of predicted entropy will be expressed as:

$$d\bar{s}^* = \omega dt + \frac{d}{dt}(\Delta \bar{s}_R) dt. \tag{5.14}$$

Consequently, it can be assumed that there exists a source of "predicted entropy" which is acting in the medium:

$$\omega^* = \omega + \frac{d}{dt}(\Delta \bar{s}_R). \tag{5.15}$$

In the above equations only irreversibilities caused by interphase transfer have been taken into account, disregarding the "wall friction" influence. As both influences may be regarded to be additive, this simplification is not essential for further developments.

For the relaxation process there is, evidently:
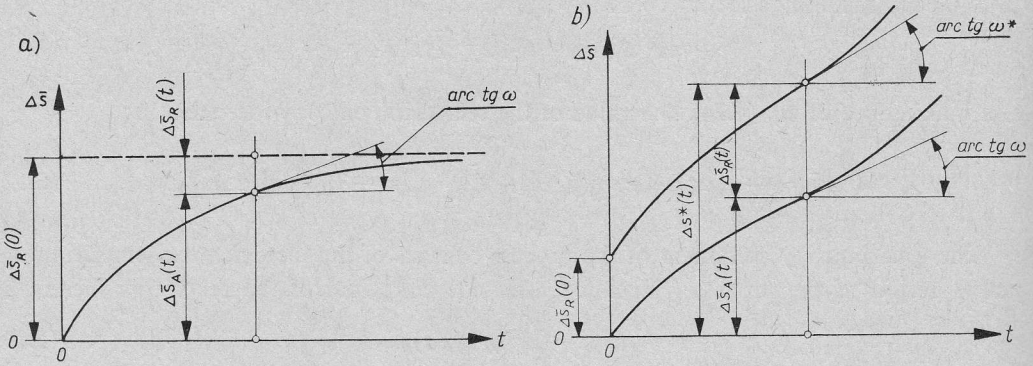
$$\omega^*_R = 0. \tag{5.16}$$

Fig. 2. Variation of entropy increments during the process (schematic diagram); a) relaxation process, b) active part of the process (general case)

If the active period of a process lasts from $t=0$ to t, the following relation is valid:

$$\int\limits_0^t \omega^*(t)\,dt = \int\limits_0^\infty \omega(t)\,dt. \tag{5.17}$$

The above considered relations are pictured in Fig. 2.

On the basis of the defining eq. (5.15), making use of (5.6) and (5.12) and of the equations governing the transfer processes and the run of deviation parameters derived in chapters 3 and 4, the source of predicted entropy can be expressed – after making the necessary transformations – as a sum of products of the deviations from equilibrium times the respective steering functions:

$$\omega^*(t) = \Psi(t)\cdot\delta(t) + \sum_m \Psi'_m(t)\cdot\delta'_m(t) + \sum_m \Psi''_m(t)\cdot u_m(t), \tag{5.18}$$

where the steering functions of the forcings associated with the respective deviations from equilibrium are:

$$\left.\begin{aligned}
\Psi(t) &= \frac{gv_s}{T_s^2}\left[\left(1 - \frac{cT_s}{h}\right)\dot{p}(t) + Q_e(t)\right], \\[2mm]
\Psi'_m(t) &= -\frac{c'v_s}{hT_s}\,g'_m\,\dot{p}(t), \\[2mm]
\Psi''_m(t) &= -\frac{gv_s}{T_s}\,g'_m\left[\frac{1}{w}\,\dot{p}(t) - P_e(t)\right].
\end{aligned}\right\} \tag{5.19}$$

With the help of the relations from chapter 4 it may be easily proved that the runs of both active entropy source ω and of the predicted entropy source ω^* are functionals of the run of the forcings and of the dynamic properties of the medium. Thus both kinds of entropy sources depend on the history of states or respective on the history of forcings; this dependence has the character of a fading memory.

Using the concept of predicted entropy increments the total losses arising in the medium may be analyzed on the basis of the time-run of the forcings. The total entropy increment

(eq. (5.13)) may be presented as:

$$\Delta\bar{s} \equiv \Delta\bar{s}_A + \Delta\bar{s}_R = \int_0^t \omega^*(t)\,dt.$$ (5.20)

5.4. Exergy Losses

The losses of exergy occurring in the medium due to irreversible interaction between the phases may be written down as:

$$\Delta\bar{b} = T_z\Delta\bar{s}$$ (5.21)

and respectively:

$$\Delta\bar{b}_A = T_z\Delta\bar{s}_A, \qquad \Delta\bar{b}_R = T_z\Delta\bar{s}_R,$$ (5.22)

where T_z is the surroundings temperature.

For calculations of adiabatic expansion-or compression processes starting from an equilibrium state, exergy loss coefficients defined as the ratio of exergy loss to the isentropic enthalpy drop of an ideal reference process (measured in the direction of the process-run) shall be introduced:

$$\zeta_A = \frac{\Delta\bar{b}_A}{\Delta\bar{i}_s}, \qquad \zeta_R = \frac{\Delta\bar{b}_R}{\Delta\bar{i}_s},$$ (5.23)

$$\zeta = \zeta_A + \zeta_R.$$ (5.24)

5.5. Results of Exergy Loss Calculations

A series of calculations of a non-equilibrium adiabatic expansion of wet steam (starting from an initial equilibrium state) have been carried out; the forcing has been assumed to be a linear variation of pressure. A typical run of temperature deviations and entropy values in such a case is shown in Fig. 3.

The initial state of the medium (H_2O) at rest has been assumed as:

$$p(0) = 0.1 \text{ bar}, \qquad g'(0) = 0.1$$

and the expansion of this medium in a guide vane of the channel length of 200 mm has been calculated assuming for each of the computational series subsequently a different size of droplets of which the liquid phase is composed and repeating the calculations for various values of the expansion rate.

The results are shown in Fig. 4, where the exergy loss coefficient (showing the total loss caused by internal irreversibilities of the two-phase medium) has been plotted as a function of the expansion rates and of the droplet radii; it may be seen that these two parameters have a substantial influence on the value of the loss. The non-equilibrium exergy loss calculated here is additive to other losses occurring in the flow.
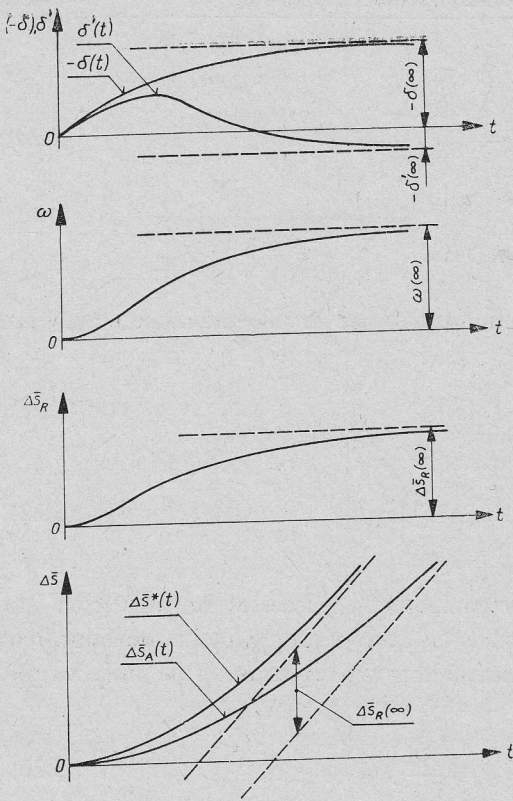
Fig. 3. Typical run of the values of temperature deviations, entropy sources and entropy increments during an expansion of wet steam (scheme)
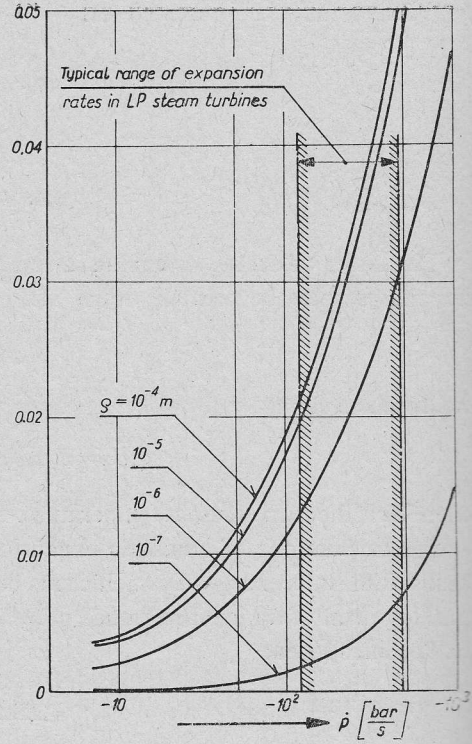
Fig. 4. Calculated values of exergy loss coefficients for wet steam flow through LP turbine nozzles as a function of the expansion rate and of the droplet's radii
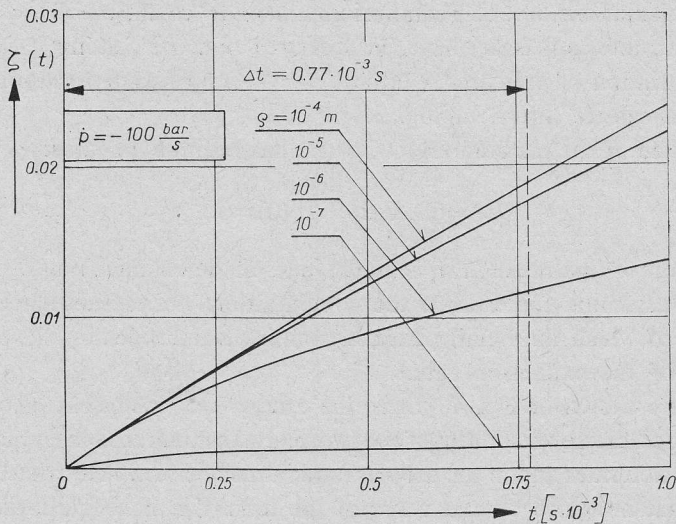

Fig. 5. Values of the local exergy loss coefficient for an expansion rate $\dot{p} = -100$ bar/s
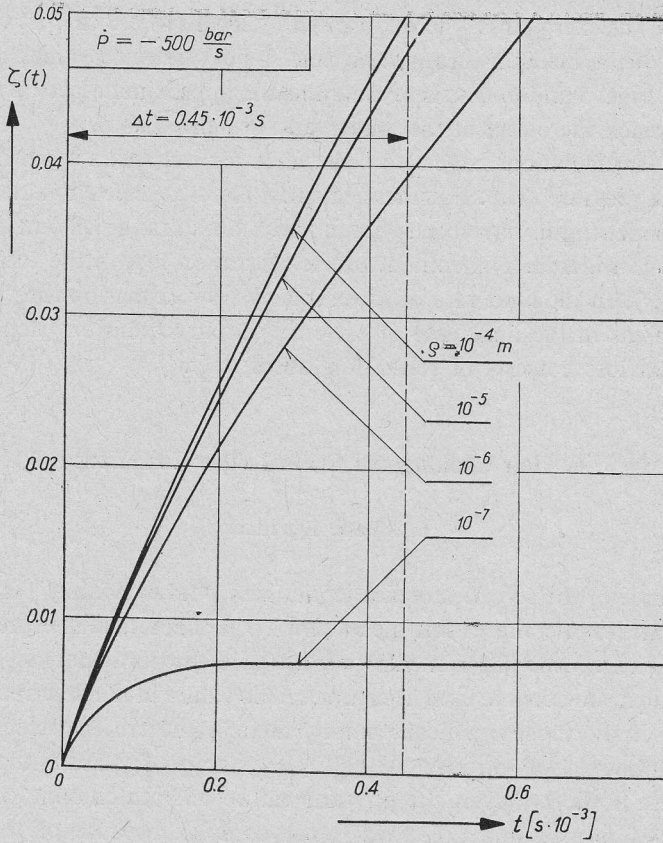
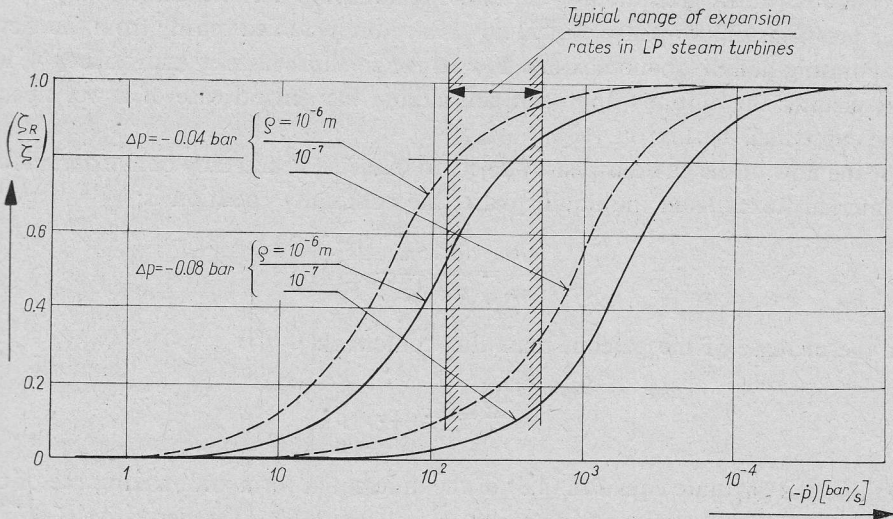Fig. 6. Values of the local exergy loss coefficient for an expansion rate $\dot{p} = -500$ bar/s

Fig. 7. The content of the relaxation loss in the total exergy loss as a function of the expansion rate

The growth of the exergy loss — here expressed by means of a local loss coefficient $\zeta(t)$ — during the flow of the medium through the duct for two selected values of the expansion rate is shown in Figs. 5 and 6. On these diagrams the approximate period of time required for the flow to reach the outlet of the guide vane is marked as Δt.

Other series of calculations have also been made for wet steam expansion of a definite magnitude of the pressure drop. For these calculations, two selected values of Δp and two values of the droplet radius have been chosen and a broad range of variation of expansion rates have been considered. The content of the relaxation loss in the total loss of exergy is shown on Fig. 7. In the limit case of an extremely slow expansion the value of this content is zero whereas in the limit case of extremely rapid expansion (step function of the forcing) the total loss consists of relaxation losses only.

6. Flow Choking and Critical Flow Conditions

6.1. Basic Equations

The appearance of the so called critical conditions ("flow choking") is one of the most characteristic features of compressible media flows. The theoretical interpretation of critical flows in homogeneous one-phase media (perfect gases in particular) is generally known and their critical parameters are used as characteristic values in the description of simple gas dynamics. One of the most significant among them is the critical velocity which in gas flows is in a well known way directly related to the velocity of sound in the medium.

In multiphase flows,. however, the physical nature of both choking conditions arising and sound waves propagation are much more complex because of the heterogeneous structure of the medium and of the internal non-equilibrium interaction between the phases. As these internal processes influence flow choking and sound propagation in different ways, there is (in the general case) no direct relationship between these two phenomena.

The problems of two-phase critical flows have been studied mostly from the viewpoint of evaporating liquid-vapour mixture flows through short nozzles and orifices [4, 14, 16]. In this chapter the author's approach considering the general case of a wet steam flow will be presented, basing on [13].

For the flow of the gaseous phase subject to influences caused by both external forcings and internal fluxes from the liquid phase, the continuity equation is:

$$\frac{dw}{w} = \frac{dg}{g} + \frac{dv}{v} - \frac{dF}{F}, \tag{6.1}$$

where the increase of the gaseous mass flux content is:

$$\frac{dg}{g} = \frac{vY}{w}dx. \tag{6.2}$$

Writing the gas state equation (4.3) in the differential form and making use of the gas phase conservation equations (4.1) yield — after eliminating the derivatives (dT/dx) and (dp/dx) — the expression for the specific volume increase. With the use of (6.1) and (6.2)

the following equation governing the flow conditions variation is obtained:

$$\left(1-\frac{w^2}{a_\infty^2}\right)\frac{dw}{w}=-\frac{dF}{F}+\left[Y+\frac{\kappa-1}{a_\infty^2}(Q+Q_e)\right]\frac{vdx}{w}-(P+P_e)\frac{vdx}{a_\infty^2},\tag{6.3}$$

where

$$a_\infty=\sqrt{\kappa p v}\tag{6.4}$$

is the sound velocity in the gaseous phase considered approximately as a perfect gas; it may also be regarded as the propagation velocity of sound waves of infinitely high frequency in the two-phase medium (owing to the freezing-up of interphase exchange processes at high time rates of pressure forcings i.e., at high frequencies).

6.2. Critical Conditions

The critical conditions of the two-phase flow (with the assumptions already made) will be defined as a situation at which the mass flow density of the two-phase medium takes on an extremal value whilst the derivative of the gas phase velocity is not equal to zero i.e.

$$F_K\equiv F(x_K)=\text{extr.},\qquad\left(\frac{dw}{dx}\right)_K\neq0.\tag{6.5}$$

The cross-section of the duct where the critical conditions occur will be called the critical cross-section and the values of the flow parameters existing at this cross-section – the critical values of the respective parameters.

Besides the definition (6.5) another characteristic situation of the flow where the mass flow density of the gaseous component only reaches its extremal value may be of interest. Thus the definition of the gas phase critical conditions is:

$$\left(\frac{F}{g}\right)_{Kg}\equiv\frac{F(x_{Kg})}{g(x_{Kg})}=\text{extr.},\qquad\left(\frac{dw}{dx}\right)_{Kg}\neq0.\tag{6.6}$$

According to these definitions the locations of both critical cross-sections of the duct, the respective critical values of the flow parameters, and of the critical flow-rate of the medium are to be determined.

It may be easily shown (cp. [13]) that the equation (6.3) can be written in the form:

$$\left(1-\frac{w^2}{w_K^2}\right)\frac{dw}{w}=-\frac{dF}{F}\tag{6.7}$$

using the quantity w_K defined as:

$$w_K^2=a_\infty^2\frac{1-\dfrac{P+Pe}{\dot p}w}{1-\dfrac{(\kappa-1)(Q+Q_e)+a_\infty^2\,Y}{\dot p}}.\tag{6.8}$$

As can be seen from eq. (6.7), the criterion (6.5) of the critical state is satisfied when $w = w_K$; thus w_K is the critical velocity with respect to the two-phase flow choking conditions.

In a similar way, the critical velocity after the criterion (6.6) — (choking of the gaseous component flow) can be obtained. For this purpose the eq. (6.3) has to be slightly rearranged making use of (6.2) and then written down in the form:

$$\left(1 - \frac{w^2}{w_{Kg}^2}\right)\frac{dw}{w} = \frac{dg}{g} - \frac{dF}{F} \tag{6.9}$$

with the critical velocity respective to the criterion (6.6) defined by:

$$w_{Kg}^2 = a_\infty^2 \frac{1 - \dfrac{P + Pe}{\dot{p}} w}{1 - \dfrac{(\kappa - 1)(Q + Q_e)}{\dot{p}}}. \tag{6.10}$$
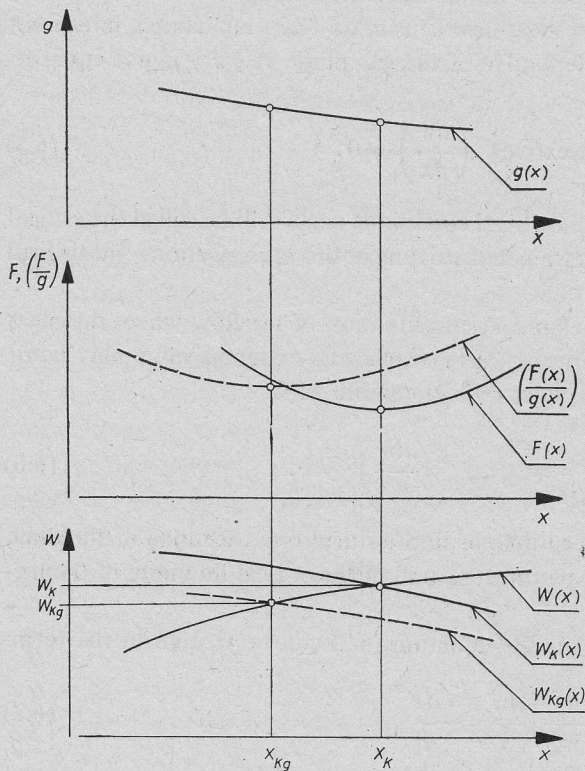

Fig. 8. Determination of critical conditions and the position of critical cross-sections (scheme)

Taking into account that at the critical cross-section

$$w = w_K \quad \text{respective} \quad w = w_{Kg} \tag{6.11}$$

the expressions for the respective critical velocities may be written down as:

$$\frac{w_K}{a_\infty} = \frac{-a_\infty(P+P_e) \pm \sqrt{a_\infty^2(P+P_e)^2 + 4\dot{p}[\dot{p}-(\kappa-1)(Q+Q_e)-a_\infty^2 Y]}}{2[\dot{}-(\kappa-1)(Q+Q_e)-a_\infty^2 Y]}, \qquad (6.12)$$

$$\frac{w_{Kg}}{a_\infty} = \frac{-a_\infty(P+P_e) \pm \sqrt{a_\infty^2(P+P_e)^2 + 4\dot{p}[\dot{p}-(\kappa-1)(Q+Q_e)]}}{2[\dot{p}-(\kappa-1)(Q+Q_e)]}. \qquad (6.13)$$

For flows in the positive direction of the co-ordinate axis the "plus" sign is valid.

As the quantities Y, Q and P are linear functions of the deviations from equilibrium, the critical velocities may be expressed as depending on the values of the deviation parameters and on the run of the forcing functions:

$$\left. \begin{array}{l} w_K = w_K(\delta, \delta'_m, u_m, \dot{p}, Q_e, P_e), \\ w_{Kg} = w_{Kg}(\delta, \delta'_m, u_m, \dot{p}, Q_e, P_e). \end{array} \right\} \qquad (6.14)$$

The relations defining the critical flow conditions and the computational method of finding the location of the critical cross-section are shown schematically in Fig. 8. Using suitable step-by-step procedures the run of the flow parameters is calculated and from their values the quantities w_K and w_{Kg} are determined for each step; the position of the critical cross-section is distinguished by satisfying the condition (6.11). In adiabatic expansions of two-phase media the critical conditions of the gaseous phase flow (defined by eq. (6.6)) occur always before the critical cross-section of the two-phase medium flow (eq. (6.5)).

It should be noted that according to the relations (6.14) and to previous considerations the values of the critical velocity and critical parameters as well as the location of the critical cross-sections also depend on the history of the flow displaying the property of "fading memory" of past states.

6.3. Limit Cases and Relations to Sound Velocity

There exist two important limit cases of two-phase medium expansion:

a. In the case of an extremely rapid expansion all the interactions between the phases are "frozen" and, consequently, the expansion process obeys exactly the laws of pure gas expansion with its critical velocity equal to the sound velocity in the gas (provided that the influences of heat supply and wall friction have been excluded).

$$w_K = w_{Kg} = a_f \quad \text{for} \quad \dot{p} \to (-\infty). \qquad (6.15)$$

b. In the case of extremely slow expansion the medium remains in the state of internal equilibrium during the whole process and the interphase fluxes have to be derived after the appropriate "thermostatic" relations.

The results of these derivations carried out in [10, 12, 13] show that in this case the critical velocity is equal to the velocity of sound waves of extremely small frequency in the two-phase medium;

$$w_K = a_s \quad \text{for} \quad \dot{p} \to 0, \qquad (6.16)$$

where

$$a_s = \sqrt{\dfrac{-gh}{2 - \dfrac{h}{RT_s} - \dfrac{T_s}{h}\left(c + \dfrac{g'}{g}c'\right)}} \qquad (6.17)$$

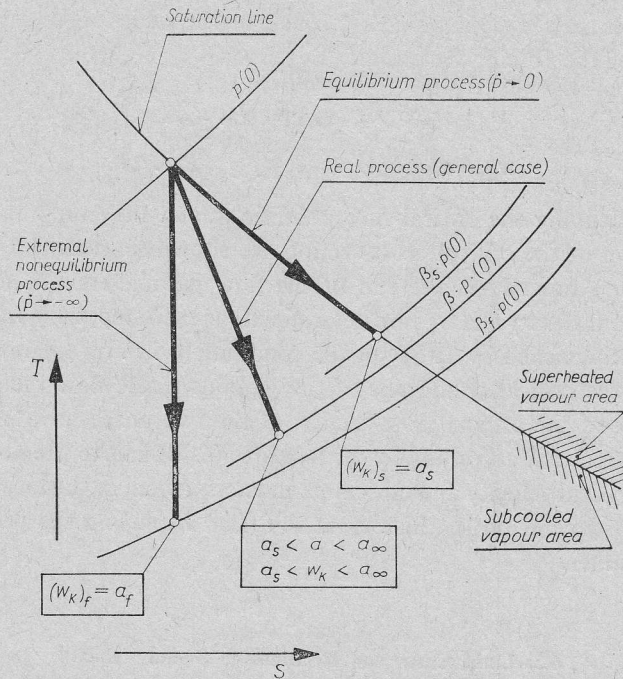is the "zero-frequency" sound velocity in a saturated wet steam.

Fig. 9. Critical expansion of the gaseous phase of a two-phase medium from an initial equilibrium state (scheme)

However in the general case, as it has been derived in the author's previous paper [12], the sound velocity in a two-phase medium depends on the frequency of the sound (dispersion of sound velocity). Although both the sound velocity and the critical velocity are influenced by the irreversible interphase processes, there is no direct relation between these two velocities. Therefore for the general case:

$$a_s \leqslant w_K \leqslant a_\infty, \qquad a_{sg} \leqslant w_{Kg} \leqslant a_\infty, \qquad (6.18)$$

and

$$a_s \leqslant a \leqslant a_\infty \qquad (6.19)$$

whilst for the limit cases, eq. (6.15) and (6.16) are valid.

In Fig. 9 the expansion of the gaseous phase of the two-phase medium (starting from an equilibrium state and proceeding, generally, into the subcooled vapour area) has been shown on a temperature-entropy diagram.

6.4. Calculations of the Critical Parameters

The values of the critical parameters — the critical velocity and the critical pressure ratio — for various cases of adiabatic expansion of wet steam have been calculated. The parameters of the initial equilibrium state of the medium at rest (zero initial velocity) have been chosen as in the flow calculations of the preceding chapter 5 and the variation of pressure has been assumed to be a linear function of time. Three separate series of calculations have been carried out, each one based on the assumption of a different value of the liquid drop radius. The purpose of the calculations has been to investigate the influence of the expansion rate and of the heterogeneous structure of the medium (represented by the values of the drop radii) on the values of the critical parameters.

For the presentation of the results two dimensionless quantities have been introduced. One of them is the ratio of the critical velocity to the velocity of sound waves of extremely high frequency in the medium (or to the sound velocity of the gas phase only) at the actual thermal state in the critical cross-section

$$\alpha_K = \frac{w_K}{a_\infty}, \qquad \alpha_{Kg} = \frac{w_{Kg}}{a_\infty} , \tag{6.20}$$
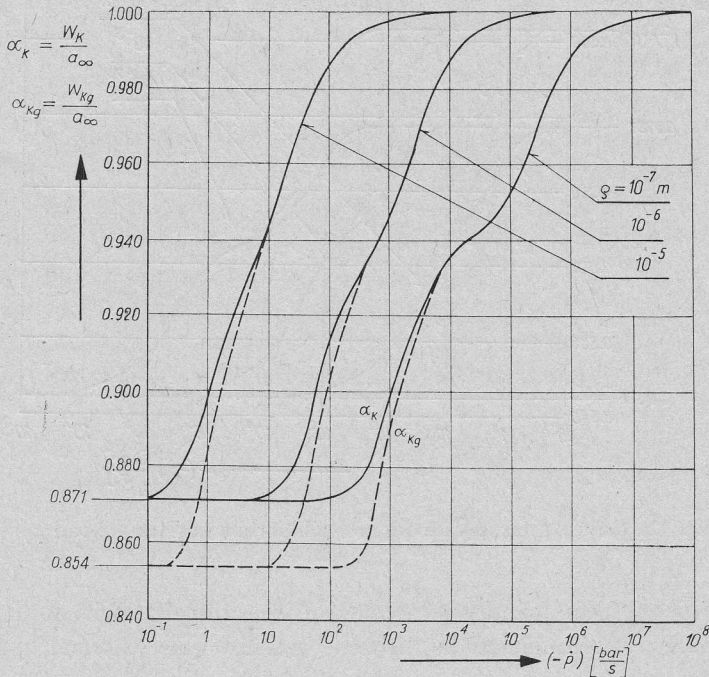
where a_∞ is defined by eq. (6.4).

Fig. 10. Dimensionless critical velocities (related to the maximal sound velocity in the critical cross-section) for wet steam expansion
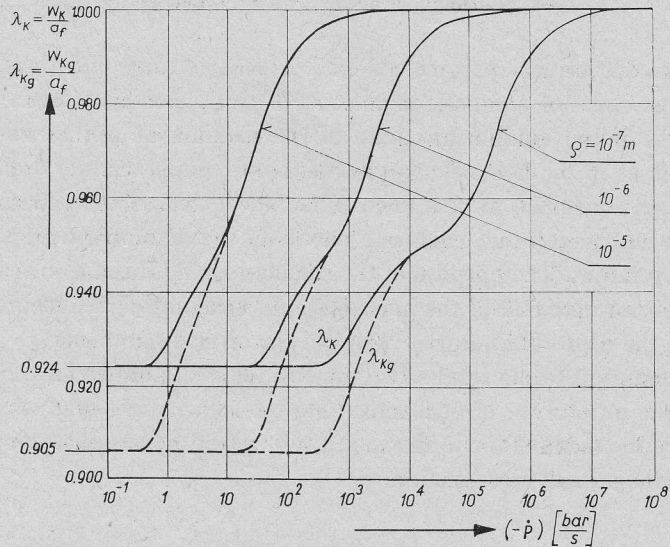
Fig. 11. Dimensionless critical velocities (related to the sound velocity in a "frozen" gas flow) for wet steam expansion
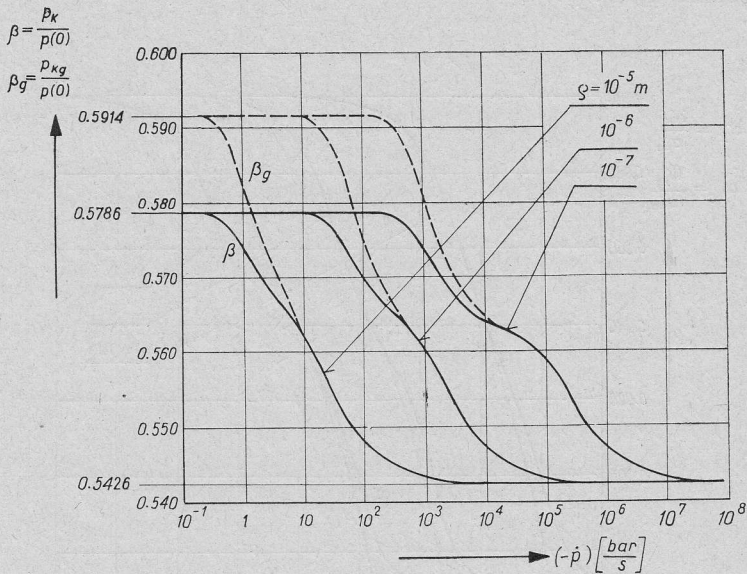

Fig. 12. Critical pressure ratio for non-equilibrium wet steam expansion

The other dimensionless quantity is the ratio of the critical velocity to the sound velocity in the critical cross-section of the "frozen" gas flow (the extremal non-equilibrium process):

$$\lambda_K = \frac{w_K}{a_f}, \qquad \lambda_{Kg} = \frac{w_{Kg}}{a_f}, \tag{6.21}$$

where

$$a_f = \sqrt{\frac{2\kappa}{\kappa+1}} \, RT(0).$$ (6.22)

On the basis of the obtained values the critical pressure ratio has been calculated as:

$$\beta = \frac{p_K}{p(0)}, \qquad \beta_g = \frac{p_{Kg}}{p(0)}.$$ (6.23)

The results have been presented in Figs. 10, 11 and 12 as functions of the expansion rate and of the assumed droplet radius. In these diagrams the functions referring to the critical conditions of the two-phase flow (definition 6.5) are shown by full lines whilst the broken lines depict critical conditions for the gaseous content of the flow.

7. The Direct Method of Flow Calculation

There are, generally speaking, two kinds of problems in flow calculation:

a. The problem of dimensioning the duct, i.e., defining the sequence of cross-section areas for a given variation of pressure and of other forcings along the path co-ordinate.

b. The reverse problem consisting in the calculation of the run of all the flow parameters along a duct of given dimensions, i.e., of a given variation of cross-section areas.

The way of solving the first problem (the "straight" one) follows immediately from the equations derived in chapter 4. With the known parameters of the flow entering the channel (including an assumption on the structure of the liquid phase, i.e., the drop radii) the set of equations (4.7) - (4.13) describes the run of the flow parameters along the duct and eq., (4.14) yields the associated variation of cross-section areas per mass flow unit of the medium. For computation a simple step-by-step method may be employed. The exergy loss for each step is most conveniently to be calculated after the "predicted entropy" increments (so that for the whole duct only a simple summation of the values from each step is necessary without adding a relaxation value at the outlet).

In the case of the reverse problem the given variation $F(x)$ has to be treated as one of the forcings, replacing the variation of pressure; the other forcings remain the same as in the preceding case. The set of equations (4.7) - (4.13) then has to be rearranged by substituting:

$$\dot{p} = \frac{Rg}{Fw} \frac{dT_s}{dt} + \frac{Rg}{Fw} \frac{d\delta}{dt} - \frac{RT_s}{Fw} \sum_m \frac{dg'_m}{dt} - \frac{gRT_s}{Fw^2} \frac{dw}{dt} - \frac{gRT_s}{F^2 w} \frac{dF}{dt}$$ (7.1)

with the help of the relations (3.5) and (4.6).

The course of the computation of the flow parameters and of the exergy losses follows the pattern of the preceding case (the appropriate procedure has been described in details in [10]).

It should be noted that for more precise calculations the values of the coefficients appearing in the equations have to be treated as dependent on the thermal state of the medium; consequently an adjusting procedure has to be built in into the computation of each step.

8. The Indirect Method of Flow Calculation

8.1. Description of the Method

In the author's previous paper [10] an indirect calculation procedure which is more convenient particularly in cases when similar calculations have to be repeated for various assumptions has been developed. The method consists in splitting up the whole procedure into two stages, the first of them being the calculation of an equilibrium process whilst the second one takes account of the non-equilibrium effects and their influence.

With the forcings given as the pressure variation, external heat supply and mass force action (the "straight" problem), the first stage of the procedure consists in calculating the run of parameters for a full-equilibrium saturated wet steam flow including the influence of energy dissipation in the boundary layer ("wall friction") which has been treated as an external forcing upon the medium. The appropriate set of equations for a step-by-step calculation is:

$$\frac{dg'_m}{dx} = \frac{-1}{\sum\limits_{j=1}^{n} \frac{g'_j}{\rho_j}} \frac{g'_m}{\rho_m} \frac{gv_s}{hw} \left[\left(1 - \frac{cT_s}{h} - \frac{g'}{g} \frac{c'T_s}{h} \right) \dot{p} + Q_e \right], \tag{8.1}$$

$$\frac{dT_s}{dx} = \frac{v_s T_s}{hw} \dot{p}, \tag{8.2}$$

$$\frac{dw}{dx} = -\frac{gv_s}{w} \left(\frac{1}{w} \dot{p} - P_e \right). \tag{8.3}$$

The resulting courses of these parameters along the path coordinate are to be used in the second stage of the procedure as "leading values" (with the subscript "zero"), after which the coefficients in the dynamic equations and the physical constants will be determined; they will also serve as a first approximation of their respective values:

$$p_0(x), \quad T_{s0}(x), \quad g'_{m0}(x), \quad w_0(x). \tag{8.4}$$

In the second stage the run of the deviations from equilibrium:

$$\delta(x), \quad \delta'_m(x), \quad u_m(x) \tag{8.5}$$

is calculated on the basis of the previously determined functions of the leading values (8.4). The calculation of the deviation parameters is carried out by means of the sets of equations (4.17) and (4.21), using them in such a manner as if the pressure variation $p_0(x)$ were the only forcing acting upon the medium; the influence of the remaining external forcings has already been accounted for indirectly through their effect on the leading values determined in the first stage of the procedure.

After determining the deviation values at a cross-section, the values of the remaining flow parameters (previously expressed by their leading values calculated on the assumption of a saturated wet steam flow) have to be adjusted to the non-equilibrium situation. In order to do so, it is considered that the non-equilibrium flow parameters actually prevailing

at a cross-section are related to the respective leading values of the equilibrium flow by satisfying the appropriate conservation equations; this is the same relationship which exists between the actual values of the flow parameters and their values in an associated relaxed flow. Consequently, with the use of eq. (4.24) and (4.25) the remaining values of flow parameters are:

$$w = w_0 - \Delta w_R, \qquad g = g_0 - \Delta g_R, \atop p = p_0, \qquad T_s = T_{s0}. \qquad \Bigg\}$$ (8.6)

If further approximations are needed this part of the procedure may be repeated using the obtained values in turn as the new leading values for the next approximation.
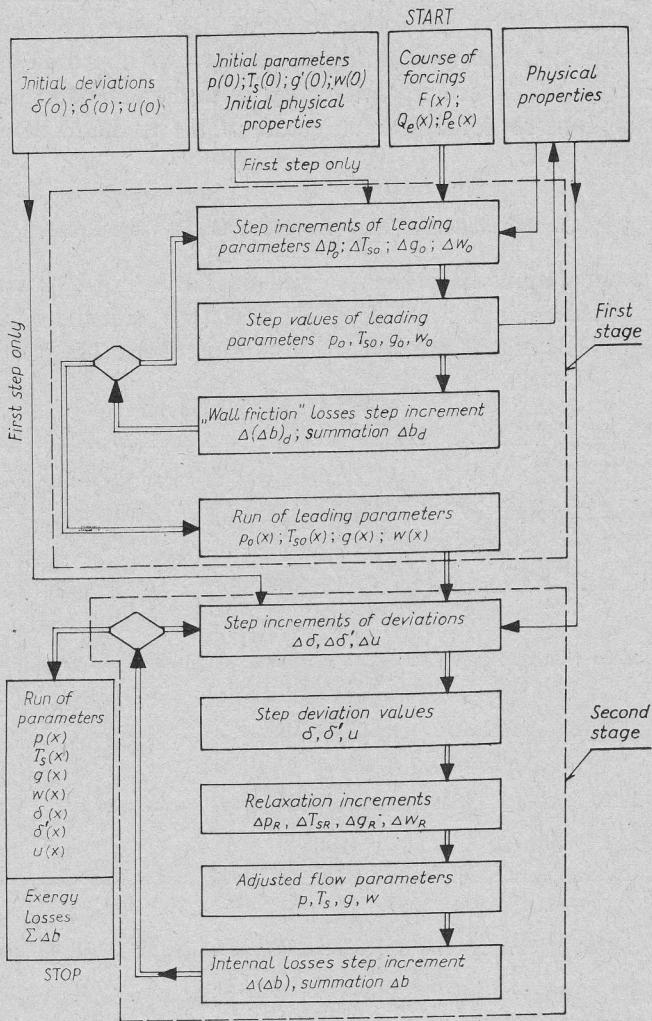

Fig. 13. Scheme of the procedure of the indirect calculation method for the reverse problem of flow calculation

Finally, at each step the required cross-section area per unit mass flux is determined after (4.14) as well as the increment of the exergy loss (calculated on the basis of the predicted entropy change to avoid the addition of a separate relaxation loss at the outlet); the exergy loss increments are subsequently summed up over the whole length of the duct.

If instead of the pressure variation the variation of the cross-section area is prescribed (the reverse problem), the first stage consists in determining the run of the leading parameters (8.4) under given conditions. In the second stage of the procedure $p_0(x)$ is treated as the only forcing and the calculations are carried out as in the "straight" problem with the only one difference being that instead of (8.6) the adjustment of flow parameters has to employ relaxation increments derived on the basis of adiabatic constant-cross-section relaxation (with $p \neq p_0$). The method of calculation of the reverse problem has been described in detail in [10]. The course of the calculations is shown schematically in Fig. 13.

The indirect method of flow calculation has the advantage of a procedure simplification without a decrease in exactness (which may be additionally improved by further approximations if necessary). It is particularly convenient when the same duct has to be calculated for various assumptions concerning the structure of the medium, the droplet radii, the transfer coefficients etc.

8.2. Calculation of Flow Around a Guide Vane Profile

A series of computations have been carried out for a non-equilibrium flow of wet steam around a profile of a LP-turbine guide vane type with the outline length on the convex and concave side of 290 mm and 250 mm respectively. The parameters of the wet
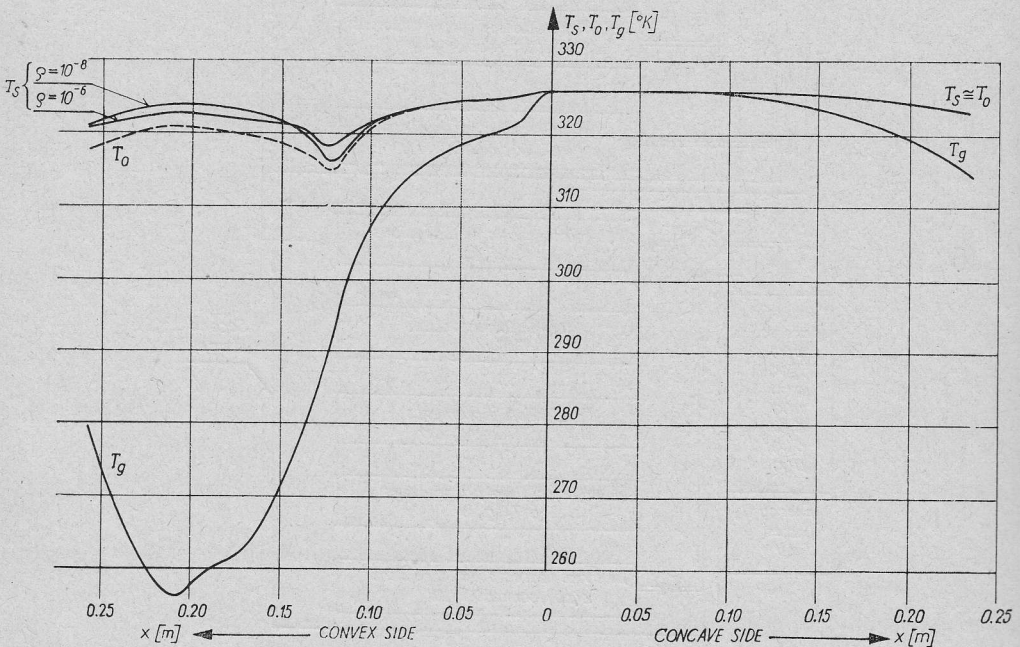

Fig. 14. Distribution of the gas flow temperature (T_g), leading temperature (T_0) and actual saturation temperature (T_s) for a non-equilibrium wet steam flow around the profile
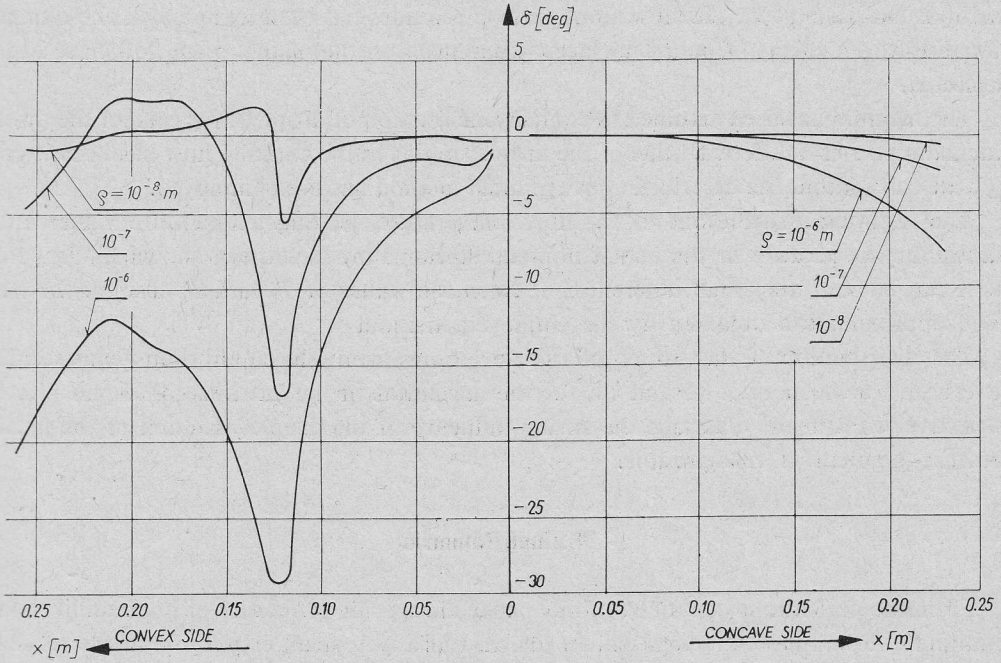
Fig. 15. Distribution of the deviation from the equilibrium temperature in the gaseous phase for wet steam flow around the profile
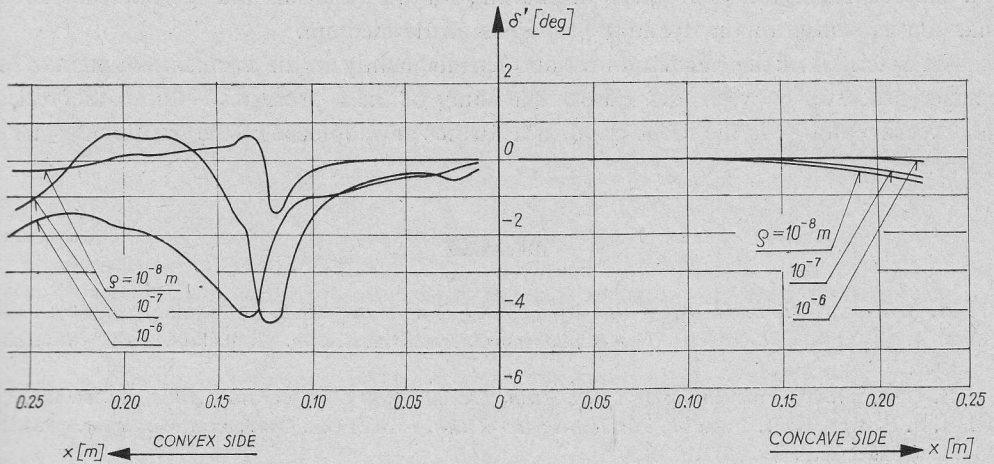
Fig. 16. Distribution of the deviation from the equilibrium temperature in the liquid phase for wet steam flow around the profile

steam flow entering the guide vane (assumed to be in equilibrium) are:

$$p(0) = 0.146 \text{ bar}, \quad g(0) = 0.976, \quad w(0) = 63.6 \text{ m/s}.$$

For the profile in question the distribution of the static pressure of a gas flow around the profile resulting either from an aerodynamic experiment or from known calculation

methods has been given. The flow along each of the sides has been treated as a one-dimensional flow; the effects of boundary layer phenomena on the main stream flow have been neglected.

The medium has been assumed to contain one fraction of droplets; for each of the three calculation series a different value of the droplet radius in the entering flow has been used. For the calculations the described above indirect method has been employed.

The resulting distributions of gas flow temperature, leading temperature values and saturation temperature in the actual non-equilibrium flow have been shown in Fig. 14. As it can be seen, the small differences between the values of T_0 and T_s account for the good approximation obtained by the employed method.

The distribution of the values of the deviations from the equilibrium temperature have been shown in Figs. 15 and 16, for the deviations in the gaseous and liquid phase respectively. From the diagrams the strong influence of the droplet size and of the local pressure gradient is recognizable.

9. Final Remarks

From the derivations contained in this paper the specific properties of non-equilibrium multiphase flows may be recognized. In the case of a wet steam expansion all the results are most substantially influenced by two main factors: the time-rate of the expansion (as the forcing exerted upon the medium) and the structure of the medium itself, i.e., the size and the concentration of droplets, accounting for the (thermal and mechanical) inertia and, consequently, for the dynamic properties of the medium.

The exactness of the calculation results depends mainly on the correct assessment of the transfer processes between the phases. The study of these processes — both theoretical and experimental — is the focus point of a further development of the multiphase media theory.

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Termodynamiczne problemy nierównowagowej ekspansji czynników dwufazowych (par wilgotnych)

Streszczenie

Artykuł zawiera wyniki szeregu prac badawczych z dziedziny termodynamiki nierównowagowych przepływów pary wilgotnej, wykonanych przez autora. Wspólnym celem tych prac było opracowanie metod opisu dynamiki procesów cieplnych w czynnikach dwufazowych i opracowanie metod obliczeniowych, uwzględniających zjawiska braku równowagi wewnętrznej w czynniku. Jedno z istotnych zastosowań tego rodzaju metod dotyczy zagadnienia przepływów pary mokrej w turbinach parowych.

Zjawiska braku równowagi wewnętrznej (termodynamicznej i mechanicznej) w czynniku wielofazowym pojawiają się w wyniku wymuszeń zewnętrznych, działających ze skończoną szybkością (np. szybka ekspansja). Uwzględnienie wpływu tych zjawisk przy obliczeniu przebiegu procesów prowadzi do poważnych różnic w stosunku do stosowanych dotychczas tzw. quasi-statycznych metod obliczeniowych.

Zawarte w pracy rozważania, oparte na metodach zbliżonych do metod stosowanych w termodynamice procesów nieodwracalnych, prowadzą do opracowania nowego sposobu opisu dynamiki procesów cieplnych w czynnikach wielofazowych; ten opis jest oparty na zaproponowanym modelu dynamiki przemian w czynniku. Do istotnych własności tego modelu należy występowanie zależności od historii stanów względnie od historii wymuszeń. Ta zależność ma charakter „zanikającej pamięci" stanów, istniejących w przeszłości.

Zastosowania wyprowadzonych metod obejmują:

— wyznaczanie przebiegu parametrów cieplnych i kinematycznych czynnika poddanego szybko działającym wymuszeniom (np. szybkiej ekspansji),

— wyznaczanie krytycznych warunków przepływu, krytycznych przekrojów i parametrów, z uwzględnieniem stanów braku równowagi,

— obliczanie dodatkowych „dynamicznych" strat energetycznych, wywołanych brakiem równowa-gi.

— wymiarowanie i ewentualnie optymalizacja kanałów przepływowych w układzie łopatkowym. W celu obliczenia dodatkowych strat ,,dynamicznych" wprowadzono pojęcie ,,przygotowanych przyrostów entropii", oparte na rozważeniu tzw. aktywnej oraz relaksacyjnej części procesu cieplnego.

Omówiono dwa rodzaje definicji warunków krytycznych w przepływie nierównowagowym i podano występujące m. in. zależności. Podano sposób obliczania krytycznych wartości parametrów przepływu oraz krytycznych przekrojów.

Zaproponowano dwie metody (,,bezpośrednią" i ,,pośrednią") przeprowadzenia kompletnych obliczeń procesów przepływu pary mokrej w kanałach kierowniczych i łopatkowych oraz występujących strat dodatkowych. Przedstawiono również sposób zastosowania tych metod do obliczania rozkładu parametrów przepływu nierównowagowego po obu stronach profilu łopatki kierowniczej.

Термодинамические проблемы неравновесного расширения двухфазных сред (влажных паров)

Резюме

В статье представлены результаты ряда исследовательских работ из области термодинамики неравновесных течений влажного пара, выполненных автором. Общей целью этих работ являлась разработка методов описания динамики тепловых процессов в двухфазных средах и расчетных методов, учитывающих явления внутреннего неравновесия в среде. Одно из важных применений методов такого рода относится к проблеме течения влажного пара в паровых турбинах.

Явления внутреннего неравновесия (термодинамического и механического) в многофазной среде появляются в результате внешних вынуждений, действующих с конечной скоростью (напр. быстрое расширение). Если учесть влияние этих явлений в расчётах хода процессов, тогда получаются значительные разности по отношению к применяемым до сих пор т. наз. квазистатическим расчетным методам.

Представленные в работе рассуждения, основанные на методах приближенных к методам, применяемым в термодинамике неотвратимых процессов, ведут к разработке нового способа описания динамики тепловых процессов в многофазных средах; это описание основано на предложенной модели динамики превращений в среде. К важнейшим свойствам этой модели можно зачислить появление зависимости от истории состояний или от истории вынуждений. Эта зависимость имеет характер ,,потухающей памяти" состояний существовавших в прошлом.

Применения выведенных методов охватывают:

— определение распределения тепловых и кинематических параметров среды, подвергнутой быстродействующим вынуждениям (напр. быстрому расширению);

— определение критических условий течения, критических сечений и параметров, с учетом состояний отсутствия равновесия;

— расчеты добавочных ,,динамических" энергетических потерь, вызванных отсутствием равновесия;

— определение размеров и возможная оптимизация проточных каналов в лопаточной системе.

С целью выполнения расчетов добавочных ,,динамических" потерь введено понятие ,,приготовленных приростов энтропии", основанное на рассуждении т. наз. активной и релаксационной частей теплового процесса.

Обсуждаются два рода определения критических условий в неравновесном течении и перечисляются зависимости, выступающие между ними. Определяется способ расчета критических значений параметров течения и критических сечений.

Предлагаются два метода (,,непосредственный" и ,,посредственный") выполнения комплектных расчетов процессов течения влажного пара в направляющих и рабочих межлопаточных каналах и выступающих добавочных потерь. Представлен также способ применения этих методов к расчетам распределений параметров неравновесного течения по обеим сторонам профиля направляющей лопатки.