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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

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Liquid and liquid-gas cooling of machine elements

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Abstract

In the work presented are thermal and hydraulic problems concerned with the liquid films formed by impinging jets. Formulated has been a simple two-dimensional model of the flow and heat transfer in the film. The model is based on simplified equations of mass, momentum and energy balance. Solution of such set of equations enables determination of velocity profile in the film as well as local heat transfer coefficients.

Keywords: Liquid-gas cooling; Impinging jet; Thin film

Nomenclature

c_p	–	heat capacity	ν	–	kinematic viscosity
Fr	–	Froude number	α	–	surface inclination angle
q	–	specific volumetric flow rate of liquid in the film	δ, h	–	boundary layer thickness, film thickness
			λ	–	thermal conductivity
Q	–	volumetric flow rate	ρ	–	density
r, z, ϕ	–	co-ordinates	σ	–	surface tension
T	–	time	v, w, u	–	velocity components: radial, normal and circumferential
T	–	temperature			

Subscripts

c	–	critical	s	–	singular point
g	–	gas	tr	–	border between the developing and fully developed regions
i	–	interface surface			
in	–	initial value	w	–	wall
k	–	droplet	0	–	initial value
m	–	mean			

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1 Introduction

Liquid (film) flow down the surface of the wall occurs in a variety of cooled power engineering installations, heat exchangers and many chemical engineering appliances. Such heat exchangers are called condensers. The process of liquid flow in spray cooled heat exchangers is different from that in condensers. Both processes have been investigated theoretically and experimentally. The flow of gravity driven liquid films formed due to vapour condensation is particularly richly represented amongst the literature. This type of flow was investigated in [1-3] for different pipe geometry including inclined cylindrical or elliptical pipes and cylindrical finned pipes. In the case of gravity driven flow, the trajectory of the liquid film motion is determined by the steepest gradients of the gravitational slope. The inertia forces are neglected then. However, in the case of impingement of a liquid jet on the surface, the inertia forces must be taken into account as they have a great effect on the trajectory of a forming liquid film. The process of a liquid film flow with a specified velocity after impingement on a flat horizontal surface was investigated both theoretically and experimentally in [4-5]. There is a lack of theoretical solutions on the flow distribution on inclined surfaces. Such solutions can serve for the development of models of liquid distribution on horizontal tubes or even more complex geometries, which can be found in cooled elements of machinery and condensers. The analysis conducted in the paper is aimed at investigation of the phenomenon of liquid distribution on the plate arbitrarily oriented in the space.

Another way of cooling of heat transfer surfaces, requiring less amount of liquid, is spray cooling using water-gas jets.

Surface wetting by means of impinging two-phase jet consisting of gas and liquid droplets is used for a more intense cooling. Particularly intense is cooling, when the film evaporates. Such type of surface cooling can be applied in heat exchangers, chemical engineering apparatus, electronic equipment with high thermal loads or computers of large power. The process of distribution of the liquid layer formed from the droplets on the wall was investigated theoretically and experimentally not by too many authors. Up till now it has yet to be fully understood. A correct mathematical description is still missing. Works known from literature [1-2] provide only very simplified models. The inertia forces are neglected, which in the case of the jet impingement cannot be disregarded as the kinetic energy of the jet approaching the wall decreases whereas the pressure increases. At the point of impingement the kinetic energy is equal zero and the pressure is at its highest. In the region of impingement of the axisymmetric jet a radial gas flow starts to develop. The pressure reduces and the radial gas velocity increases. Therefore there exists the radial pressure gradient, which influences the liquid film motion. The atomised liquid phase contained in gas in the form of droplets separates on the wall due to bigger than gas inertia, which lead to deviation with respect to gas streamlines. The droplets form a liquid film, which flows out radially. In the vicinity of stagnation the flow is laminar. In the work considered are thin liquid films moving with the laminar character on the wall. The process of impinging liquid distribution on a horizontal surface was investigated both theoretically and experimentally in [4-5]. The aim of the present work is to understand phenomena of liquid distribution in the film formed by the impingement of a jet of gas with liquid droplets on the surface. Developed has been model of the phenomenon based on integral equations of liquid film motion on the wall. Numerical calculations have been performed to illustrate the method.

2 Motion of liquid film resulting from liquid and gas-liquid jet

2.1 Motion of liquid film produced by the liquid jet

A flat surface, inclined by the angle α, is presented together with the co-ordinate system in Fig. 1. A cylindrical system of co-ordinates has been assumed. The axial co-ordinate is perpendicular to the inclined surface. A two-dimensional flow is considered described by the co-ordinates r and ϕ. Both co-ordinates are in the plane of considered surface, where the jet with the volumetric rate Q impinges.

It has been assumed in the analysis that

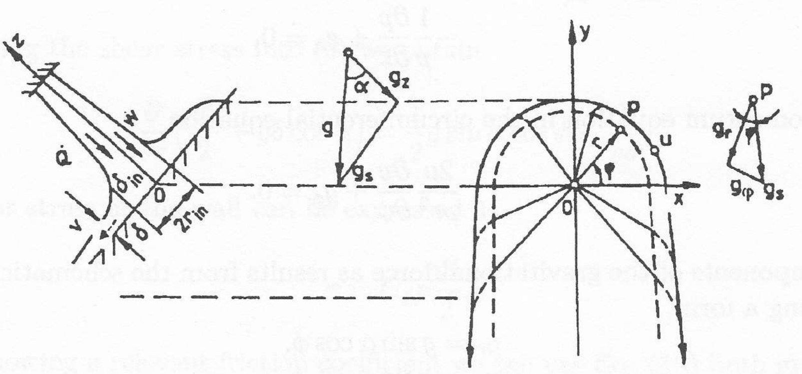

Figure 1. Impingement of a liquid jet on an flat inclined surface: a) physical situation, b) system of coordinates.

- The flow of the liquid film is steady, laminar and governed solely by the action of the gravity and inertia forces;

- The surrounding medium is at rest and there is no shear stress at the interfacial surfaces;
- Liquid-wall and liquid-vapour interfacial surfaces are smooth;
- Physical properties of the liquid are constant;
- The liquid film thickness is small, which justifies simplifications in the mass and momentum conservation equations;
- Velocity across the film is constant and equal to the average velocity in the film. The average velocity is a function of the co-ordinates r and ϕ;
- The circumferential and vertical (normal) components of velocity vanish ($u = w = 0$). That means that the radial component of the velocity and the film thickness are functions of r, ϕ.

In accordance with outlined above assumptions a full set of conservation equations of mass and momentum written in cylindrical co-ordinates, after implementation of the above simplifications yields:

- continuity equation

$$\frac{1}{r}\frac{\partial(rv)}{\partial r}+\frac{\partial(w)}{\partial z}=0,$$

- momentum equation in radial direction

$$v\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}=g_r-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\frac{\partial^2 v}{\partial z^2}, \tag{1}$$

- momentum equation in the normal direction

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z=0,$$

- momentum equation in the circumferential equation

$$\frac{2\mu}{\rho r^2}\frac{\partial v}{\partial \phi}+g_\phi=0.$$

The components of the gravitational force as results from the schematic in Fig. 1 are having a form

$$\begin{aligned} g_r &= g\sin\alpha\cos\phi,\\ g_\phi &= g\sin\alpha\sin\phi,\\ g_z &= g\cos\alpha. \end{aligned} \tag{2}$$

The mass balance in the limits of liquid film thickness is obtained by integration of the first equation in (1) in the limits of the liquid layer

$$\frac{\partial(rv\delta)}{\partial r}=0. \tag{3}$$

Integrating and comparing against the volumetric flow rate of impinging jet we obtain

$$Q = 2\pi r v \delta. \tag{4}$$

On the other hand, by integration in the z direction we get the pressure distribution

$$p = \rho g \delta \cos\alpha + p_0, \tag{5}$$

where, after differentiation we get

$$\frac{\partial p}{\partial r} = \rho g \frac{\partial \delta}{\partial r} \cos\alpha, \tag{6}$$

From the momentum equation in the circumferential equation we get

$$\frac{\partial v}{\partial \phi} = -\frac{g\rho}{2\mu} r^2 \sin\alpha \cos\phi, \tag{7}$$

Differentiating (7) we obtain

$$\frac{\partial^2 v}{\partial \phi^2} = \frac{g\rho}{2\mu} r^2 \sin\alpha \sin\phi. \tag{8}$$

Substituting (6) and (8) into the radial equation of motion integrated in the z direction and incorporating (2) we obtain after some re-arrangements

$$\frac{\partial}{\partial r}\Big(\frac{v^2}{2} + g\delta\cos\alpha\Big) = \frac{1}{2} g \sin\alpha\cos\phi - \frac{1}{\rho\delta}\mu\Big(\frac{\partial v}{\partial z}\Big)_w. \tag{9}$$

Introducing the shear stress into (9) we obtain

$$\frac{\partial}{\partial r}\Big(\frac{v^2}{2} + g\delta\cos\alpha\Big) = \frac{1}{2} g \sin\alpha\cos\phi - \frac{1}{\rho\delta}\tau. \tag{10}$$

The shear stress at the wall can be expressed as

$$\tau = C_f \rho \frac{v^2}{2}. \tag{11}$$

Then, knowing a relevant friction coefficient we can use Eq. (10) both in laminar and turbulent flows.

The relation (10) together with the mass balance Eq. (4) forms a closed set of equations enabling determination of velocity field as well as local film thickness. According which quantity we want to determine, i.e. either velocity or film thickness, we can formulate adequate boundary conditions to the problem. Both, velocity and film thickness at the initial radius r_{in} are assumed as known and equal to δ_{in} and v_{in}. Let's introduce the film thickness from the mass balance Eq. (4) and the relation (11) to the momentum balance Eq. (10). In effect, after some transformations we get

$$\frac{dv}{dr} = \frac{\dfrac{gQ\cos\alpha}{2\pi r^2 v} + \dfrac{g\sin\alpha\cos\phi}{2} - \dfrac{C_f v^3 \pi r}{Q}}{v - \dfrac{gQ\cos\alpha}{2\pi r v^2}}, \tag{12}$$

where C_f – is a friction coefficient which can be determined from the Blasius relation in the case of turbulent boundary layer as

$$C_f = \frac{0.664}{\sqrt{\dfrac{v(r - r_{in})\rho}{\mu}}}, \tag{13}$$

where r_{in} – is the inlet radius of the liquid layer.

Integration of Eq. (12) gives a relationship for the velocity as a function of the radius r or expressing the velocity in terms of the film thickness from Eq. (4), a relationship between the film thickness and the film radius can be found. The direction of the integral curve is determined either by the value of the velocity derivative or the film thickness derivative with respect to the radius. The derivatives tend to infinity if the denominator in Eq. (12) approaches zero. This situation is characteristic for the so-called *turning point.* Physically it refers to the critical flow conditions. Let us find the criticality conditions equating the denominator in Eq. (12) to zero. Then we have

$$v_c = \left(\frac{gQ\cos\alpha}{2\pi r_c}\right)^{1/3}, \tag{14}$$

or in the dimensionless form

$$\frac{2\pi r v_c^3}{gQ} = \frac{v_c^2}{g\delta_c} = Fr_c^2 = \cos\alpha. \tag{15}$$

The relation (15) for the flat horizontal surface was obtained in [5]. It follows from (15) that the parameters of the critical flow are local and depend neither on the integration path nor on the friction coefficient. Therefore, they are the same also in the case of inviscid flow without friction.

In the case when both the numerator and denominator in (12) are simultaneously equal to zero, then the derivative, which locally determines the direction of the integration path is indefinite and we obtain a *singular point.* That situation also refers to the critical conditions given by Eq. (14).

Equating the numerator and denominator of (12) to zero, one can obtain the dimensionless co-ordinates of the singular point as follows

$$r_s = \left(\frac{2Q^2 \cos^2 \alpha}{\pi^2 g (C_f \cos\alpha - sin\alpha \cos\phi)^3}\right)^{1/5}, \tag{16}$$

$$v_s = \left(\frac{gQ\cos\alpha}{2\pi r_s}\right)^{1/3}. \tag{17}$$

The co-ordinates of the singular point in the dimensionless form have a following form

$$Fr_s = \sqrt{\cos\alpha}, \tag{18}$$

$$R_s^+ = \frac{r}{\frac{6}{C_f}\left(\frac{v^2}{g}\right)^{1/3} Re_s^{2/3}} = \frac{1}{3}\frac{\cos^{2/3}\alpha}{\cos\alpha - \sin\alpha\frac{\cos\varphi}{C_f}}, \tag{19}$$

where $Re_s = \frac{v_s \delta_s \rho}{\mu}$.

It results from (16) that a singular point can appear only in the gravity driven flow, even in the case of an ideal liquid flow without friction. The dependence between the Froude number and the dimensionless radius of the film, as a function of the inclination angle, are given in Fig. 2a and Fig. 2b. The relation between the Froude number and the dimensionless film radius is presented in Fig. 2c. The behaviour of the integral curves in the vicinity of the singular point can be

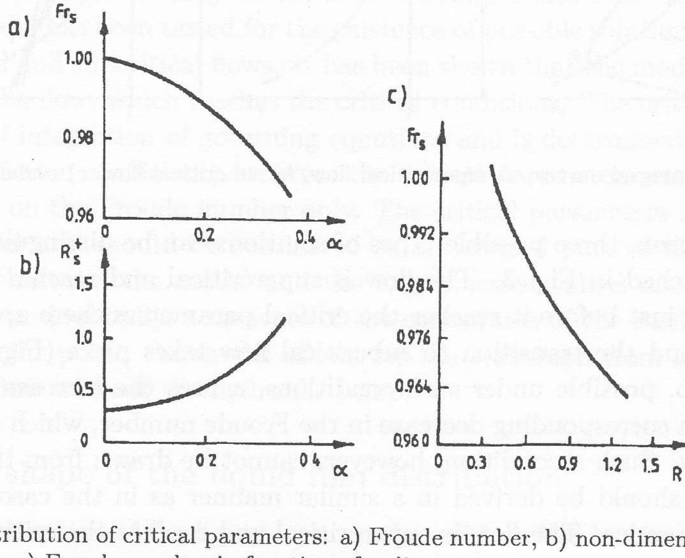

Figure 2. Distribution of critical parameters: a) Froude number, b) non-dimensional film thickness, c) Froude number in function of radius.

investigated by linearisation of the left hand side of Eq. (27). In order to calculate the slope of the integral curve in the singular point $\left(\dfrac{dv^+}{dr^+}\right)_s = \eta$ let's unveil the function into the Taylor series, both the nominator V and the denominator R, (12) and then apply the de l'Hospital theorem

$$\eta = \left(\frac{dv^+}{dr^+}\right)_s = \frac{\dfrac{dV}{dr^+} + \dfrac{dV}{dv^+}\dfrac{dv^+}{dr^+}}{\dfrac{dR}{dr^+} + \dfrac{dR}{dv^+}\dfrac{dv^+}{dr^+}}. \tag{20}$$

The above relation is identical with the characteristic equation

$$\frac{dR}{dv^+}\eta^2 + \left(\frac{dR}{dr^+} - \frac{dV}{dv^+}\right)\eta - \frac{dV}{dr^+} = 0. \tag{21}$$

According to values of the roots of (21) we get different types of singular points. These are: the saddle point, nodal point and spiral point. An example of the nodal point has been shown in Fig. 3c.

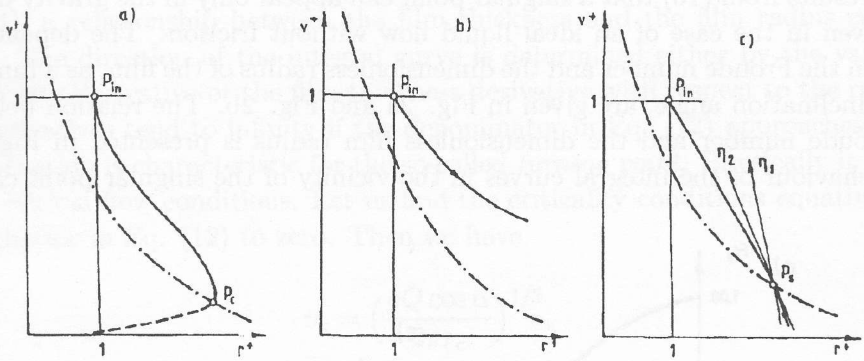

Figure 3. Integral curves; a) supercritical flow, b) subcritical flow, c) nodal point.

As can be seen, three possible types of solutions can be distinguished, which have been sketched in Fig. 3. The flow is supercritical and reaches the critical parameters, or just before it reaches the critical parameters there appears a hydraulic jump and the transition to subcritical flow takes place (Fig. 3a). The hydraulic jump, possible under such conditions, causes the increase in the film thickness and a corresponding decrease in the Froude number, which leads to the subcritical flow. Such a condition, however, cannot be drawn from the analysed equations and should be derived in a similar manner as in the case of a shock wave in gasdynamics. The flow is supercritical and reaches the critical parameters at a singular point. Depending on the nature of the singularity the flow can turn into a subcritical one (Fig. 3c). The flow is subcritical and never reaches the critical parameters, (Fig. 3b).

The supercritical flow can reach critical parameters at a turning point. In this case, in order to integrate Eq. (12) within the whole range of supercritical and subcritical flow it is necessary to reverse the status of the dependent and independent variable. Then the turning point becomes the extremum. The curve referring to the critical parameters is also shown in the figure. The curve divides the plane of solutions into regions of supercritical and subcritical flow. Starting from the parameters corresponding to the supercritical conditions, it is possible to reach the critical parameters. It follows from the figure that the integral curve has a maximum for a certain value of the film radius r, which from the physical point of view means that the flow can be supercritical up to a maximum value of radius (the critical radius). The hydraulic jump can occur in response to a variety of disturbances before the flow reaches the critical conditions and the flow will convert to a subcritical flow, described by the part of solution towards left from the extremum. The subcritical part of the curve has the physical meaning only when the transition from supercritical to subcritical flow takes place due to the hydraulic jump as a decrease in the film radius otherwise is not physically possible.

A formulated simple model of liquid jet impingment on an inclined surface consisting of conservation equations of mass and momentum with closing equations may serve, with some modifications, in the development of the liquid jet distribution on the cylindrical surface and subsequently in calculations of heat transfer on such surfaces. As mentioned earlier that is concerned with calculations of performance of spray cooled heat exchangers and other installations.

The model has been tested for the existence of possible solutions in the case of supercritical and subcritical flows. It has been shown that the model is capable of describing the flow, which reaches the critical conditions. The critical film radius is a result of integration of governing equations and is determined by the closure equation (friction coefficient), whereas the critical parameters are the local ones and depend on the Froude number only. The critical parameters are determined by the model and should be considered as an integral part of the model. The consistency between the model and the experimental values of the critical parameters is, undoubtedly, a measure of the correctness of the model. In the case with a singular point, the model allows for the transition from supercritical to subcritical flow without the hydraulic jump.

2.2 The shape of the liquid film distribution

The distribution of the liquid layer on the solid surface has its specifics. That will be presented below. The momentum of the jet impinging on the surface is recovered by the friction forces, and finally balanced by the surface tension forces. In such way the wetted surface area is limited.

Let's assume that the liquid flows into the sector tangentially, picks up the liquid flowing in the sector and leaves the sector tangentially, as shown in Fig. 4.

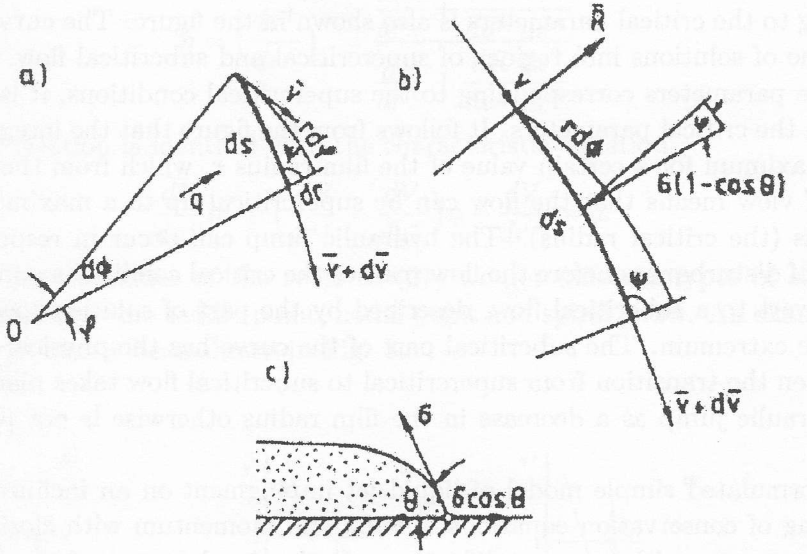

Figure 4. A schematic of the liquid distribution in the elementary segment $d\phi$; a) velocity distribution at the jet border, b) surface tension acting on the element of the jet border, c) balance of surface tensions acting on the liquid film

Let's consider the momentum balance on the elementary distance of the liquid border (Fig. 4)

$$v\delta\rho\Delta\vec{v}rd\phi = \vec{\sigma}(1-\cos\theta)ds. \tag{22}$$

On the left hand side there exists the momentum flux, whereas on the right hand side there are surface tension forces of liquid and gas as well as of the solid, as shown in Fig. 4. The balance of forces in the direction normal to the jet border gives an equation

$$q\rho v\cos\psi rd\phi = \sigma(1-\cos\theta)\sin\psi ds, \tag{23}$$

where $q = Q/2\pi r$ is a specific volumetric flux of liquid in the film.
The length of the element of the border ds can be expressed as:

$$ds = \sqrt{1+\left(\frac{dr}{rd\phi}\right)^2}, \tag{24}$$

and

$$\operatorname{ctg}\psi = \frac{dr}{r d\phi}. \tag{25}$$

Introducing the length of the arc (24) and $ctg\psi$ (25) into (23) we get

$$\frac{\rho q}{\sigma(1-\cos\theta)} = \frac{\sqrt{1+\left(\frac{dr}{rd\phi}\right)^2}}{v\left(\frac{dr}{rd\phi}\right)^2}. \tag{26}$$

Solving Eq. (26) with respect to $\frac{dr}{rd\phi}$ we get

$$\frac{dr}{d\phi} = \pm\frac{r}{\sqrt{\left(\frac{\rho q}{\sigma(1-\cos\theta)}\right)^2 v - 1}}. \tag{27}$$

Assuming that the liquid layer thickness does not vary a lot and can be regarded as constant as well as that the entire liquid flows into the sector with the central angle ϕ and then flows tangentially to the border, then the continuity equation in the following form holds

$$Q = 2\pi r_{in} q_{in} = \phi r v \delta \cong \phi r_{in} q. \tag{28}$$

Introducing the continuity equation into (27) we get

$$\frac{dr}{d\phi} = \pm\frac{r}{\sqrt{\left(\frac{2\pi\rho q_{in} v_{in}}{\sigma\phi(1-\cos\theta)}\right)^2 v^2 - 1}}. \tag{29}$$

The boundary condition for that equation is a value of the radius r_b

$$\text{for} \qquad \phi = \pi/2, \qquad r^+ = r_b^+.$$

Due to the symmetry of impinging jet a value of the limiting radius can be determined from the integration until the specified velocity is reached

$$v_b^+ = \frac{1}{v_{in}}\sqrt{\frac{\sigma(1-\cos\theta)}{\rho}}. \tag{30}$$

That velocity results from the balance between the momentum and surface tension forces.

A simple model of the shape of the jet includes the influence of principal parameters influencing the liquid distribution, for example surface tension of solid and liquid, and liquid and gas or the surface tension of liquid and gas and the wetting angle. As mentioned earlier this is connected with calculations of spray cooled heat exchangers. In the case of supercritical film flow the wetted surface is often determined by the hydraulic jump, as seen from experiments.

2.3 Two-phase liquid-gas jets

A two-phase jet impinging normally a flat surface, together with the co-ordinate system is schematically shown in Fig. 5. A cylindrical co-ordinate system has been assumed. The z co-ordinate is perpendicular to the surface. Considered is a two-dimensional axially symmetrical flow of gas and liquid flowing out from the nozzle with the rate Q, described by the co-ordinates r and z.

In the analysis it has been assumed that:

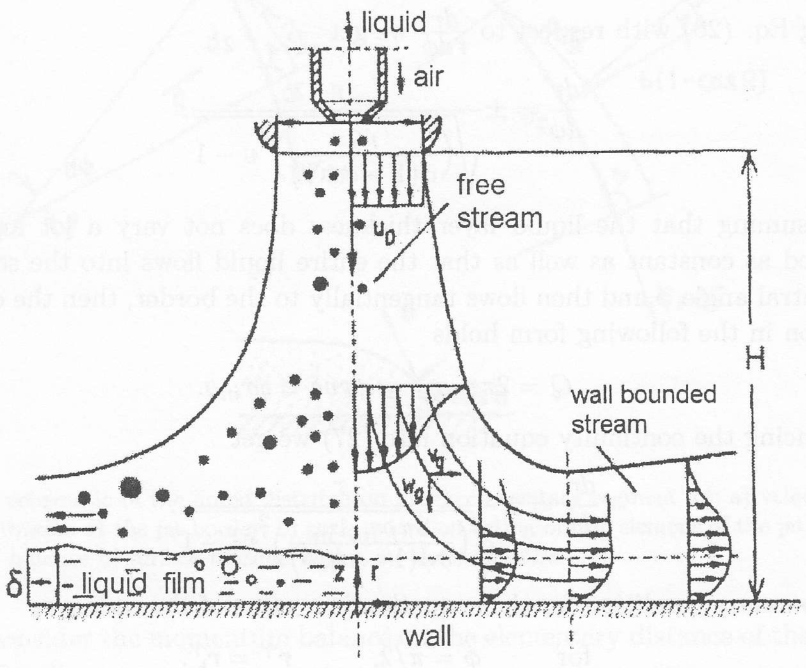

Figure 5. Schematic of a two-phase jet wetting the surface

- Liquid film motion is laminar and fully developed, and caused by the motion of the gas phase;
- Gas phase flowing axially symmetric out from the nozzle causes the shear stress at the liquid film surface and feeds that film with liquid at the rate q;
- Phase interfaces, liquid-wall and liquid-vapour are smooth;
- Physical properties of liquid are constant;
- The liquid film thickness is small, which enables assumption of a linear velocity profile in the z direction and the assumptions of the boundary layer in the equation of motion;
- Gravitational force is neglected;

- The circumferential component of velocity is equal zero due to symmetry ($u = 0$). That means, that the radial component and the film thickness are functions of the radius r.

In accordance with assumptions a full set of conservation equations in cylindrical co-ordinates, after simplifications can be cast in the form

$$\frac{1}{r}\frac{\partial(rv)}{\partial r} + \frac{\partial(w)}{\partial z} = 0,$$

$$v\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial z^2}, \tag{31}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = 0.$$

Boundary conditions for the problem are as follows

$$\begin{aligned} &z = 0, \quad w = v = 0, \\ &z = \delta, \quad v = v_i, \quad \frac{\partial v}{\partial z} = \frac{\tau_i}{\mu} \cdot \end{aligned} \tag{32}$$

From the latter Eq. (31) it results, that pressure across the liquid film is constant, just as in the boundary layer, and depends only on the r co-ordinate. The pressure gradient and shear stresses at the interface surface, in the case of film, can be determined from the radial flow of the gas stream under the assumption that the film is thin and its thickness can be neglected in the gas flow. Analytical solution for the axially-symmetric gas flow has been given by White (1974) and Schlichting (1980), [6]. Here we consider the flow in the boundary layer with the pressure gradient. Pressure distribution in the flow is described by the relation (Schlichting)

$$p_g = p_0 - \frac{1}{2}\rho_g a^2 (r^2 + f(z)). \tag{33}$$

Velocity components of gas are described by the relations (Martin, 1977), [6]

$$\begin{aligned} &v_g = arF'(\eta), \quad w_g = -2az \\ &a \approx \Big(\frac{V_D}{D}\Big)\Big(1.04 - 0.034\frac{H}{D}\Big), \quad \eta = z\sqrt{\frac{a\rho_g}{\mu_g}} \cdot \end{aligned} \tag{34}$$

Martin stated, that in the case of axially-symmetric nozzle a value of the coefficient a can be expressed by the geometric parameters as from the Fig. 5.

Shear stress at the border between gas and liquid have been calculated based on the gas flow. These are

$$\tau_i = \mu_g \frac{\partial v_g}{\partial z}\Big|_0 = 1.312\sqrt{a^3 \rho_g \mu_g r}, \tag{35}$$

as: $F''(0) = 1.312$.

In order to find an approximate solution to the problem, an integral form of balance equations was decided to be developed [7].

The mass balance in the limits of the liquid film thickness can be obtained by integration of the first equation from (31) in the limits of the liquid layer. Differentiating the integral according to the rules of the integral differentiation with the parameter and utilising the boundary conditions we get

$$\frac{\partial}{\partial r}\int_0^\delta (rv)dz + \frac{\partial \delta}{\partial t} r = 0. \tag{36}$$

Assuming

$$\frac{\partial \delta}{\partial t} = -q = const. \tag{37}$$

Introducing (37) into (36) we obtain the mass balance equation in the form

$$\frac{\partial}{\partial r}\Big[r\int_0^\delta v dz\Big] - qr = 0. \tag{38}$$

Integrating (38) in the limits from 0 to r and multiplying by 2π we obtain a form of mass balance equation provided in [1]

$$2\pi r\int_0^\delta v dz = \pi r^2 q. \tag{39}$$

An integral momentum equation is obtained by integration of particular terms of the second equation from the set of Eq. (31) and then

$$\frac{\partial}{\partial r}\int_0^\delta v^2 dz - v_i q + \frac{1}{r}\int_0^\delta v^2 dz = \frac{\tau_i - \tau_w}{\rho} - \frac{1}{\rho}\frac{\partial p}{\partial r}\delta. \tag{40}$$

In order to solve the above problem let's assume the simplest, linear velocity profile in the form

$$v = C(r)z. \tag{41}$$

The coefficient $C(r)$ and the film thickness can be determined from the integral balance equations,

$$C(r) = \frac{qr}{\delta(r)^2}, \tag{42}$$

and next the wall shear stress can be determined in the form

$$\tau_w = \mu \frac{\partial v}{\partial z} = C(r)\mu = \frac{qr\mu}{\delta(r)^2}. \tag{43}$$

Let's assume initially, that the inertia forces are small and can be neglected (left hand side of (40)). Then the following relation can be obtained

$$q = 1.312\sqrt{\frac{a^3 \rho_g \mu_g}{\mu^2}\delta^2 + \frac{a^2 \rho_g}{\mu}\delta^3}, \tag{44}$$

analogical to the one obtained in [6]. The difference is that in the second term of the right hand side in [6] there appears the coefficient equal to 2/3.
From the full momentum Eq. (40) we get

$$\frac{d\delta}{dr} = -\frac{3}{q^3 r}\left(\frac{1.312\sqrt{a^3 \rho_g \mu_g \delta^2}}{\rho} + \frac{\rho_g a^2 \delta^3}{\rho} - \frac{q\mu}{\rho}\right). \tag{45}$$

Introducing the non-dimensional variables

$$r^+ = \frac{r}{r_0}, \qquad \delta^+ = \frac{\delta}{r_0}, \tag{46}$$

where r_0 – a characteristic radius.
The relations (44) and (45) have a following form

$$q = 1.312 r_0^2 \sqrt{\frac{a^3 \rho_g \mu_g}{\mu^2}\delta^{+2} + \frac{a^2 \rho_g}{\mu} r_0 \delta^{+3}}, \tag{47}$$

$$\frac{d\delta^+}{dr^+} = -\frac{3}{q^2 r^+}\left(\frac{1.312\sqrt{a^3 \rho_g \mu_g r_0 \delta^2}}{\rho} + \frac{\rho_g a^2 r_0^2 \delta^{+3}}{\rho} - \frac{q\mu}{\rho}\right). \tag{48}$$

In order to solve (48), we need to know the film thickness for the zero radius or the film thickness after the stabilisation distance, where the inertia forces can be neglected. As shown from the experiments the film thickness at the centre of the jet is not equal zero. The film thickness after stabilisation is determined by the relation (47).

The shear stress at the wall can be generally expressed as

$$\tau_w = C_f \rho \frac{v^2}{2}. \tag{49}$$

Then, knowing the friction coefficient we can use (40) both in the laminar and turbulent flow.

Relation (40) together with the mass balance Eq. (38) is a closed set of equations enabling determination of local film thickness.

Integrating (48) we can obtain the dependence of liquid layer thickness on the radius.

Performed have been calculations for the parameters close to those, at which conducted have been experimental investigations in [6], i.e. a jet of water droplets atomised by the nozzle in air, which approximately, based on relation (36), can be characterised by the parameter $a = 1500\ s^{-1}$. Assumed in calculations has been the air pressure of $p_g = 240$ kPa and the characteristic radius equal to $r_0 = 2{\cdot}10^{-4}$ m. For the assumed data the stabilised liquid film thickness δ_s has been determined from (47) in dependence from the wetting rate q. The results are presented in Fig. 6.

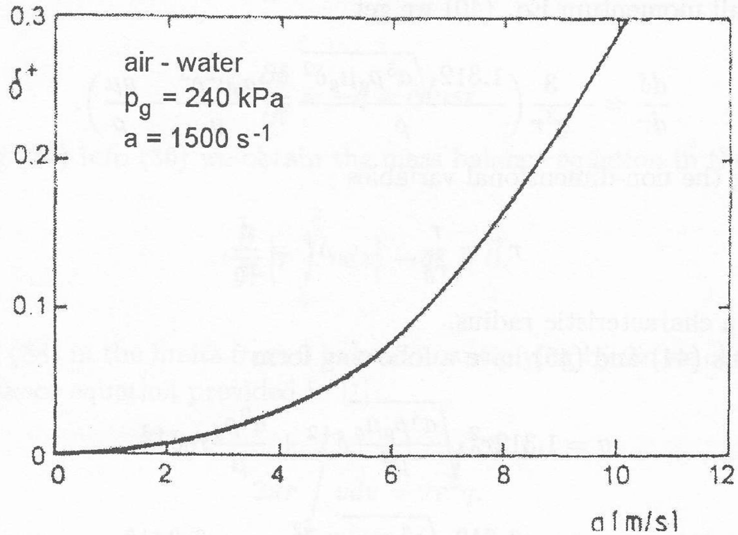

Figure 6. Dependence of the non-dimensional film thickness on the unit volumetric wetting rate.

Knowing the wetting rate, we are able to determine the stabilised thickness of liquid film. Assuming a unit volumetric flow rate of water equal to $q = 1\ \dfrac{l}{hcm^2} = \dfrac{1}{360}\dfrac{m}{s}$, we obtain the film thickness $\delta_s = 200{\cdot}10^{-6}$ m. In the case of that volumetric flow rate calculated from (48) has been the film thickness prior to stabilisation. Calculations have been performed by means of the Runge-Kutta method of the fourth order. The results of calculations are presented in Fig. 7.

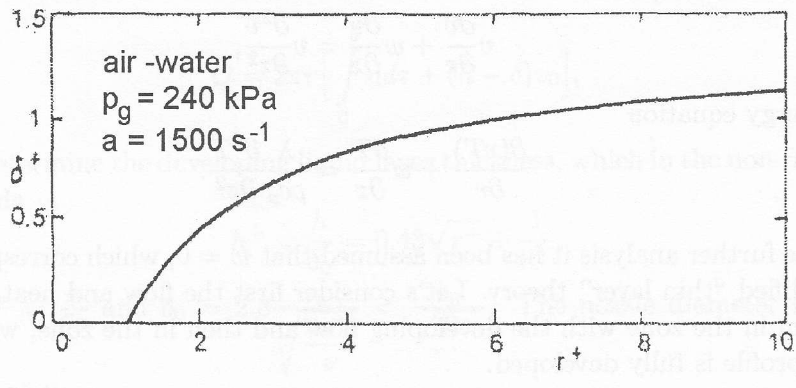

Figure 7. Change of non-dimensional liquid film thickness in function of dimensionless co-ordinate.

3 Modelling of heat transfer in liquid film

Let's consider a single phase liquid jet impinging on a flat surface, which produces a thin liquid film distributed on the solid surface. In such film three zones can be discerned: a vicinity of the stagnation point, non-developed film, where the boundary layer grows and has yet to reach the thickness of the developed film and the fully developed film, where the boundary layer has reached the thickness of the fully developed film. In the present work two latter cases will be considered. In the case of the first zone there is an exact solution presented by Schlichting [5]. The model presented below provides analytical solutions approximated to the case where the film flow is laminar. An approximate solution has been obtained on the basis of so called thin layer approximation. In such theory neglected is velocity in the transverse direction of the moving liquid layer. Hydrodynamic aspects of impinging single phase jets have been considered by the author in [10-11, 13].

3.1 Heat transfer in impinging single phase jets

Equations describing the behaviour of non-compressible two-dimensional liquid films flowing on the solid body, written in the cylindrical co-ordinates with the assumptions relevant to the boundary layer can be written as

- Continuity equation

$$\frac{\partial(rv)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0, \tag{50}$$

- Momentum equation

$$v\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}, \tag{51}$$

- Energy equation

$$\frac{\partial(vT)}{\partial r}+w\frac{\partial T}{\partial z}=\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial z^2}. \tag{52}$$

In the further analysis it has been assumed that $w=0$, which corresponds to the simplified "thin layer" theory. Let's consider first the flow and heat transfer in the film in the zone with the developing flow and then in the zone, where the velocity profile is fully developed.

3.2 Analysis of the developing film

Let's assume the first approximation of the velocity profile in the linear form

$$v=\frac{v_0 z}{\delta}. \tag{53}$$

The above profile obeys the condition of zero velocity at the wall and takes a value of undisturbed velocity at the boundary layer border v_0. Substituting the above velocity profile to the left hand side of momentum equation (51), using $w=0$, and then integrating using the boundary conditions of $v=0$ at $z=0$

$$z=\delta,\qquad v=v_0,\qquad \frac{\partial v}{\partial z}=0, \tag{54}$$

we obtain a subsequent approximation of the velocity profile in the form

$$v=-\frac{v_0^2}{12\nu\delta^3}\frac{d\delta}{dr}\left(z^4-\delta^3 z\right)+\frac{v_0 z}{\delta}, \tag{55}$$

where δ is the boundary layer thickness. Using the second condition at the border of the boundary layer we obtain a following differential equation

$$\frac{1}{4}\frac{v_0}{\nu}\frac{d\delta}{dr}=\frac{1}{\delta}. \tag{56}$$

This equation can be solved by means of separation of variables and the boundary condition that for $r=0\Rightarrow\delta=0$. This enables to determine the distribution of the boundary layer thickness in liquid with respect to the radius r, which reads

$$\delta=2.8\sqrt{\frac{\nu r}{v_0}}. \tag{57}$$

Defining the volumetric flow rate for the entire film in the form

$$Q = 2\pi r\Big[\int_0^{\delta} v dz + (h-\delta)v_0\Big], \tag{58}$$

we can determine the developing liquid layer thickness, which in the non-dimensional form reads

$$h^+ = \frac{h}{\delta_0} = 0.43\sqrt{r^+} + \frac{1}{r^+}, \tag{59}$$

where $r^+ = \frac{r}{r_0}$ and $\delta_0 = 2.8\frac{r_0}{\sqrt{\frac{v_0 r_0}{\nu}}} = \frac{r_0}{\sqrt{Re_0}}$. The nozzle diameter has been assumed as r_0.

The limiting radius of the boundary layer, where it reaches the liquid film thickness can be determined from the relation

$$\sqrt{r^+} = 0.43\sqrt{r^+} + \frac{1}{r^+}, \tag{60}$$

which leads to the relation $r^+_{tr} = 1.4$. It results further that $h^+_{tr} = 1.187$, or $h_{tr} = 3.35\frac{r_0}{\sqrt{Re_0}}$.

Let's define the mean film temperature in the form:

$$T_m = \frac{\int_0^h vTdz}{\int_0^h vdz} = \frac{2\pi r}{Q}\int_0^h vTdz. \tag{61}$$

The denominator in (61) can be determined from the continuity equation in the form

$$2\pi r\int_0^h vdz = Q, \tag{62}$$

where Q is the volumetric flow rate. From integration of (62) in the limits from 0 to h, using the definition of mean temperature (61) and the boundary conditions we get

$$\frac{\partial}{\partial r}\Big(\frac{T_m}{r}\Big) = \frac{2\pi}{Q\rho c_p}(q_w - q_i), \tag{63}$$

where q_w is the heat flux at the wall and q_i is the heat flux at the phase separation surface. Integrating (63), assuming that for $r = r_0$, $T_m = T_0$ we obtain a distribution of the mean temperature along the radius

$$T_m = \frac{2\pi q_w r_0^2}{Q\rho c_p}\Big(1 - \frac{q_i}{q_w}\Big)(r^2 - 1) + T_0. \tag{64}$$

In order to determine the temperature distribution across the film we use Eq. (52) and a definition of the mean temperature (61). From (61) it results that

$$(vT)_m = \frac{QT_m}{2\pi rh}. \tag{65}$$

Let's use also the approximation

$$\frac{\partial(vT)}{\partial r} \cong \frac{\partial(vT)_m}{\partial r} = \frac{Q}{2\pi}\frac{\partial}{\partial r}\left(\frac{T_m}{rh}\right). \tag{66}$$

Expressing the left hand side of the energy equation by (66) we obtain a differential equation

$$\frac{\partial^2 T}{\partial z^2} = \frac{c_p\rho}{\lambda}\frac{\partial(vT)_m}{\partial r} = A(r), \tag{67}$$

where

$$A(r) = \frac{Qc_p\rho}{2\pi r_0^2\lambda\delta_0}B(r^+), \tag{68a}$$

$$B(r^+) = \frac{5(10Mr^2 + M(r^+)^{5/2} + 3M\sqrt{r^+} - 3\sqrt{r^+})}{(5 + 2(r^+)^{3/2})^2}, \tag{68b}$$

$$M = 0.357\frac{\sqrt{Re_0}}{K}\left(1 - \frac{q_i}{q_w}\right), \tag{68c}$$

$$K = v_0 c_p \rho \frac{T_0}{q_w}. \tag{68d}$$

Integrating (67) at the condition, that we have at the wall $T = T_w$ and $\frac{dT}{dz} = -\frac{q_w}{\lambda}$ we obtain a temperature distribution across the film. From that we determine the temperature difference $T_m - T_w$, which enables determination of the heat transfer coefficient α as well as the Nusselt number

$$\alpha = \frac{q_w}{T_m - T_w}, \tag{69}$$

$$\mathrm{Nu} = \frac{\alpha\delta_0}{\lambda} = \frac{1}{N(r^{+2} - 1) + PB(r^+)r^+ + \sqrt{r^+}}, \tag{70}$$

where $N = \frac{0.13}{Pr}\left(1 - \frac{q_i}{q_w}\right)$ and $P = 1.4\frac{K}{\sqrt{Re_0}}$.

3.3 Analysis of a fully developed film

An approximate value of heat transfer coefficient can be calculated using the thin layer theory neglecting the influence of transverse velocity. The continuity Eq. (58) is fulfilled by the following velocity distribution

$$v = \frac{Q}{2\pi h}\frac{1}{r}. \tag{71}$$

Substituting the above velocity profile into the momentum Eq. (51), and then integrating it using the boundary conditions that for $z = 0 \Rightarrow v = 0$ and for $z = h \Rightarrow \frac{\partial v}{\partial z} = 0$ we obtain a following velocity distribution

$$v = \frac{Q^2}{4\pi^2 \upsilon} f(r)\Big(\frac{z^2}{2} - hz\Big), \tag{72}$$

where h is the film thickness and $f(r) = \frac{1}{hr}\frac{d}{dr}\Big(\frac{1}{rh}\Big)$.
Based on the relation (58), in the case of a constant film flow in the film we obtain

$$\frac{dh}{dr} + \frac{h}{r} = 6\pi\upsilon\frac{r}{Q}, \tag{73}$$

which integrated at the boundary condition

$$r = r_{tr} \qquad h = h_{tr}, \tag{74}$$

gives us the distribution of film thickness with respect to the radius (from the beginning of the fully developed film)

$$\frac{h}{h_{tr}} = S\Big(r^{+2} - \frac{1}{r^+}\Big) + \frac{1}{r^+}, \tag{75}$$

where $S = \frac{r_{tr}^2}{\delta_0 Re_0}$, superscripts + denote non-dimensional quantities. Energy Eq. (67) can now be considered, which requires consideration of the relation (64) and the assumption (66). Incorporating (64) and (66) as well as (75) into the energy equation we get

$$\frac{\partial^2 T}{\partial z^2} = \frac{c_p \rho}{\lambda}\frac{\partial(vT)_m}{\partial r} = A(r), \tag{76}$$

where

$$A(r) = \frac{Q c_p \rho T_0}{2\pi\lambda r_{tr}^2 h_{tr}} B(r^+), \tag{77a}$$

$$B(r^+) = -r^+ \frac{(Sr^{+3} + 2S - 2 - 3Sr^+)}{(Sr^{+3} - S + 1)^2} M - \frac{3Sr^{+2}}{(Sr^{+3} - S + 1)^2}, \tag{77b}$$

$$M = K\Big(1 - \frac{q_i}{q_w}\Big)\frac{r_{tr}^2}{r_0\delta_0}. \tag{77c}$$

Integrating (76) with account of the boundary condition, that at the wall we have $T = T_w$, we obtain the temperature distribution. Subsequently we get the temperature difference $T_m - T_w$, which enables determination of a local heat transfer coefficient and the Nusselt number

$$\mathrm{Nu} = \frac{\alpha h_{tr}}{\lambda} = \frac{1}{\dfrac{9MBr^+}{40\Big(1-\dfrac{q_i}{q_w}\Big)}h^{+2} + \frac{5}{8}h^+}. \tag{78}$$

4 Results

In order to illustrate the performance of the model conducted have been calculations for different values of K and q_i/q_w. The parameter K was set values of 10 and 50 respectively, whereas the parameter $q_i/q_w = 0.5$ and 1.0 respectively. Additionally assumed were $\mathrm{Pr} = 7$, $\mathrm{Re}_0 = 1000$. Physical properties of water corresponded to 20°C. Calculations have been performed on a commercial version of MATHCAD2000. Results of the influence of K on the Nusselt number and the non-dimensional film thickness are presented in Fig. 8 and 9.

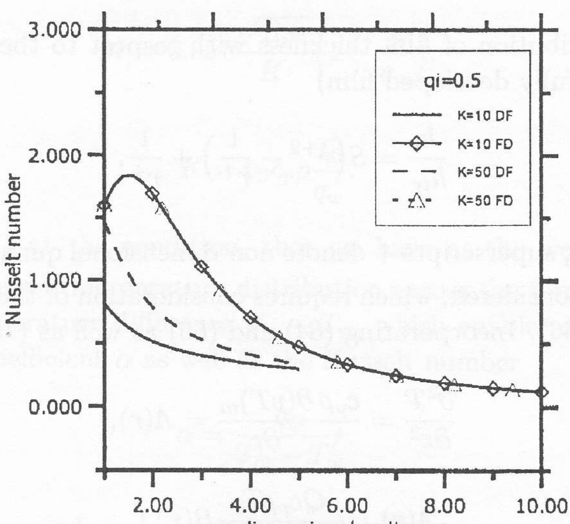

Figure 8. Distribution of Nusselt number for various values of K and $q_i/q_w = 0.5$: DF – developing film zone, FD – fully developed film.

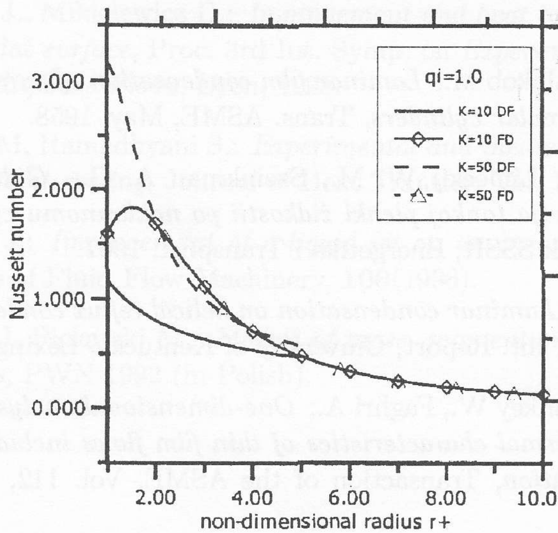

Figure 9. Distribution of Nusselt number for various values of K and $q_i/q_w = 1.0$: DF – developing film zone, FD – fully developed film.

Increase of K from 10 to 50 (reduction of q_w or increase of temperature) causes the Nusselt number to increase in the case of developing flow (DF) formulation. No visible changes are perceptible in the case of fully developed calculations (FD). Increasing the ratio of interface to wall heat flux, q_i/q_w, we obtain much better agreement between the formulations, i.e. a smoother transition.

5 Conclusions

In the work presented has been an approximate solution of the flow and heat transfer problem in the case of a film formed by the jet impingement on a surface. Formulated has been a simple two-dimensional model of the flow and heat transfer in the film. The model is based on simplified conservation equations of mass, momentum and energy. Solution of that system of equations at adequate boundary conditions enables determination of the velocity profile in the film and local values of heat transfer coefficient and Nusselt numbers. The model enables analysis of the influence of heat transfer at the free surface on heat transfer from the wetted wall.

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