POLSKA AKADEMIA NAUK
INSTYTUT MASZYN PRZEPŁYWOWYCH

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

83-84

WARSZAWA – POZNAŃ 1983

PAŃSTWOWE WYDAWNICTWO NAUKOWE

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

*

THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

---

REDAKCJA—EDITORIAL OFFICE

Instytut Maszyn Przepływowych PAN
ul. Gen. Józefa Fiszera 14, 80-952 Gdańsk, skr. pocztowa 621, tel. 41-12-71

ISBN 83-01-04553-1
ISBN 0079-3205

PAŃSTWOWE WYDAWNICTWO NAUKOWE – ODDZIAŁ W POZNANIU

Nakład 340+90 egz.	Oddano do składania 17 VI 1982 r.
Ark. wyd. 21,5. Ark. druk. 16,75+1 wkl.	Podpisano do druku 22 IV 1983 r.
Pap. druk. sat. kl. V, 70 g	Druk ukończono w maju 1983 r.
Nr zam. 489/184.	E-9/215. Cena zł 200,–

DRUKARNIA UNIWERSYTETU IM. A. MICKIEWICZA W POZNANIU

HYDROFORUM

KONFERENCJA NAUKOWO-TECHNICZNA

na temat

PROBLEMY ROZWOJU HYDRAULICZNYCH MASZYN WIROWYCH ZE SZCZEGÓLNYM UWZGLĘDNIENIEM POTRZEB ENERGETYKI

Porąbka-Kozubnik, 20 - 23, września 1980 r.

*

HYDROFORUM

SCIENTIFIC-TECHNICAL CONFERENCE

on

DEVELOPMENT PROBLEMS OF HYDRAULIC TURBOMACHINES WITH SPECIAL ACCOUNT OF THE NEEDS OF POWER ENGINEERING

Porąbka-Kozubnik, September 20 - 23, 1980

*

ГИДРОФОРУМ

НАУЧНО-ТЕХНИЧЕСКАЯ КОНФЕРЕНЦИЯ

на тему

ПРОБЛЕМЫ РАЗВИТИЯ ГИДРАВЛИЧЕСКИХ РОТОРНЫХ МАШИН С ОСОБЫМ УЧЕТОМ НУЖД ЭНЕРГЕТИКИ

Поромбка-Козубник, 20 - 23 сентября 1980 г.

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH
1983 Zeszyt 83 - 84

OLIVÉR FÜZY, ANDRÁS SEBESTYÉN

Budapest*

A Possible Model of Pump's Dynamic Work

In examination of nonstationary behaviour of pipe networks the knowledge of the pump's dynamic behaviour is a crucial question. In the following an approximate simulation calculation method is intended to be introduced for the examination of pump running out, a transient working state important for practice, where from the knowledge of the t moment state the characteristics of the $t+\Delta t$ moment state can be determined.

Symbols

u – peripheral velocity,
c – absolute velocity,
γ – tangential component of c,
Γ – blade circulation,
Q – volumetric current of flow,
ω – angular velocity of rotor,
w – relative velocity,
Γ_u – circulation of u,
A_l – profile contour area,
Θ – inertia of rotation masses,
M – impeller moment,
M'_T – disc friction drag moment,
M'_M – mechanical frictional moment,
Re – Reynolds number,
ρ – density,
N – number of blades,
L – length of pressure tube,
a – medium sound velocity,
v – flow velocity in the pressure tube,
H – delivery head,
He – theoretical delivery head,
φ – quantitative number,
η_h – hydraulic efficiency,
g – gravitational acceleration,
g_w – potential gradient of circuital field,
b – rotor disc housing gap,
p – pressure.

1. Introduction

In course of calculation aimed at determining the velocity distribution developed in straight blade cascade the well-known procedure (published in 1969) based on the method of singularities [1] is used. The form of the integral equation serving the determination of the velocity distribution, with the assumption that the liquid arrives into the pump without prerotation, is the following:

$$\gamma(s)+\oint_G K(s,s')\cdot\gamma(s')\,ds'=P(s)\cdot Q+R(s)\cdot\omega, \tag{1}$$

* Department of Hydraulic Machinery, Technical University, Budapest.

where s is a fixed locus coordinate on the blade profile, s' is the running point of integration, and $P(s)$, $R(s)$ and $K(s, s')$ functions depend only on the geometrical formation of the rotor. In the course of solution application of the functions $Q=Q(\Gamma(t))$ and $\omega= =\omega(\Gamma(t))$ forms seems to be expedient.

2. The Uniqueness Condition

To ensure the uniqueness of the solution of integral equation (1) one more condition should be specified. To define it, the time variation of the circulation of relative speed along a flowing off curve $S(t)$ surrounding blade profile and changing in time is examined:

$$\frac{d}{dt}\oint_{S(t)} \underline{w}\, d\underline{r} = \oint_{S(t)} \left(\frac{d\underline{w}}{dt} + \underline{w}\frac{\partial \underline{w}}{\partial \underline{r}}\right) d\underline{r}. \tag{2}$$

The Euler equation for the relative flow:

$$\frac{d\underline{w}}{dt} + \frac{1}{\rho}\frac{\partial p}{d\underline{r}} - \underline{g}_w + (\nabla \times \underline{u}) \times \underline{w} = 0. \tag{3}$$

From the comparison of (2) and (3):

$$\frac{d}{dt}\oint_{S(t)} \underline{w}\, d\underline{r} = -2\oint_{S(t)} \underline{\omega}' \underline{w}\, d\underline{r}.$$

During the examination we work with given time step Δt. If in moment t $S(t)$ itself is the blade profile, then according to Figure 1 after Δt it can be written:

$$\frac{l}{\Delta t}\left[\oint_{S(t+\Delta t)} \underline{w}\, d\underline{r} - \oint_{S(t)} \underline{w}\, d\underline{r}\right] = -2\omega(t+\Delta t)\cdot\delta\cdot w_k(t) \tag{4}$$

that is on the contour $\underline{w} \times d\underline{r} = 0$.

Decomposing the left hand side of equation (4) for the circulation of the tangential components of absolute and peripheral velocities, as well as considering the fact that in radial flow machines (with the application of the Stokes theorem):

$$\Gamma_u = \oint_{S(t)} \underline{u}\, d\underline{s} = 2\int_{A_1} \underline{\omega}\, d\underline{A} = 2\omega \cdot A_1$$

in every moment we arrive at the following equation, used as a uniqueness condition applied for a nonstationary flow:

$$\Gamma(t+\Delta t) + (\gamma_A - \gamma_B)\Delta s = \Gamma(t) + \omega(t+\Delta t)\cdot[2\cdot A_1 - 4\delta\Delta s] - 2\omega(t)\cdot A_1. \tag{5}$$

Equation (5) in case of standing cascade placed into a stationary flow becomes the uniqueness condition of Kutta: $\gamma_A = \gamma_B$.

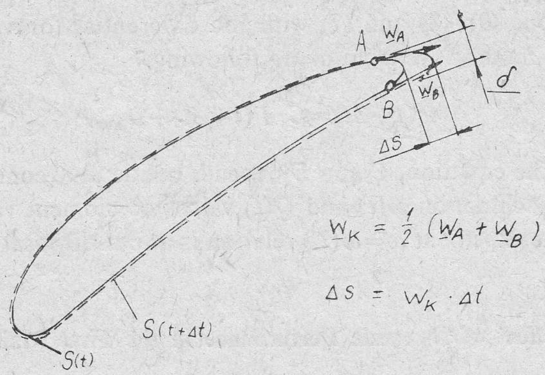

Fig. 1

3. The Kinematic Equation of the Rotor Motion

The kinematic equation of motion of the rotor:

$$\Theta \frac{d\omega}{dt} = M_{\text{mot}} - M_{\text{pump}} \tag{6}$$

where in course of the running out of the pump $M_{\text{mot}}=0$. In the calculation of M_{pump} we start from the relation $M_{\text{pump}}=M_{\text{hydr}}+M'_T+M'_M$. The moment of disc friction drag is calculated on the basis of [2], as the examined rotors had so-called smooth discs, or discs that can be regarded smooth. Theoretically it is possible to take into account the disc friction drag moment appearing on the disc of the rotor of radial relief blades on the basis of [3]. Thus, according to [2],

$$M'_T(\omega)=C'_T(Re, D_2, b)\frac{\rho}{2}\omega^2. \tag{7}$$

The loss moments caused by mechanical friction have been considered on the basis of [4] with the aid of the suitably reduced relation

$$M'_M(\omega)=\left[C'_{M1}+\left(\frac{C'_{M2}}{\omega(t)}\right)^2\right]M_{\text{pump}}, \tag{8}$$

where C'_{M1} and C'_{M2} are the well-known values depending on nominal work parameters, constant during the examination. Equation (6) is rewritten into differential equation for simulation with the time mean values of the respective moments taken into account. Relation (7) of disc friction drag moment is linearized with the aid of the first two terms of the binomial series with respect to ω. The time mean value of hydraulic moment is calculated in the following way:

$$M_{\text{hydr}}=\frac{\rho N}{2\pi}\cdot Q(t)\cdot \Gamma(t+\Delta t). \tag{9}$$

Combining equations (9), (8) and (7) with the differential form of equation (6) according to the above method we obtain the following:

$$\omega(t+\Delta t)=C\cdot\Gamma(t+\Delta t)+D, \tag{10}$$

where, after reducing the equation, C and D depend, beside the geometrical sizes and the nominal data of the machine, on $\omega(t)$ and $Q(t)$ values of moment variables regarded as well-known. With it the expedient $\omega=\omega(\Gamma)$ relation is at our disposal in form (10).

4. The Effect of Dynamic Perturbance in the First Main Time

The running out of a pump is a physical phenomenon occurring when during operation at $t=0$ a depression wave starts in the pressure tube because of the current absence. Its magnitude in $0\leqslant t\leqslant 2L/a$ interval is:

$$p(t)-p_0=\rho a\,[v(t)-v_0]$$

i.e.

$$H(t)-H_0\cong\frac{a}{gA_1}[Q(t)-Q_0], \tag{11}$$

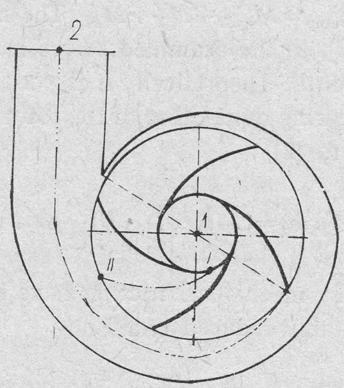

Fig. 2

where the index 0 refers to the initial state, supposing that relation $\eta_h=\eta_h(\varphi)$ does not change substantially during the examination. It is obvious that

$$H=\eta_h H_e-h_a, \tag{12}$$

where h_a is the effective delivery head decrease resulting from the speed change (deceleration). The value of h_a consists of two parts: one related to the quantities written for the average streamline of the spiral-housing according to Figure 2 and a part associated with the average streamline between the suction connection and admission cross-section of the rotor. The quantities written for the absolute streamline in the rotor according

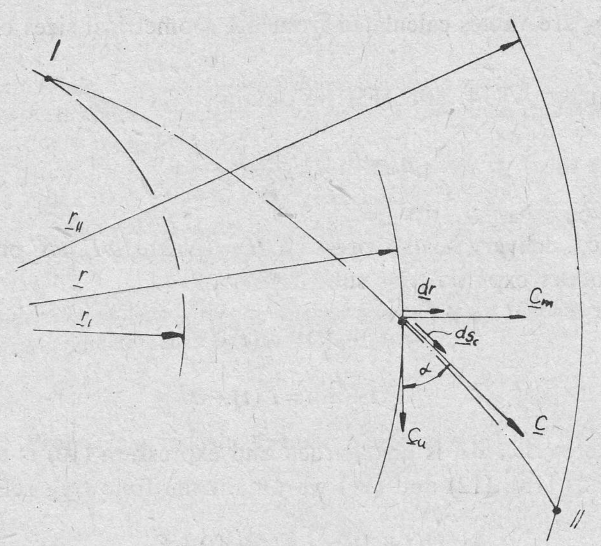

Fig. 3

to Figure 2:

$$h_a = \frac{1}{g} \cdot \int_1^2 \frac{\partial c}{\partial t} ds + \frac{1}{g} \cdot \int_I^{II} \frac{\partial c}{\partial t} ds. \tag{13}$$

Let's examine the integral expression in the second term of the right-hand side of equation (13):

$$\int_I^{II} \frac{\partial c}{\partial t} ds = \int_I^{II} \frac{\partial c}{\partial t} ds_c = \int_I^{II} \frac{\partial c}{\partial t} \frac{ds_c}{dr} dr.$$

In Figure 3 it can be seen that $dr/ds_c = \sin\alpha(r)$ while in $\partial c/\partial t \approx \Delta c/\Delta t$ approximation for calculation of c values with peripheral integral averages the following can be written: $c = c_m \sqrt{1 + \mathrm{ctg}^2\,\alpha(r)}$. Hence

$$\frac{1}{g} \int_I^{II} \frac{\partial c}{\partial t} ds \cong B_{11}\, Q(t + \Delta t) + B_{12}, \tag{14}$$

where B_{11} and B_{12} are values depending on the geometrical sizes of the pumps and the pressure tube, and also on known values of $\omega(t)$ and $Q(t)$ for the machine. Similarly for the first term of expression (13) one can write:

$$\frac{1}{g} \int_1^2 \frac{\partial c}{\partial t} ds = B_{21}\, Q(t + \Delta t) + B_{22}, \tag{15}$$

where B_{11} and B_{22} are values calculated from the geometrical sizes of the machines and the pressure tube.

Comparing equations (14) and (15) we obtain

$$h_a = B_1 Q(t+\Delta t) + B_2, \tag{16}$$

where $B_1 = B_{11} + B_{21}$, $B_2 = B_{12} + B_{22}$.

In the theoretical delivery head expression $H_e = (N/2\pi g)\,\omega\Gamma$, $\omega\Gamma$ product is linearized, namely in the product expressed by substitution of

$$\omega(t+\Delta t) = \omega(t) + \Delta\omega,$$

$$\Gamma(t+\Delta t) = \Gamma(t) + \Delta\Gamma$$

the second-order term $\Delta\omega.\ \Delta\Gamma$ is disregarded and expression (10) is taken into account. Thus, combining eqs (16), (12) and (11) we obtain the following relation:

$$Q(t+\Delta t) = E\Gamma(t+\Delta t) + F, \tag{17}$$

where E and F can be calculated from the initial state, from the geometrical data of the machine and the pressure tube connected to it, from the physical characteristics of the delivered medium and from the known values $\omega(t)$ and $Q(t)$ for the moment t. With it the expedient $Q = Q(\Gamma)$ is at our disposal, in relation (17).

On the basis of it, substituting equations (10) and (17) into equation (1) and applying relation $\Gamma = \oint_G \gamma(s)\,ds$ we obtain the expression suitable for the simulation of pump running out

$$\gamma(s) + \oint_G K^* \cdot \gamma(s')\,ds' = Z, \tag{18}$$

where

$$K^* = K(s, s') - [P(s) \cdot E + R(s) \cdot C],$$

$$Z = P(s) \cdot F + R(s) \cdot D.$$

Rewriting integral equation (18) to a linear equation system, in the knowledge of the values belonging to the moment t as introduced in [1], solved together with the uniqueness condition (5) the outline $\gamma = \gamma(s)$ circulation distribution belonging to the moment $t + \Delta t$ is obtained as the result.

So in a given moment $\Gamma(t+\Delta t) = \oint_G \gamma(s)\,ds$ can be calculated and then upon substituting it into equations (10) and (17) $Q(t+\Delta t)$ and $\omega(t+\Delta t)$ also can be determined. Then using the values of $\Gamma(t+\Delta t)$, $Q(t+\Delta t)$ and $\omega(t+\Delta t)$ with Δt time step values $\Gamma(t+2\Delta t)$, $\omega(t+$ $+2\Delta t)$ and $Q(t+2\Delta t)$ are obtained from new simultaneous solution of equations (18) and (5). With this step we go on until $t = 2L/a$, the $\Gamma(t)$, $\omega(t)$ and $Q(t)$, and $H(t)$ relations can be determined in discrete moments.

At present the work is just before completion that on the basis of the above method makes possible computer simulation of the running out of a given radial-flow pump from the known stationary state.

References

[1] O. Füzy, A. Thuma, *Calculating the velocity distribution of a plane airfoil cascade given by its geometry*. III. Áramlástechnikai és Áramlástechnikai Gépek Konferencia (Fluid Mechanics and Fluid Mechanic Machines Conference), Budapest 1969.

[2] J. W. Daily, R. E. Nece, *Chamber Dimension Effects in Induced Flow and Frictional Resistance of Enclosed Rotating Disks*. Journal of Basic Engineering, March 1960, p. 217.

[3] S. Wilk, *The effect of the radial relief blades on pressure distribution behind the impeller in centrifugal pumps*. IV. Áramlástechnikai Gépek Konferencia (Fluid Mechanic Machines Conference), Budapest 1972.

[4] J. Csemniczky, *Approximate method for the determination of actual characteristics curve of radial and semiaxial pumps*. Candidate's dissertation. Budapest 1975.

Propozycja modelu dynamicznej pracy pompy

Streszczenie

Znajomość dynamicznej pracy pompy odgrywa poważną rolę w badaniach stanów nieustalonych w rurociągach. Celem referatu jest zaprezentowanie teoretycznej metody opisu dynamicznej pracy pompy na ważnym z punktu widzenia praktyki przykładzie. Przy rozwiązywaniu tego zagadnienia bierze się pod uwagę wpływ wiru spływowego wywołanego zmianami dynamicznymi oraz wpływ bezwładności wirnika i pompowanej cieczy. Pierwszy model opracowano dla badania dynamicznej pracy pompy podłączonej do stosunkowo długiego rurociągu; w referacie zwraca się jednak uwagę na zasadnicze odmiany modelu pozwalające na badanie dynamiki pracy pompy podłączonej do rurociągu o dowolnej długości.

Предложение модели динамической работы насоса

Резюме

Знание динамической работы насоса играет серьезную роль в исследованиях неустановившихся состояний в турбопроводах. Целью работы является представление теоретического метода описания динамической работы насоса на важном с практической точки зрения примере. В решении этого вопроса учитывается влияние выходного завихрения, вызванного динамическими изменениями, а также влияние инертности ротора и перекачиваемой жидкости. Первая модель обработана для исследования динамической работы насоса соединенного с относительно длинным трубопроводом. В работе обращается однако внимание на основные разновидности модели, позволяющие исследовать динамику работы насоса соединенного с трубопроводом произвольной длины.