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exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machinery

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Gdańsk

The Influence of Thermoelastic Deformations of Bearing Bush and its External Fixings on Static and Dynamic Properties of Journal Bearings

Part I. Thermoelastohydrodynamic Model of Bearing*

A complex thermoelastohydrodynamic model of slider bearing is presented. This model includes the heat transfer between the oil film, the bush and the surroundings. Bush deformations are calculated with the aid of finite element method. The dynamic properties of the bearing are described with the use of the equation of motion for small displacements of the centre of the journal. The equations describing the theoretical model are adjusted to the requirements of numerical calculations.

Nomenclature

b_o, B_o – dimensional and non-dimensional viscosity-temperature coeffcients, respectively b_0 [1/K],

$c_{w\tau}$ – specific oil heat, [m^2/(s^{2}K)],

D – diameter of the bearing journal, $D=2r_c \cong 2r_w$ [m],

$E, \bar{E}$ – dimensional and non-dimensional modulus of elasticity, respectively, E [N/m^2], $\bar{E}=E(\Delta r/r_w)^3/(\mu_0\omega)$,

$\{F_p\}$ – non-dimensional loadings vector transformed to node forces,

$\{F_t\}$ – non-dimensional loadings vector due to thermal interactions,

g – acceleration of gravity,

h, H – dimensional and non-dimensional thickness of oil film, respectively, h [m], $H=h/\Delta r$,

h_k, H_k – dimensional and non-dimensional thickness of oil film for a rigid bush; for bearings with circular cylindrical clearance:

$$h_k=\Delta r - e\cos(\psi-\gamma),$$

$$H_k=1-\varepsilon\cos(\psi-\gamma),$$

h_δ, H_δ – dimensional and non-dimensional deformation of a bush in the radial direction, h_δ [m], $H_\delta=h_\delta/\Delta r$,

$[K]$ – global stiffness matrix (non-dimensional),

k_{ol} – heat transfer coefficient for oil, [kg m/s^3 K)],

k_p – heat transfer coefficient for bush material, [kg m/(s^3 K)],

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K_{op} – relation of heat transfer coefficient for oil to heat transfer coefficient for bush material, $K_{op}=k_{ol}r_w/(k_p\Delta r)$,

L – bearing width, [m],

M – non-dimensional viscosity, μ/μ_0,

Nu – Nuselt number, $Nu=\alpha_{0t}r_w/k_p$,

P_E – Peclet number (inverse), $P_E=k_{ol}/(\rho c_{wt}\omega\Delta r^2)$,

P_{st} – static bearing load, [N],

Q_k – non-dimensional hot oil carry over flow,

$$Q_k=q_k LD\omega\Delta r,\ q_k\,[\mathrm{m^3/s}],$$

Q_p – non-dimensional oil flow at the leading edge,

Q_s – fresh oil flow (non-dimensional) fed to axial oil grove,

q_w, Q_w – dimensional and non-dimensional side leakage from axial oil grooves, $Q_w=Q_{w1}+Q_{w2}+\ldots+$,

r, R – dimensional and non-dimensional radial coordinate for a bush, r[m], $R=r/r_w$,

r_c – journal radius, [m],

r_w – radius of the bush inner surface, [m],

r_z, R_z – dimensional and non-dimensional radius of the outer surface of bush, r_z[m], $R_z=r_z/r_w$,

Δr – radial clearance, $\Delta r=r_w-r_c$ [m],

s, S – dimensional and non-dimensional radial coordinate for oil film, s[m], $S=s/h$,

S_L – ratio of the axial oil grove width to the bearing width,

t, T – dimensional and non-dimensional temperature of oil film, t[K], $T=t\rho c_{wt}(\Delta r/r_w)^2/(\mu_0\omega)$,

t_c, T_c – dimensional and non-dimensional temperature of the journal,

t_o, T_o – dimensional and non-dimensional inlet temperature of oil,

t_p, T_p – dimensional and non-dimensional temperature of the bush,

t_z, T_z – dimensional and non-dimensional temperature of ambient,

W_{x0}, W_{y0} – dimensional components of load capacity,

α_t^* – non-dimensional thermal expansion coefficient,

$$\alpha_t^*=\alpha_t\mu_0\omega/(\rho c_{wt}(\Delta r/r_w))^3,$$

γ – attitude angle, [°],

ε – eccentricity ratio, $\varepsilon=e/\Delta r$,

ψ – circumferential angular coordinate, $\psi=l/r_w$,

ψ_p – angular circumferential coordinate of the leading edge,

ψ_k – angular circumferential coordinate of the trailing edge,

ρ – density of oil, [kg/m³],

Π – non-dimensional hydrodynamic pressure in oil film;

$$\Pi=p(\Delta r/r_w)^2/(\mu_0\omega),$$

Π_o, Π_z – non-dimensional feeding pressure of oil and non-dimensional pressure due to external fixings, respectively,

ω, ω_r – angular velocity of rotor (of journal), [rad/s],

ω_{gr} – angular velocity determining the stability limit of the system, [rad/s],

ω_0 – reference parameter $\omega_0=\sqrt{g/\Delta r}$ [rad/s].

1. The subject of the paper

Despite the undisputable progress in the research dealing with properties of oil films, the problem of heat transfer and thermoelastic deformations of the bush has not been fully examined. This refers particularly to the dynamic properties of oil films. The systems: rotor-journal-lubricating film-bush-external fixing form a tribologic system which is a very difficult one for theoretical description. There are great, often qualitative, discrepancies between the results of theoretical calculations and experimental data.

In the literature one can find numerous examples of theoretical models describing heat exchange in the oil film [1, 2]. Examples of calculations of elastic and thermal deformations of bushes or pads caused by hydrodynamic pressure are given in [3, 4, 5, 6]. However, the majority of these works is devoted to the static properties of lubricating film and only simple models of deformable elements of bearing are applied (e.g. a beam or rigs rigidly fixed on the external surface. In the extensive literature on the dynamics of rotors [7, 8, 9] one can find descriptions of the dynamical properties of the rotor-bearing system which are also based on a simple model of the bush. Nevertheless, one can find works [10] in which the authors try to define the influence of bush deformation and the type of its fixing on the dynamic properties of a rotor-bearing system (however, the thermal deformations are usually neglected).

A thermoelastohydrodynamic model of a bearing has been developed in the Institute of Fluid-Flow Machinery, Polish Academy of Science. It accounts for the problem of heat transfer in the oil grooves, the oil film, the bush and the heat exchange with the surroundings. The model includes elastic as well as thermal bush deformations. The finite element method enables one to model reactions of the external fixings of the bush in the support of the bearing. A system consisting of a rigid, single mass, symmetric rotor has been considered. Such a simple system allows one to determine the dynamic characteristics for the bearing only.

The term "dynamic characteristics" is used here for the characteristic quantities, calculated with the use of damping and stiffness coefficients, such as stability limits, damping (logarithmic decrement) and the free vibration frequency of a rotor-bearing system.

The basic aim of the work was to create a theoretical tool consisting of a model and a computer program which would enable one to perform more precise calculations of the properties of slider bearings. The research on the influence of external fixings of bushes on the static and dynamic characteristics of the bearing presented in this paper can be treated as an illustration showing the possibilities as well as limitations of the proposed theoretical model. To build a model of an external fixing of a bush is not an easy task. This is due to the complicated mechanical and thermal interactions. It is very difficult to construct a model which would, within good approximation, describe the real, practically encountered, designs of external fixings of the bush, and the conditions of the heat transfer. Thus, in the present work only several simple cases of external fixings have been considered. Further, simple conditions of heat exchange have been assumed (heat transfer to the surrounding by free convection).

With those assumptions in mind, the following question has been considered. Does a change of the design of the bush fixing, for typical operating conditions, imply significant changes in the dynamic and static properties of the bearing and the whole system? In the literature one can find very little information on this subject. Part I of this paper describes basic assumptions and equations of the theoretical model. On the other hand Part II is devoted to the presentation of an experimental verification of the model and theoretical investigations of the influence of external fixings on the properties of the bearing.

2. Theoretical model

The processes occuring in a non-isothermal oil film and in a deformable lubricating gap are described by a number of inter-conjugated equations. Only the final basic equations and the most important simplifying assumptions will be presented here. All classical assumptions of the hydrodynamic theory of lubrication will be tacitly assumed to hold.

2.1. Reynolds equation

This equation has been derived basing on the fundamental assumption that oil viscosity μ is the function of perimeter coordinate l and film thickness s (the diathermic model)

$$\mu = f(l, s),$$

and is not dependent on the coordinate along bearing width z_p (Fig. 1). This is equivalent to the assumption of constant temperature along the bearing width. The shape of lubricating gap h is a function of perimeter coordinate l and of bearing width z_p

$$h = f(l, z_p).$$

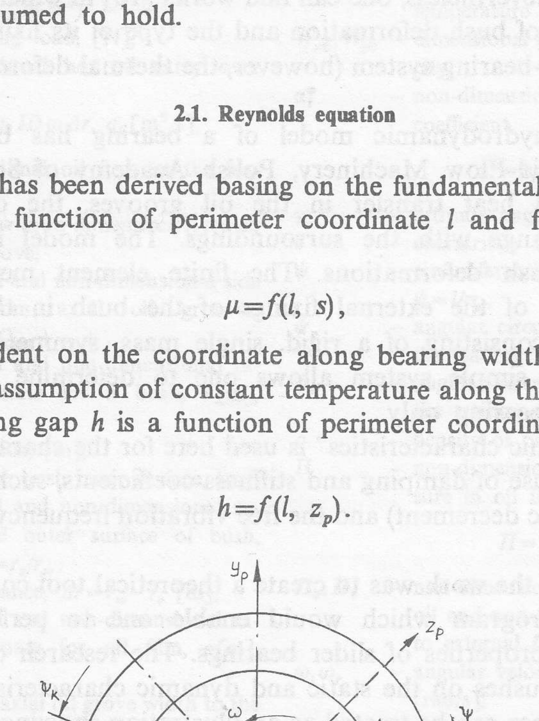


Fig. 1. System of coordinates

One can introduce the following non-dimensional quantities

$$\psi = \frac{l}{r_w}, \quad Z_p = \frac{z_p}{r_w}, \quad S = \frac{s}{h}, \quad H = \frac{h}{\Delta r}, \quad \tau = \omega t, \quad M = \frac{\mu}{\mu_0} \text{ and } \Pi = p\frac{(\Delta r/r_w)^2}{\mu_0\,\omega}$$

The Reynolds equation takes the following form

$$\frac{\partial^2 \Pi}{\partial \psi^2}+\frac{\partial \Pi}{\partial \psi}\left[\frac{3}{H}\frac{\partial H}{\partial \psi}+\frac{\frac{\partial}{\partial \psi}\int_0^1 A dS}{\int_0^1 A dS}\right]+\frac{\partial^2 \Pi}{\partial Z_p^2}+\frac{3}{H}\frac{\partial H}{\partial Z_p}\frac{\partial \Pi}{\partial Z_p}=$$

$$=\frac{1}{H^3\int_0^1 A dS}\left[\frac{\partial H}{\partial \psi}\left(\int_0^1 B\, dS-1\right)+H\frac{\partial}{\partial \psi}\int_0^1 B dS-\frac{\partial H}{\partial \tau}\right], \tag{1}$$

where

$$A=\int_0^S \frac{S}{M}dS-\frac{\int_0^1 \frac{S}{M}dS}{\int_0^1 \frac{1}{M}dS}\int_0^S \frac{1}{M}dS,$$

$$B=\frac{\int_0^S \frac{1}{M}dS}{\int_0^1 \frac{1}{M}dS}. \tag{1a}$$

If one employs the non-dimensional notation, the components of the hydrodynamic capacity of the hydrodynamic capacity of the bearing $\bar{\Pi}_{x0}$, $\bar{\Pi}_{Y0}$ are

$$\bar{\Pi}_{x0}=\frac{w_{x0}}{r_w^2}\frac{(\Delta r/r_w)^2}{\mu_0\,\omega}=-\int_{-L/D}^{L/D}\int_{\psi p}^{\psi k}\Pi \cos\psi\, dZ_p\, d\psi\,,$$

$$\bar{\Pi}_{Y0}=\frac{w_{Y0}}{r_w^2}\frac{(\Delta r/r_w)^2}{\mu_0\omega}=-\int_{-L/D}^{L/D}\int_{\psi p}^{\psi k}\Pi \sin\psi\, dZ_p\, d\psi\,.$$

The total capacity $\bar{\Pi}_0$ is the geometrical sum of the components $\bar{\Pi}_{x0}$ and $\bar{\Pi}_{Y0}$. If the components $\bar{\Pi}_{x0}$, $\bar{\Pi}_{Y0}$ are known one can easily calculate the capacity corresponding to the classical definition of the Sommerfeld number

$$So=\frac{P_{st}}{LD}\frac{(\Delta r/r_w)^2}{\mu_0\,\omega}=\frac{1}{4\frac{L}{D}}\sqrt{\bar{\Pi}_{x_0}^2+\bar{\Pi}_{Y_0}^2}=\frac{1}{4\frac{L}{D}}\bar{\Pi}_0.$$

The iterative solution of Eq. (1) has been obtained with the use of a five-points difference approximation method and the classical Reynolds boundary condition.

2.2 Equation for energy

The energy equation which determines the temperature distribution in the oil film, has been derived basing on the assumption that in the perimeter direction of the bearing l heat is transferred only by convection, and along the film thickness S by conduction. The assumption of constant temperature along the bearing width z_p implies the use of mean values of velocity u_{sr} and their derivatives $(\partial u/\partial s)_{sr}$, $(\partial w/\partial s)_{sr}$ in energy equation. These values can be obtained from their distribution along the bearing width. After introducing of the following additional nondimensional quantities

$$T = t\rho c_{wt}\frac{(\Delta r/r_w)^2}{\mu_0\,\omega}, \qquad U_{sr} = \frac{u_{sr}}{u_{cz}} = \frac{u_{sr}}{\omega\, r_w},$$

$$W_{sr} = \frac{w_{sr}}{u_{cz}} = \frac{w_{sr}}{\omega\, r_w}, \qquad P_E = \frac{k_{ol}}{\rho c_{wt}\,\omega \Delta r^2},$$

one obtains energy equation in the following form

$$U_{sr}\frac{\partial T}{\partial \psi} - \frac{P_E}{H^2}\frac{\partial^2 T}{\partial S^2} = \frac{M}{H^2}\left[\left(\frac{\partial U}{\partial S}\right)^2_{sr} + \left(\frac{\partial W}{\partial S}\right)^2_{sr}\right], \tag{2}$$

where

$$U_{sr} = H^2_{sr}\left(\frac{\partial \Pi}{\partial \psi}\right)_{sr} A + 1 - B,$$

$$\left(\frac{\partial U}{\partial S}\right)_{sr} = \frac{1}{M}\left[H^2_{sr}\left(\frac{\partial \Pi}{\partial \psi}\right)_{sr}\left(S - \frac{\int_0^1 \frac{S}{M}\,dS}{\int_0^1 \frac{1}{M}\,dS}\right) - \frac{1}{\int_0^1 \frac{1}{M}\,dS}\right],$$

$$\left(\frac{\partial W}{\partial S}\right)_{sr} = \frac{H^2_{sr}}{M}\left(\frac{\partial \Pi}{\partial Z_p}\right)_{sr}\left(S - \frac{\int_0^1 \frac{S}{M}\,dS}{\int_0^1 \frac{1}{M}\,dS}\right),$$

whereas A and B are defined by Egs (1a).

The boundary conditions for energy equation (2) are as follows:

– temperature distribution on the inner surface of the bush (calculated from conduction equation)

$$T(S, \psi)_{s=1} = T_p(R, \psi)_{R=1},$$

– temperature distribution of the journal surface

$$T(S, \psi)_{s=0} = T_c.$$

In the model under consideration the journal temperature is assumed to be constant. Its value can be obtained by averaging the temperature distribution on the inner bush surface. It can be also estimated in simplified calculations, or tested experimentally.

The initial condition for Eq. (2) is defined by the temperature distribution along the thickness of lubricating film at the inlet edge of oil groove. This can be calculated from the equation of heat balance in the oil grooves

$$T(S,\psi)_{\psi=\psi_p}=aS^2+bS+c,$$

where the coefficients *a*, *b*, *c* can be determined from the equation of heat balance (mixing).

The energy equation (2) has been solved numerically with the use of a six points scheme of difference approximation of Crank-Nicholson and the Gauss elimination method.

2.3. Equation of conduction

In order to calculate the heat transfer through the elements of the bearing, a model of the bush (or pad) has been assumed in form of a thick-walled complete (or partial) ring, however, without such structural elements as pitch planes and application oil grooves. For a constant temperature along the bearing width (axis Z_p) equation of conduction takes the form:

$$\frac{\partial^2 T_p}{\partial R^2}+\frac{1}{R}\frac{\partial T_p}{\partial R}+\frac{1}{R^2}\frac{\partial^2 T_p}{\partial \psi^2}=0. \tag{3}$$

The boundary conditions for the equation of conduction (3) in radial direction R are defined in the following way:

– on the inner surface of bush $R=1$ the heat fluxes in oil film and bush are assumed equal. i.e.

$$\left.\frac{\partial T_p}{\partial R}\right|_{R=1}=\frac{K_{op}}{H}\left.\frac{\partial T}{\partial S}\right|_{S=1},$$

– on the outer surface of bush $R=R_z$ the heat transfer to the surroundings takes place by the free convection:

$$\left.\frac{\partial T_p}{\partial R}\right|_{R=R_z}=Nu\left(T_z-T_p|_{R=R_z}\right).$$

In the perimeter direction the boundary conditions for Eq. (3) are equivalent with a periodical solution (for a complete ring) or with the condition of free convection (for a partial ring).

In the case of a complete ring and the condition for periodical solution an analytical solution of equation of conduction has been used. On the other hand in the case of a part of bush (pad) the iterative solution of Eq. (3) was obtained with the use five-points difference approximation method.

2.4. Equation of heat balance of oil grooves

The process of heat exchange between the supplied cold oil and the recirculating hot oil, which takes place in the oil grooves, highly influences the obtained solutions. It determines the temperature distribution on the inlet edge of the oil groove, and hence it determines the initial condition in the energy equation (2).

It has been assumed that the fresh oil fed to the oil groove at pressure Π_o and temperature T_o mixes completely with the hot oil of the average temperature T_K recirculating from the oil groove. This mixing process allows the temperature T_M in the oil groove to be constant. Temperature T_M can be obtained from the following balance equation

$$T_M(Q_k+Q_s)=Q_k T_k+Q_s T_0, \tag{4}$$

where

$$Q_s=Q_p-Q_k+Q_w,$$

whereas the amount of oil dribbling out through the side edges Q_w is given by

$$Q_w=\frac{1}{6M}\frac{\partial \Pi}{\partial Z_p}\int_{\psi_k}^{\psi_p} H^3 d\psi, \quad \frac{\partial \Pi}{\partial Z_p}=\frac{\Pi_0}{\frac{L}{D}(1-S_L)}.$$

The temperature distribution at the inlet has been approximated with a polynomial of the second order using the condition:

$$\int_0^1 T(S,\psi)|_{\psi=\psi_p}=\int_0^1 (aS^2+bS+c)\,dS=T_M, \tag{4a}$$

which means that the amount of heat supplied to the bush Q_p T_M equals the amount of heat calculated from the assumed parabolic temperature distribution. Coefficients a, b, c have been calculated with the use of the two other boundary conditions, i.e.

$$T(S,\psi)|_{\substack{s=0\\ \psi=\psi_p}}=T_c \quad \text{and} \quad T(S,\psi)|_{\substack{s=1\\ \psi=\psi_p}}=T_p(R,\psi)|_{\substack{R=1\\ \psi=\psi_p}}.$$

2.5. Equation of thermoelastic deformations. Dynamic characteristics

In order to calculate thermoelastic deformations of the bush the finite element method (FEM) has been utilized. The space of the bush is digitized with finite spatial elements of different number of nodes (8 to 21).

Elastic and thermal deformations of the bush $\{U\}$ are obtained from the solution of a general matrix equation of the following form:

$$\bar{E}[K]\{U\}=\{F_p\}+\alpha_t^*\bar{E}\{F_t\}. \tag{5}$$

By fixing the arbitrary nodes on the external surface of the bush and following arbitrary external loads Π_z one can model various types of bush fixing in the bearing support, and then calculate dislocations in each of the nodes of the FEM net corresponding to the type of fixing and the hydrodynamic loads. If the components of the dislocation vector $\{U\}$: U_x, U_y, U_z are known the equantity describing the deformation in the cylindrical system of coordinates can be easily determined as

$$H_\delta(\psi,Z_p)=U_x(Z_p)\cos\psi+U_Y(Z_p)\sin\psi.$$

The shape of lubricating gap $H(\psi, Z_p)$ can be described by the following relation

$$H(\psi, Z_p) = H_k(\psi) + H_\delta(\psi, Z_p). \tag{6}$$

Let us consider the case of small oscillations of the centre of the journal around the point of static equilibrium. The assumption of small vibrations allows one the linearize the relations between the hydrodynamic reaction Π and the displacement. Thus, one can use the so-called stiffnes and damping coefficient.

For an arbitrary rotor-bearing system one can determine, basing on the calculated stiffness and damping coefficients, its basic dynamic properties such as the stability limits, damping and frequency of self-vibrations.

For further considerations let us assume a simple system consisting of two bearings and a stiff, symmetrical and single mass rotor of a non-dimensional mass α. Such a system simplifies the diagnostics of dynamic properties of the oil film itself and hence it is particularly suitable for the kind of analysis presented in the following paper.

The solution of the homogenous equation of motion of the centre of the journal which includes the previously derived relations enables one to get a highly complex description of the dynamic properties of the system. E.g., the stability limit of the operation of the system is a function of the following parameters:

$$\frac{\omega_{gr}}{\omega_0} = f\left(S_0, \frac{L}{D}, H_k, \Pi_0, \bar{E}, \alpha_t^*, \Pi_z, Nu, P_E, K_{op}, B_0, T_0, T_z, T_c\right).$$

Now, it is clear that the stability limits of the system depend on such a great number of parameters that any general theoretical analysis of the influence of non-dimensional quantities is an arduous task.

From the technical point of view it can be interesting to analyse the dynamic properties of a system for a set of geometries of the bearing, given the type of bush fixing, given the cooling and lubricating mode and for known constant values of material coefficients and heat transfer coefficients. The only variable parameters are rotor revolutions and the load of bearing (rotor mass). According to Glienicke [10], if one denotes the relative Sommerfeld number by

$$So_0 = S_0 \frac{\omega}{\omega_0} = \frac{P_{st}(\Delta r/r_w)^2}{lD\,\mu_0\,\omega_0},$$

the related damping value of a given system u_n/ω_0 can be presented in a form of the following, respectively simple function

$$\frac{u_n}{\omega_0} = f\left(So_0, \frac{\omega}{\omega_0}\right).$$

For a given external load of the bearing P_{st} (rotor mass) the value So_0 is constant and independent of ω. One can trace the distribution of the damping value of a system u_n/ω_0 and the distribution of the self-vibration frequency for various rotational velocities of the rotor ω/ω_0. The received damping curves allow one not only to determine the stability limits (the point of zero damping) but also, for a known operational velocity ω_r/ω_0 the so called stability (or damping) reserve. The possibility of knowing this reserve is particularly useful for optimisation calculations.

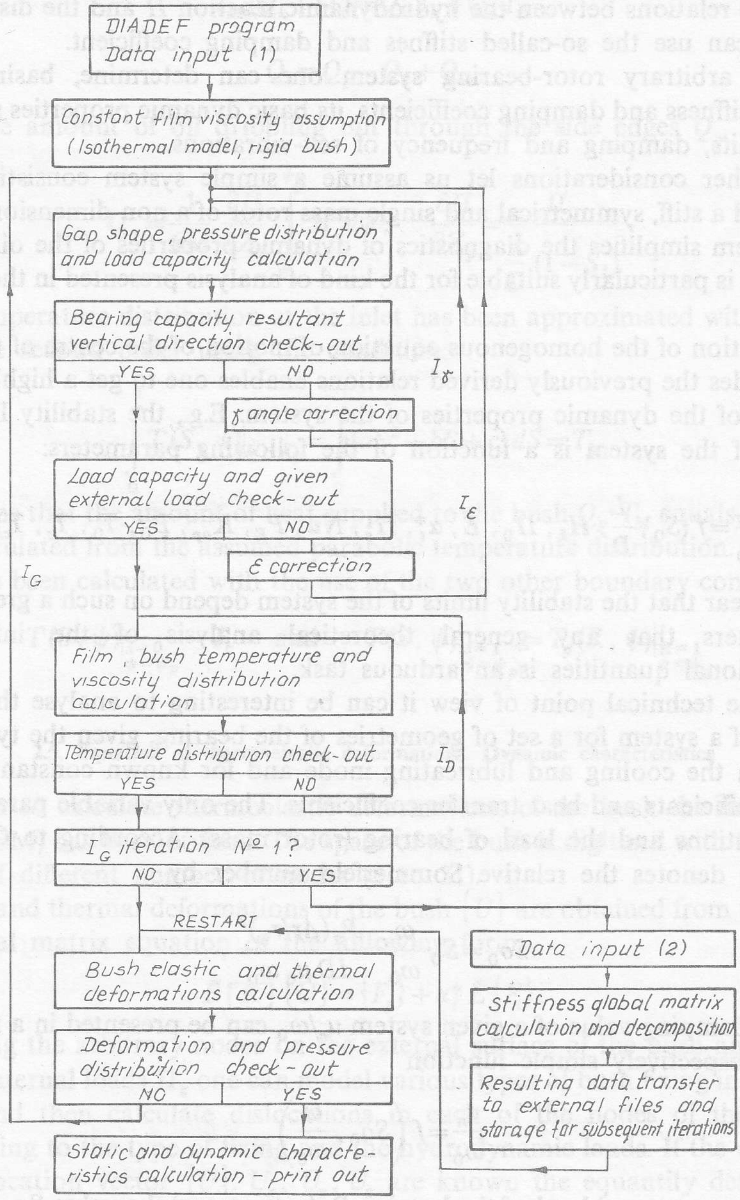
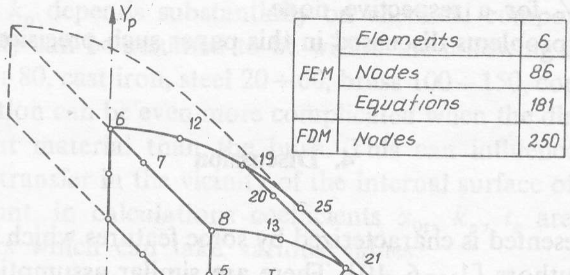


Fig. 2. Block diagram of calculation procedure for DIADEF program (I_γ – angle γ calculation loop, I_ε – ε calculation loop, I_D – diatermic loop, I_G – main iteration loop)

3. Algorithm of calculations

Equations (1)÷(6) form an inter-conjugated set of partial differential equations. The solution of this set can be obtained only by an iterative method choosing the most efficient system of iteration loops.

As it has been mentioned at the beginning of the paper, at the Institute of Fluid Flow Machinery a suitable algorithm and a computer program named DIADEF have been developed. These enable one to solve the set of equations and to determine the static and dynamic characteristics of the rotor-bearing system.



FEM	Elements	6
	Nodes	86
	Equations	181
FDM	Nodes	250

Fig. 3. Scheme of digitizing a bush with a finite element net (FEM)

Fig. 2 shows the calculation procedure for a given bearing geometry, given fixing and feeding mode, given material coefficients and set values of load and rotor revolutions, i.e. So_0 and ω_r/ω_0.

The solutions $X^{\nu+1}$ (distributions of the pressure, temperature and deformations) have been obtained with the use of the relaxation method of the following type

$$X^{\nu+1} + X^{\nu-1} + \xi(X^{\nu} - X^{\nu-1}).$$

The best convergence of the iterative process has been obtained for the loop determining the temperature with the relaxation coefficient ξ equal to 0.02÷0.1, and for the loop determining the deformation with $\xi = 0.2 \div 1.0$.

In the following paper the calculation has been performed using a 6-element digitalization of a bush. (For the half of a bush both in $Y_p - Z_p$ and in $Y_p - X_p$ plane – the principle of division in symmetric and antisymmetric parts was utilized). The scheme of such digitalization (FEM net) together with nodes numbering is shown in Fig. 3. The authors of ref. [11] have performed an analysis of the influence of the number of elements on the results. This analysis has been carried out for a model load and two versions of the external fixing (free bush and a cross fixing). The calculations have been made using discretization with 6, 36 and 72 elements. The biggest discrepancies in the computed deformation have been obtained within the limits of 2 – 10% for a respective node.

In the case of problems discussed in this paper such precision seems to be quite sufficient.

4. Discussion

The model presented is characterized by some features which can be found in the works of other authors [1 – 6, 10]. There are similar assumptions concerning the heat exchange in the oil film, and the deformable parts of the bearing, as well as the exchange of heat with the surroundings (free convection). However, the distinctive feature of the present model is a different set of initial conditions for the equation of energy. These conditions have been established basing on an attempt to obtain the heat balance in lubricating grooves which requires the assumption of the total mixing of the oil. The computer program based on this model is a coherent synthesis of elements connected with the diatermic heat model and of thermoelastic deformations in the bearing.

With the above considerations in mind, one may be tempted to make several more general remarks about the model presented as well as other models which are known from the literature.

Despite its complexity, the presented theoretical model is only an approximation of a real tribologic system: the journal-lubricating film-bush-external fixing. The model is based on the assumption that the total heat in the system is generated only by the lubricating film. In real systems heat can be supplied to bearing e.g. through the journal. There are cases when the amount of supplied heat can be evident (for example: thermal rotary machines with slide bearings). The assesment of the mean temperature of the journal can pose a difficult problem. The assumption of average temperature on internal surface of the bush is doubtful in such a case. The choice of temperature t_c can be based only on experimental measurements (if such ones exist) or on an estimation.

The condition, commonly used in the literature, of heat transfer on the external surface of the bush by free convection is scarcely encountered in real situations.

In laboratory research one can have a free bush. However, in practical applications there ase usually bush fixing elements in bearing supports. Hence, it is difficult to determine the heat transfer process.

The condition of free convection heat transfer to the surroundings cannot be applied. Besides, the fixing elements can exchange heat with external sources.

The proper determination of such coefficients as α_{0t} (the coefficient of free convection heat transfer) or k_p (the heat transfer coefficient in bush material) is a significant problem. Their values are strongly dependent on actual conditions of heat transfer and the chemical composition of the material used. E.g., in the literature concerning the two-phase flow one can find values ranging between 50 and 10 000 kg/s^3 K. Some authors use $\alpha_{0t} = 80$ kg/s^3 K when dealing with the problem of slider bearings, [5].

The coefficient k_p depends substantially on chemical composition of the bush material and usually can be assumed to be within the following limits: white metal $20 \div 40$, bronze $25 \div 80$, cast iron, steel $20 \div 60$, brass $100 \div 150$, copper $350 \div 390$ [kg m/s^3 K]. The situation can be even more complicated when the direct sliding layer is made of a different material than the bush. This can influence and change the conditions of heat transfer in the vicinity of the internal surface of the bush. Taking all this into account, in calculations coefficients α_{0t}, k_p, t_c are often treated as variable parameters which can take various values.

As far as the thermal model is concerned, the bush bearing has been treated as a thick-walled complete or partial ring without such structural elements as dividing planes, grooves, drillings and oil offtake channels. This obvious simplification of the real structure is necessary for a theoretical description of problems described by the conduction equation.

The mixing of cold and hot oil poses another difficult question. In the present model the following assumption has been made the cold oil is completely mixed with the hot one and as a result the temperature in the oil groove is constant and equal to t_M. However, the complexity of all the phenomena involved restricts the range of validity of such a simple model.

Even a very expanded theoretical model, based on the scheme that the only heat source is the lubricating film and the heat is transferred by the simple structural elements of the bearing, gives only a simplified description of the reality. Thus, some authors find complex theoretical models useless, as these are often based on general assumptions which are difficult to check. Instead, simple models based on experimental data valid within particular operating conditions are thought to be more useful.

The above conclusions, which point at the complex nature of phenomena taking place in slider bearing and emphasize the problems and the limitations connected with formulating a theoretical descriptions, cannot diminish enormous advantages and possibilities offered by a theoretical model of this kind. It is particularly useful for comparison or an optimisation calculus. Experimental data are very expensive to obtain and sometimes simply not available from the technological point of view. A theoretical description is in many cases the only source of information both for designers and researchers of bearings.

Results of an experimental verification of the presented model are given in the second part of this paper. The influence of the external fixings of the bush on the static and dynamic properties of a simple rotor-bearing system are presented there. The results give a good illustration of the theoretical description using the proposed model and the computer program.

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Wpływ termosprężystych deformacji panwi oraz jej utwierdzeń zewnętrznych na statyczne i dynamiczne własności poprzecznych łożysk ślizgowych

Część I. Termoelastohydrodynamiczny model łożyska

Streszczenie

W pracy przedstawiono termosprężysto-hydrodynamiczny model łożysk ślizgowych. Model obejmuje wymianę ciepła między filmem olejowym, panwią i otoczeniem. Przedstawiono model równania równowagi cieplnej w osiowym rowku olejowym oraz równania odkształceń sprężystych i cieplnych panwi. Odkształcenia panwi liczone są metodą elementów skończonych. Możliwość ustalenia dowolnych węzłów na zewnętrznej powierzchni panwi oraz zmiennego obciążenia umożliwia modelowanie mocowania panwi w podporach łożyska. Dynamiczne własności łożyska są opisane za pomocą równania ruchu dla małych przemieszczeń środka czopa. Równania opisujące model teoretyczny zostały dopasowane do wymagań stawianych obliczeniom numerycznym. Algorytm obliczeń i program komputerowy zostały również opisane w pracy.

Влияние термоупругих деформации вкладыша и его внешних утверждений на статические и динамические свойства поперечных подшипников скольжения

Часть I. Термоэластогидродинамическая модель подшипника

Резюме

В работе представлена термоупругогидродинамическая модель подшипников скольжения. Модель охватывает теплообмен между масляной плёнкой, вкладышем и окружностью. Представлена модель – уравнение теплового равновесия в осевой масляной канавке, а также уравнения упругих и термических деформаций вкладыша. Расчёт деформаций вкладыша производится методом бесконечных элементов. Возможность устанавливания произвольных узлов на внешней поверхности вкладыша а также переменной нагрузки обеспечивает моделирование крепления вкладыша в опорвх подшипника. Динамические свойства подшипника описываются управнениями движения для небольших перемещений центра цапфы. Уравнения описывающие теоретическую модель приспособлены к требованиям предъявляемым численным расчётам. В работе описываются также алгоритм и программа на ЭВМ.