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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

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Fluid mechanics in machine construction and exploitation – examples of relations

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Abstract

In the work presented have been fundamental equations of hydrodynamic theory of lubrication (the Reynolds equation and energy equation) as a specific case of classical equations of fluid-mechanics, i.e. Navier-Stokes equations. On such basis conducted has been an attempt to describe the phenomena of hydrodynamic instability in the rotor-bearings system followed by the description of vortices and oil whip. Indicated have been the relations between the theory and equations of fluid mechanics and the practical aspects resulting from the analysis of entire machine with the view to its exploitation and diagnostics.

Keywords: Fluid mechanics; Hydrodynamic; Lubrication; Journal bearings

1 Introductory remarks

In recent years the integration tendencies in the area of many scientific disciplines can be observed. The differences between, for example, tribology, diagnostics, safety and machine reliability simply disappears as well as between the design and operation. In the present time we talk about operation oriented design and the term co-current engineering is a chic issue, which focuses on the product maintenance after its selling hence directed on the market and sociological processes.

Many scientists and engineers started their professional carrier in the disciplines closely related to fluid mechanics, and then continued their development in the design of various kinds of machinery and equipment or designing their control and operation systems. Such tendency can be regarded as fully natural

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and even forced by the contemporary development of technology and information technology in particular. Fluid mechanics has formed the mathematical and numerical basis for the description and identification of models of various phenomena subsequently widely implemented in machine construction and exploitation. Mentioned here can be the works from the area of turbulent flows, widely understood cavitation, hydraulic jumps or finally hydrodynamic theory of lubrication. The results of fundamental research regarding these problems, conducted in the frame of fluid mechanics, have been directly implemented in the process of design, construction and exploitation of machinery and power engineering equipment, marine propellers, pipeline networks in district heating or the bearings nodes of various kind of machinery. In the present work, in more detail, presented will be the hydrodynamic aspects of lubrication theory and their subsequent influence on the theory and practice of design of bearings nodes and then their influence on the theory and practice of design of bearings nodes in various kinds of machinery. Such considerations are a classical example of interactions, which take place between fluid mechanics and the machine exploitation.

2 General Reynolds equations

The Reynolds equation is the most fundamental equation of a hydrodynamic theory of lubrication. It describes the distribution of hydrodynamic pressure in the lubrication slot and also indicates possible sources of hydrodynamic bearing load capacity. The form of that equation depends directly on the assumed theoretical model. Almost each contemporary publication referring the theory of slide bearings provides, for its own use, various simplified versions of that equation adequate for the considered case [1-3]. Most often these are typical, published in the classical literature, models [2, 3]. A variety of the forms of the Reynolds equations can sometimes be the reason for the difficulty in their implementation in non-typical cases as well as in adaptation to the assumed co-ordinate system.

Prior to writing an appropriate Reynolds equation let's assume the co-ordinate system in the selected point of the lubrication slot, as shown in Fig. 1, together with the following assumptions:

- The thickness of the lubrication film h (in the direction y) is small compared with the remaining dimensions of the bearing (surface co-ordinates x, z). This means that the changes of the component of the liquid velocity v in the y direction are also small, similarly as all remaining derivatives of the u, w components of velocity with exception of $\partial u/\partial y$ and $\partial w/\partial y$.
- Hydrodynamic pressure p along the thickness of the film is constant, which renders $\partial p/\partial y = 0$.

- The particle of liquid adheres directly to the constraining surface, has the same velocity as that surface, i.e. there is no slip condition at that surface.
- Mass forces are negligible in comparison with viscosity forces.
- The lubrication liquid is a Newtonian liquid as well as incompressible.
- The flow through the slot is laminar.

Then the classical equation of fluid mechanics, i.e. the Navier-Stokes equation, can be written in the following form:

$$\begin{aligned}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}A\right)+\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial x}A\right) = -\bigg[(U_2-U_1)\left(\frac{\partial B}{\partial x}-\frac{\partial y_2}{\partial x}\right)+\\ +(W_2-W_1)\left(\frac{\partial B}{\partial z}-\frac{\partial y_2}{\partial z}\right)+B\frac{\partial}{\partial x}(U-U)+B\frac{\partial}{\partial z}(W_2-W_1)+\\ +h\left(\frac{\partial U_1}{\partial x}+\frac{\partial W_1}{\partial z}\right)+\Delta v^*+\frac{\partial h}{\partial t}\bigg],\end{aligned} \tag{1}$$

where

$$\left.\begin{aligned}A &= \int\limits_0^h\left[\int\limits_0^y \frac{y}{\mu}dy - \frac{\int_0^h \frac{y}{\mu}dy}{\int_0^h \frac{1}{\mu}dy}\int\limits_0^y \frac{1}{\mu}dy\right]dy\\ B &= \int\limits_0^h\left[\frac{\int_0^y \frac{1}{\mu}dy}{\int_0^h \frac{1}{\mu}dy}\right]dy\end{aligned}\right\}, \tag{2}$$

$$\Delta v^* = U_2\alpha_2^* + W_2\beta_2^* - U_1\alpha_1^* - W_1\beta_1^*,$$

The co-ordinate system and nomenclature are consistent with the Fig. 1.

Equation (1), according to the assumptions, is the most general form of Reynolds equations encompassing all possible cases of relative motion of both sliding surfaces of the bearing.

The right hand side of that equation covers possible sources of the bearing hydrodynamic load capacity. We can easily notice that two first terms correspond to the principle of so called geometrical lubrication wedge, and hence the principle of operation of all two-dimensional thrust bearings. Subsequent three terms indicate the possibility of formation of a positive hydrodynamic load capacity by an appropriate change of velocities U and V in specified points of the lubrication slot. In most cases, attainment of the hydrodynamic load capacity is not possible on that path, as this would mean 'dilatation' of slide surfaces. We

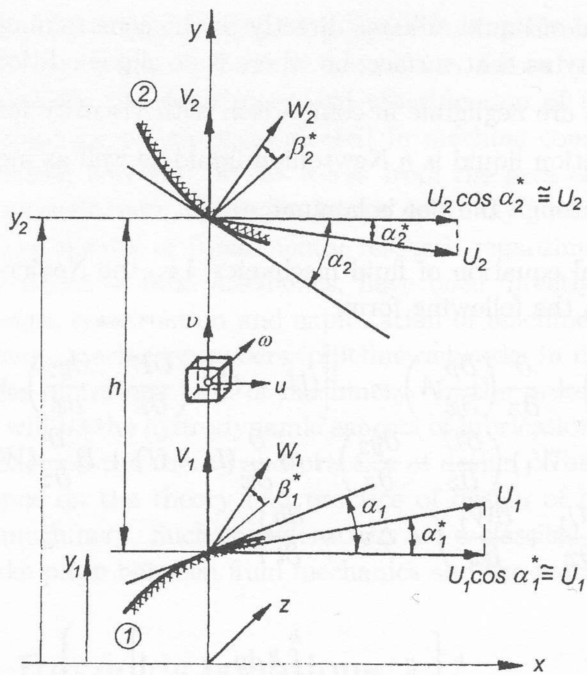


Figure 1. Co-ordinate system in a selected point of the lubrication slot.

ought to notice, however, that in the case of segmental thrust bearings, of for example Michell type, velocities U and W are dependent on co-ordinates x, y, even in the case of constant angular velocity of rotating disc w as well as assumption of stiff 'non-dilatable' slide surfaces. Above terms are therefore important in the analysis of operation of that type of bearings.

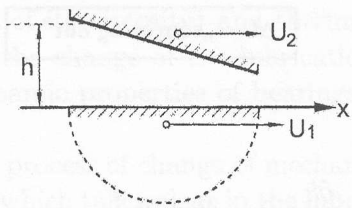
The terms denoted by the symbol Δv^*, on the right hand side of Eq. (1), correspond to the load resulting from the slope of U, W velocities with respect to the surface co-ordinates x, z. These are the terms, which describe operation of all journal slide bearings. The last term on the right hand side of the Reynolds equation describes the changes in time along the thickness of the slot, and indicates the possibility of achieving the load capacity through the so called 'squeezing out' effect. This is a very important term in the analysis of the problems of dynamics of all kinds of objects.

In Fig. 2 and 3 schematically are presented all possible sources of positive hydrodynamic load capacity in slide bearings. This has a paramount importance from the point of view of understanding of the principles of their correct operation and the adequate design of bearing nodes at the same time.

HYDRODYNAMIC LOAD CAPACITY – DISCUSSION OF THE RIGHT HAND SIDE OF REYNOLDS EQUATION

① LUBRICATION WEDGE

Terms of the type: $(U_2 - U_1)\dfrac{\partial h}{\partial x}$



$U_1 > U_2$

$\dfrac{\partial h}{\partial x} < 0 \quad P > 0$

Thrust Bearings

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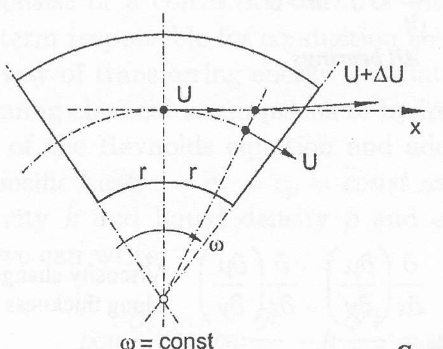
② „SHRINKING" OF THE SURFACE

Terms of the type: $h\ \dfrac{\partial}{\partial x}(U_2 - U_1)$



$U_2 - U_1 = U$

$\dfrac{\partial U}{\partial x} \neq 0$

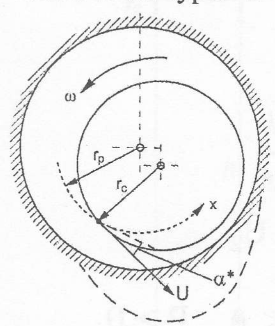
$P > 0$

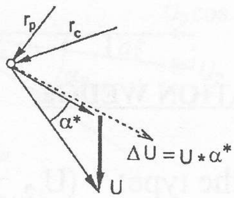
Segment Thrust Bearings

Figure 2. Sources of additional hydrodynamic load P of slide bearings.

(3) „SLOPE" OF THE CIRCUMFERENTIAL VELOCITY

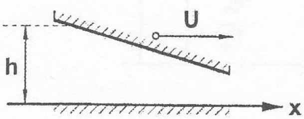
Terms of the type: $U \cdot \alpha^*$





$2P > 0$!

$\boxed{\Delta U \text{ – „squeezing out”}}$

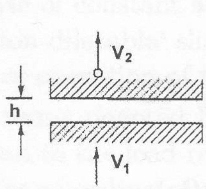


$U > 0 \qquad \dfrac{\partial h}{\partial x} < 0 \qquad \underline{P < 0\ !}$

Journal bearings

(4) DYNAMIC TERM – SQUEEZING OUT

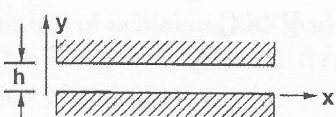
Terms of the type: $\dfrac{\partial h}{\partial t}$



$\dfrac{\partial h}{\partial t} < 0 \quad P > 0$

All bearings

(5) THERMAL WEDGE



$\dfrac{\partial}{\partial x}\left(\dfrac{\partial \mu}{\partial y}\right), \quad \dfrac{\partial}{\partial z}\left(\dfrac{\partial \mu}{\partial y}\right)$ Viscosity change along thickness

All bearings

Figure 3. Continuation of Fig. 2.

3 Energy equation

The shearing process of liquid, which takes place in the lubrication slot of the bearing, renders the increase of the lubrication medium temperature, as the performed work of internal friction is completely converted to heat. The amount of produced in such a way thermal energy can be significant and that regards particularly to the large and fast-rotating bearings. The friction power of the slide bearing measured in such cases reaches values of even of several megawatts. Such significant amounts of heat are removed mainly through the leaking oil, rendering its increase in temperature, and the bearing walls, which confine the lubrication slot.

This phenomenon renders a significant change of the lubrication medium (change of its viscosity) and thermal deformations of the bearing's structure as well as the change of the lubrication slot energy at the same time. The static and dynamic properties of bearings change then, sometimes even in qualitative manner.

The process of change of mechanical work (shearing friction) into the thermal energy, which takes place in the lubrication slot, can be described by means of the energy equation. The details connected with derivation of that equation and the underlying assumptions can be found without problems in numerous monographs and publications [2, 3]. For that reason these will not be repeated here. For our requirements let's recall only some general principles.

It is apparent, that in the case of steady-state conditions, it results from the energy conservation equation that the amount of energy exchanged in the control volume of fluid (Fig. 1) and the amount of mechanical work performed by the surface stress and mass forces (in the unit of time) must be equal. Transferred energy consist of a convection term of internal energy of the element of fluid and the term responsible for conduction according to the Fourier law. The third possible way of transferring energy – radiation – is neglected here.

Assuming classical assumptions of hydrodynamic theory of lubrication, as in the case of the Reynolds equation and additionally assuming a constant value of the specific heat $c = c_v = c_p = const$ as well as a constant value of thermal conductivity k and liquid density ρ and expressing them according to the SI system, we can write:

$$\rho c u \frac{\partial t}{\partial x} + \rho c w \frac{\partial t}{\partial x} - k \frac{\partial^2 t}{\partial y^2} = \mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] . \tag{3}$$

Accordingly to the assumed co-ordinate system (Fig. 1) there is:

$$\left.\begin{aligned}
u &= \frac{\partial p}{\partial x} a + (U_2 - U_1) b + U_1 \\
w &= \frac{\partial p}{\partial z} a + (W_2 - W_1) b + W_1 \\
\frac{\partial u}{\partial y} &= \frac{1}{\mu}\frac{\partial p}{\partial x}\left[y - \frac{\int_0^h \frac{y}{\mu} dy}{\int_0^h \frac{1}{\mu} dy} \right] + (U_2 - U_1)\frac{1}{\mu \int_0^h \frac{1}{\mu} dy} \\
\frac{\partial w}{\partial y} &= \frac{1}{\mu}\frac{\partial p}{\partial z}\left[y - \frac{\int_0^h \frac{y}{\mu} dy}{\int_0^h \frac{1}{\mu} dy} \right] + (W_2 - W_1)\frac{1}{\mu \int_0^h \frac{1}{\mu} dy}
\end{aligned}\right\} \tag{4}$$

Relation (5) describes a and b.

$$\left.\begin{aligned}
a &= \int_0^y \frac{y}{\mu} dy - \frac{\int_0^h \frac{y}{\mu} dy}{\int_0^h \frac{1}{\mu} dy} \int_0^y dy \\
b &= \frac{\int_0^y \frac{1}{\mu} dy}{\int_0^y \frac{1}{\mu} dy}
\end{aligned}\right\} \tag{5}$$

It is easy to notice that two first terms on the left hand side of Eq. (3) represent convection of heat in x and z directions, whereas the third term – the conduction of heat in the direction y (over the oil film thickness).

Equation (3), in the case of our assumptions, is an equation describing the full diathermal model of a lubrication film and hence a model, where the spatial variation of temperature t and viscosity in the lubrication slot are included:

$$t = f(x, y, z) \; ; \quad \mu = f(x, y, z)$$

4 The system bearings-machine

Reynolds (1) and energy (3) equations describe the hydrodynamic and thermal phenomena, which take place in the lubrication film of slide bearings. These are both a present topic of investigations and the achievement of fluid mechanics.

Let's see now how, in a simple way, we can approach the problems connected with the construction and development of machinery, and in that context with the dynamics of bearings nodes and bearings-machine systems. Let's assume in our investigations the model of machine with slide bearings.

Let's assume in our considerations a simple model of rotor-bearings system with a single mass-rotor founded on two identical bearings. Assuming the coefficients of stiffness and damping of oil film as a basis of considerations we can obtain interesting and practical information regarding the dynamic properties of such system as: natural frequency, the threshold of stability or the parameters of the vibrations ellipse in the case of various kinds of external excitations.

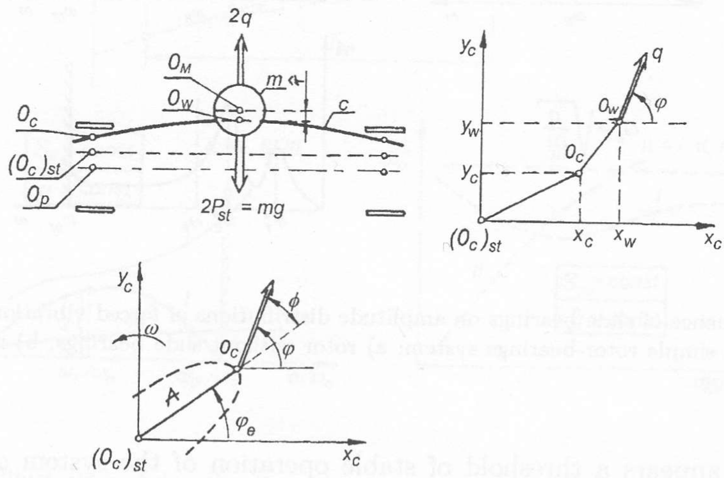


Figure 4. The rotor-bearings model.

Let's assume in our considerations a model of the rotor-bearings system as shown in Fig. 4. Under the influence of excitation force $2q$ imposed in the centroid of rotor the bearing shaft performs small oscillations with respect to the static equilibrium point $(O_c)_{st}$. The required equations of dynamics of such system can be found in [1].

Let's consider the behaviour of assumed model of rotor at external excitations rendered by the imbalance of rotating mass m, where the radius of imbalance let be r. Let's compare that with the behaviour of the same rotor without slide bearings, supported in stiff prisms. That will enable us to distinct clearly the influence of slide bearings. Such example is presented in Fig. 5.

We can see that the influence of bearings on the operation of rotor (amplitudes of vibrations) has not only a qualitative but also a quantitative character. It can primarily be concluded that:

- slide bearings render the shift in resonance velocity toward lower angular velocities of rotor, i.e. $\omega^* < \omega_k$,

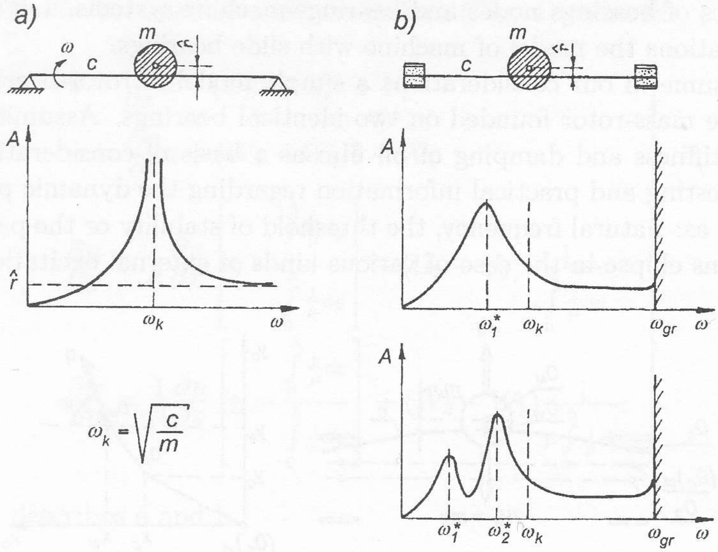


Figure 5. Influence of slide bearings on amplitude distributions of forced vibrations in the case of a simple rotor-bearings system; a) rotor without slide bearings, b) rotor-bearings system.

- there appears a threshold of stable operation of the system ω_{gr}, beyond which the amplitudes of vibrations can achieve permanent, maximally large values (limited only by values of clearances),
- possible is the appearance of two resonance velocities ω_1^*, ω_2^*,
- resonance amplitudes have limited values (due to the damping effect of lubrication film.

In the case of free vibrations of such a system, in the stable region, the system will perform damping with time vibrations such as shown in Fig. 6. We can easily discern here characteristic parameters such as natural frequency u_n and corresponding frequencies of system damping un, where $n = 1, 2, 3$ (or $n =$ I, II, III) denotes the numbers of subsequent free vibrations frequencies. In Fig. 6 presented is a distribution of the component of amplitude of vibrations X_n for one natural frequency.

From the above comparison it results that the equations of fluid mechanics (1) and (3) as well as equations of dynamics of the rotor-bearings system (neglected in the present paper) provide several practical information about the behaviour of the entire machine. Even more spectacular are the considerations about hydrodynamic instabilities and so called vortices and oil whip.

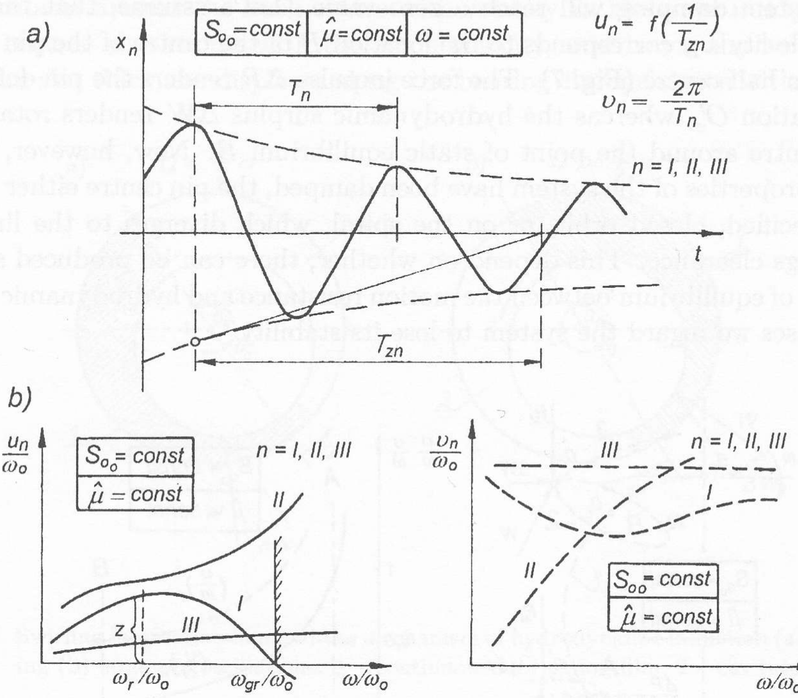


Figure 6. Illustration of damping u_n and natural frequency ϑ_n in rotor-bearings system.

5 Hydrodynamic instability – the physics

Let's try to trace the conditions, where the loss of a system can be attained. At small angular velocities of the rotor wA the centre of the pin has a location A in the lower part of the equilibrium half-circle, as shown in Fig. 7. Damping of the system $(u/\omega)_I$, corresponding to that angular velocity, is very significant. Let's assume now that the bearing's pin is influenced by the force ΔP (an impulse), which can depict possible disturbances of the ideal static equilibrium state. The pin will deflect to the location O_c', which has a corresponding hydrodynamic reaction W. The reaction generates some hydrodynamic surplus ΔW, which is not balanced by the external load. The surplus ΔW renders motion of the pin centre around the point of static equilibrium A with velocity balancing the drag force and the force ΔW. Because damping of the system (u_n/ω) is very large, hence the motion has a character of a deteriorating spiral, tending to A. We say, that the system is stable (Fig. 7). With the increase of angular velocity ω, the point of bearing's static equilibrium is displaced toward upper part of the equilibrium half-circle, whereas the damping of the system decreases. At velocity

ω_B the system damping will reach a zero value. Let's assume, that the rotor angular velocity ω_B corresponds to the location B of the centre of the pin on the equilibrium half-centre (Fig. 7). The force impulse ΔP renders the pin deflection to the location O'_c, whereas the hydrodynamic surplus ΔW renders rotation of the pin centre around the point of static equilibrium B. Now, however, as the damping properties of the system have been damped, the pin centre either travels on the specified, closed orbit, or on the spiral, which diverges to the limits of the bearings clearance. This depend on whether, there can be produced specific conditions of equilibrium between the motion resistance and hydrodynamic forces. In both cases we regard the system to lose its stability.

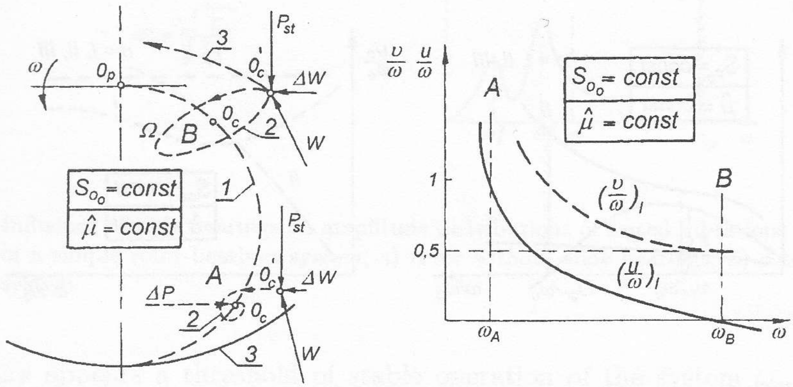


Figure 7. Explanation of the mechanism of hydrodynamic instability in the rotor-bearings system. 1 – semicircle of static equilibrium, 2 – trajectory of the pin, 3 – circle of clearances.

Let's notice that the frequency of system natural vibrations $(\vartheta_n/\omega)_I$ at the velocity equal ω_B is 0.5 (Fig. 7). This means, that the pin centre, which has departed from equilibrium, due to the disappearance of excitation forces, will immediately fall into the some kind of resonance and will move with respect to the point B with angular velocity $\Omega = \omega/2$.

The motion of the pin centre around some point with velocity Ω, independently of the angular velocity around its axis ω, will render and additional flow of oil (pumping effect) in the region of the lubrication slot. The phenomenon of such kind is often described in the literature as the oil whirl. As in the discussed case, the angular velocity of the pin centre around the point of static equilibrium is $\Omega = \omega/2$, hence such a whirl is sometimes called a half-whirl. Often introduced are also half-vibrations.

The case of large oscillations of the pin centre is a good example explaining natural vibrations rendered by hydrodynamic forces. Let's assume that the pin centre rotates around the bush centre with velocity Ω, as shown in Fig. 8a. The line of centres also rotate with the same velocity, and hence the location of a minimum slot thickness displaces along the bearing circumference with the whirl.

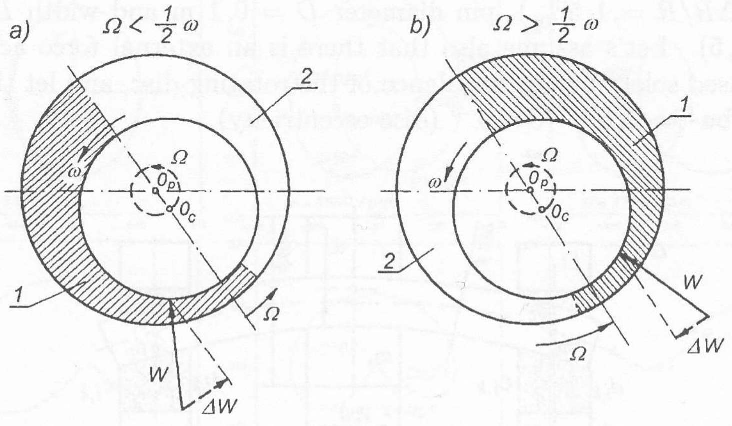


Figure 8. Swirling lubrication film and the mechanism of hydrodynamic induction (a) and damping (b) large pin oscillations. 1 – continuous lubrication film, 2 – cavitated zone.

Solving the Reynolds equation for that case it will turn out, that in the case when $\Omega < 1/2\omega$ then the lubrication film (region of positive hydrodynamic pressure) will form only in the convergent part of the lubrication slot and the component of reaction W will be inclined with respect to a movable line of centres with the angle corresponding to a tangent component ΔW in the direction of pin rotation (Fig. 11a). This sustains vibrations with high amplitude and increases the system tendency to unstable performance.

If $\Omega > 1/2\omega$, then the lubrication film will build in the divergent part of the lubrication slot and the resulting reaction W will produce the component ΔW in opposite direction to the pin rotation (Fig. 8b). Hence the lubrication film damps vibrations. The above explains the fact, why synchronous excited vibrations ($\Omega = \omega$) does not induce directly vibrations with the natural character.

For the sake of completeness, let's note that in the case of $\Omega = 1/2\omega$ we will obtain from the Reynolds equation, in the discussed case, a zero value of the hydrodynamic load capacity.

6 Simulation of whirls and oil whip

Let's try to trace the development of oil whirls and oil whip on the example of a two-supported rotor presented in Fig. 8. Let its theoretical mapping will be a system consisting of 13 masses presented in Fig. 9b. The slide bearing bushes with masses $m_p = 50$ kg each let be fixed in the susceptible elements. Let's assume in our investigations a slide bearing with two lubrication pockets, with the circular-cylindrical clearance of absolute value $\Delta R = 75 \cdot 10^{-6}$ m (relative clearance $\Delta R/R = 1,5^0/_{00}$), pin diameter $D = 0,1$ m and width $L = 0,05$ m ($L/D = 0,5$). Let's assume also that there is an external force acting on the system caused solely by the imbalance of the rotating disc, and let the radius of imbalance be $\hat{r} = \Delta R = 75 \cdot 10^{-6}$ (disc eccentricity).

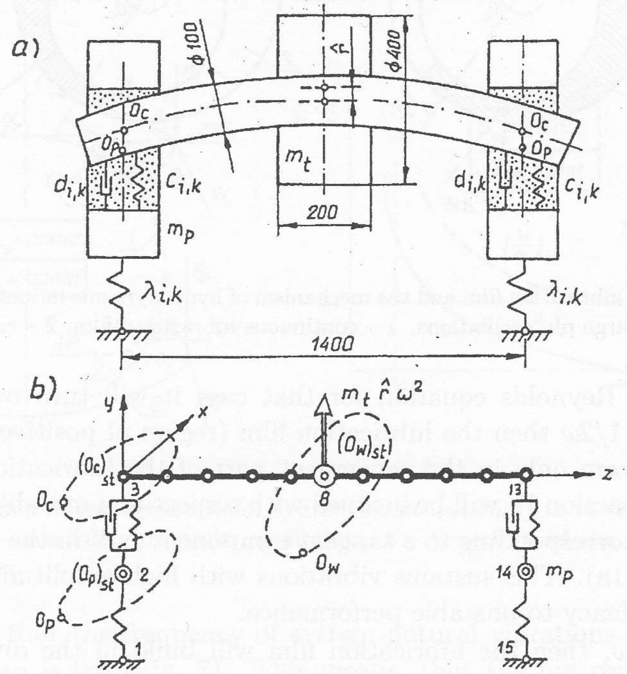


Figure 9. Assumed model of rotor-bearings system (a) and its accompanying discretisation schematic (b) using finite elements.

In Fig. 10a recorded is the moment of formation of the oil film and its subsequent development (Fig. 10b, c). It is characteristic, that the oil whirl forms through an arbitrary trajectory split with the period of $\theta^* = \theta_w$ into two slightly different distributions, where one with increase of rotational velocity n significantly reduces and produces at the same time a characteristic 'half-loop'. Now, the period of the entire trajectory is approximately equal to $\theta^* = 2\theta_w$. The development of oil whirls occurs in the direction from the larger to smaller 'half-loop', hence in the other direction than sometimes assumed. This takes place under conditions of sudden change of values of phase angles Φ_w (Fig. 10b and 10c).

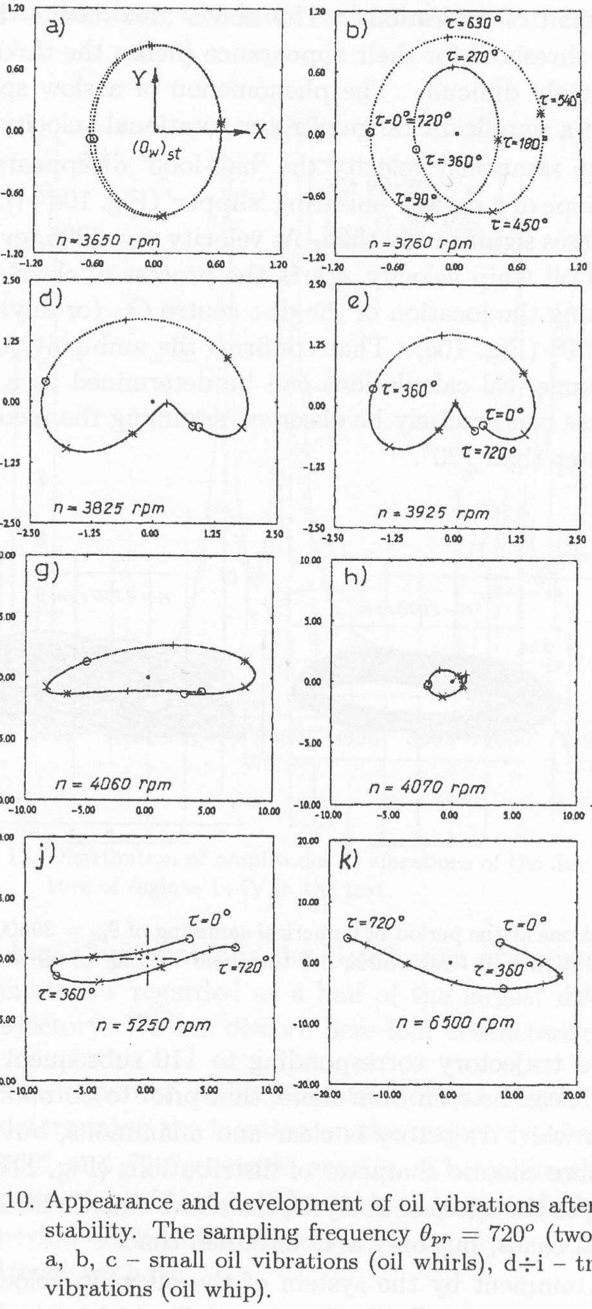


Figure 10. Appearance and development of oil vibrations after passing the threshold of system stability. The sampling frequency $\theta_{pr} = 720^o$ (two subsequent revolution of rotor). a, b, c – small oil vibrations (oil whirls), d÷i – transitional phase, j, k – large oil vibrations (oil whip).

The fact of appearance of oil whirls (Fig. 10a, b, c) does not necessarily mean the danger to the operation of the entire system. Their amplitude increases in the beginning stage at a small rate only. Hence, such vibrations can be qualified to the group of so called small oil vibrations. The above also means that a precise determination of the threshold for their appearance (hence the threshold of system stability) is extremely difficult. The phenomenon of a slow split of trajectories can take place in a significant range of rotor rotational velocity.

At further increase of the rotational velocity the 'half-loop' disappears and the trajectory assumes the shape of a rapidly pulsating 'slipper' (Fig. 10d÷i). The amplitude of vibrations increases significantly then. At velocity $n = 3925$ rev/min, which will be described as a oil whip velocity, starts the process of clear divergence of the points determining the location of the disc centre O_w (or any other node) for $\tau = 0^o$ and $\tau = 720^o$ (Fig. 10e). That confirms the ambiguity of the trajectory itself (which in numerical calculations can be determined as a solution divergence). Such process can precisely be observed assuming the recording period O_{pr} significantly greater than 720^o.

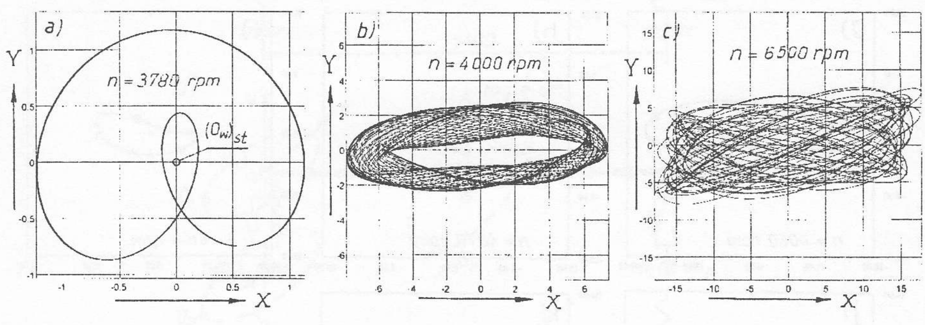


Figure 11. Trajectory of oil vibrations at the period of numerical sampling of $\theta_{pr} = 39600^o$ (110 subsequent rotor revolutions). a – oil whirls, b – threshold velocity of oil whip, c – fully developed oil whip.

In Fig. 11 presented is a trajectory corresponding to 110 subsequent rotor revolutions ($\theta_{pr} = 39600^o$). It can be seen from there, that prior to obtaining the oil whip n_b (Fig. 11a). The whirl trajectory is clear and unanimous, but when $n > n_b$ then it assumes a more chaotic character of distributions (Fig. 11b, and particularly 11c). According to the spectral analysis, the above chaotic behaviour of distributions is not a total chaos, but only a 'determined chaos'.

We can assume, that attainment by the system of the oil whip velocity n_b completes the range of existence of small oil vibrations (oil whirls). At higher rotational velocities of the rotor the system enters the phase of significant and dangerous oil vibrations, known as 'oil whip'. During sampling period of $\theta_{pr} = 720^o$ the points on the trajectory corresponding to the location of the imbalance vector for $\tau = 0^o$ and $\tau = 720^o$ propagate more strongly and the trajectory assumes the shape of a more and more open curve.

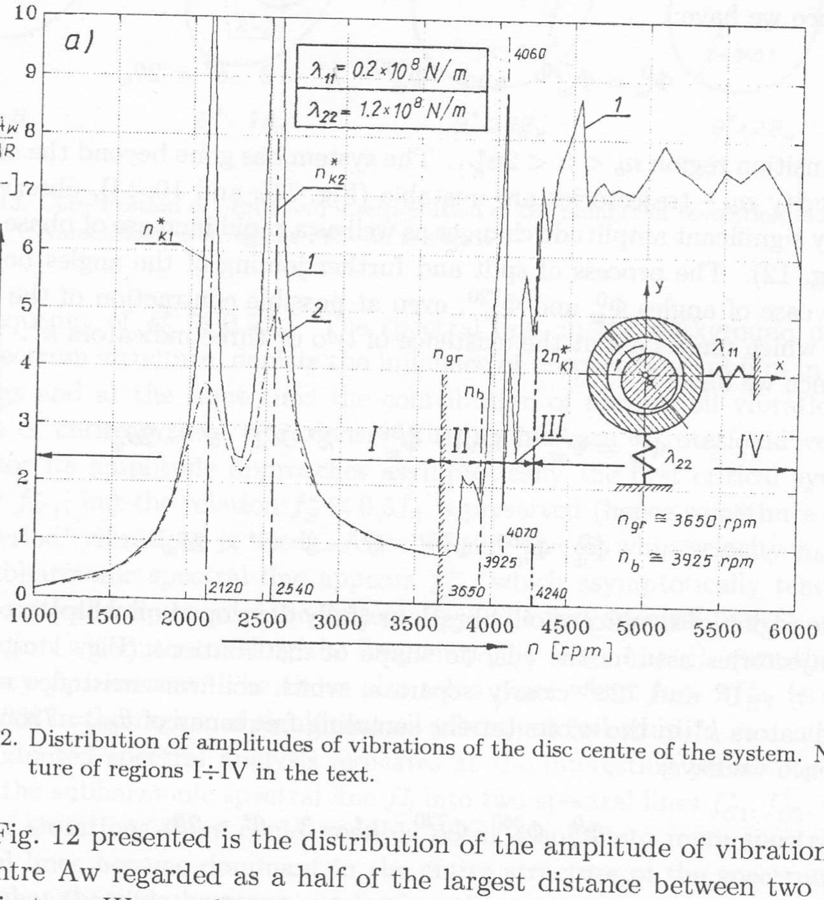


Figure 12. Distribution of amplitudes of vibrations of the disc centre of the system. Nomenclature of regions I÷IV in the text.

In Fig. 12 presented is the distribution of the amplitude of vibrations of the disc centre Aw regarded as a half of the largest distance between two point of the trajectory. We can discern here four characteristic regions:

I. The region of stable operation of the system $0 < n < n_{gr}$. The points determining the location on the trajectory of an arbitrary node for $\tau = 0^o$, 360^o and 720^o coincide exactly. The orbits show only a one indicator of revolutions $k^* = 1$. The trajectory period θ^* corresponds exactly to the period of excitation force θ_w.
Hence we have:

$$\Phi_w = \Phi_w^0 = \Phi_w^{360} = \Phi_w^{720} \quad k^* = 1 \quad \theta^* = \theta_w$$

II. The region of small oil vibrations (oil whirls) $n_{gr} < n < n_b$. Trajectories have a characteristic 'half-loop' similar to those presented in Fig. 10a, b, c. The system has, however, gone beyond the threshold of stability n_{gr}, but the trajectories, even though of different shape and period θ^*, still remain stable. A clear separation of phase angles takes place for $\tau = 0^o$ and 360^o, which renders that the orbit exhibits existence of two indicators k^*. Hence we have:

$$\Phi_w^0 = \Phi_w^{720} \quad \text{and} \quad \Phi_w^{360} \quad k^* = 2 \quad \theta^* = 2\theta_w$$

III. Transition region $n_b < n < 2n_{K1}^*$. The system has gone beyond the oil whip velocity n_b – trajectories are unstable (Fig. 11b and 10e÷j), observed are very significant amplitude changes as well as a rapid increase of phase angles (Fig. 12). The process of split and further joining of the angles begins in the case of angles Φ_w^0 and Φ_w^{720}, even at possible contraction of the period θ^*, which means about the existence of two or three indicators k^*. Hence we have:

$$\Phi_w^0 = \Phi_w^{720} \quad \text{and} \quad \Phi_w^{360} \quad k^* = 2 \quad \theta^* = 2\theta_w$$

or

$$\Phi_w^0,\ \Phi_w^{360},\ \Phi_w^{720} \quad k^* = 3 \quad \theta^* < 2\theta_w$$

IV. The region of significant oil vibrations (fully developed oil whip) $n > 2n_{K1}^*$. Trajectories assume the chaotic shape of distributions (Fig. 11c), points for $\tau = 0^o$ and 720^o clearly separate, which confirms existence of three indicators k^* in the orbits (at the sampling frequency of $\theta_{pr} = 720^o$). Hence we have:

$$\Phi_w^0,\ \Phi_w^{360},\ \Phi_w^{720} \quad k^* = 3 \quad \theta^* > 2\theta_w$$

It results from the conducted simulations that by classification of adequate orbits we can conclude about the conditions of system operation. The number of indicators k^* and a corresponding characteristic shape of the orbits can be a convenient determinant of the dynamic state of the system, as presented in Fig. 13.

The spectral analysis of trajectory provides a lot of practical information. Due to a fact of assumed sensitivity of analysis $\Delta f = 1$ Hz the recording period between recordings was $\theta_{pr} = 39600^o$ (which corresponds to 110 subsequent revolutions of the rotor). The results obtained by means of the Fast Fourier Transform (FFT) are presented in Fig. 14. After passing through the stability threshold (Fig. 14 c, d) a characteristic subharmonic spectral line appears, with

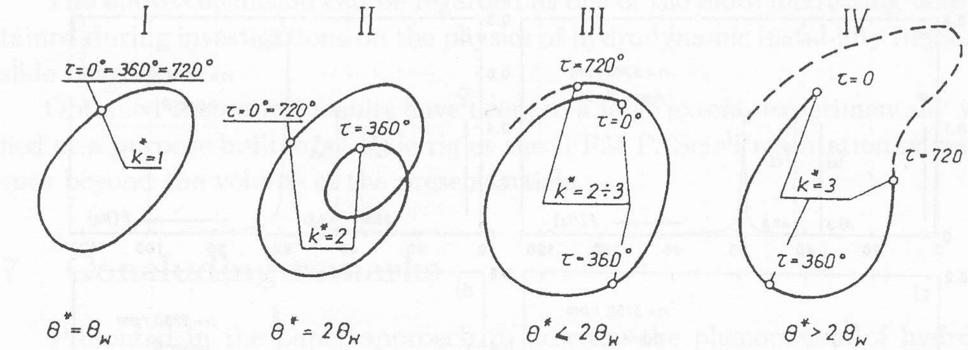


Figure 13. Classification of orbits and interpretation of the number of revolution indicators k^*. Nomenclature of regions I÷IV in the text.

the frequency of $f_S^* \cong 0.5 f_n$. The spectral line, in the background of the entire spectrum structure, depicts the influence of properties of oil film in the slide bearings and at the same time the contribution of natural oil vibration in the picture of entire system vibrations. With an increase of rotational velocity of the rotor its amplitude approaches asymptotically the first critical system frequency f_{K1}^*, but the relation $f_S^* \cong 0.5 f_n$ is preserved (hence sometimes the term 'half-period' vibrations is used). After passing the oil whip velocity n_b the second subharmonic spectral line appears f_{S2}^*, which asymptotically tends to the second critical frequency of the system f_{K2}^*. The first spectral line f_{S1}^* stabilises its location without exceeding the frequency f_{K1}^* (Fig. 14g÷j), even though the frequency f_n increases. The above also takes place when $f_n > 2f_{K1}^*$ ($n > 2n_{K1}^*$) and hence in the region of significant oil vibrations (oil whip).

Conducted spectral analysis indicates at the interesting possibility of splitting of the subharmonic spectral line f_S^* into two spectral lines f_{S1}^*, f_{S2}^*, (in more complex situations there could possibly occur a split into more spectral lines). Spectral lines become dominant in the entire structure of the spectrum, which means that the slide bearings, predominantly, generate energy sustaining vibrations of the entire system.

The fact turns attention, that despite operation of the system under conditions of significant unstable conditions (oil whip) and unstable trajectories (Fig. 11c) reminding of chaotic distributions, the principal characteristic subharmonic spectral lines are clearly showing off in the picture of undetermined precisely noise (Fig. 14g÷j). It can therefore be assumed that the distributions of trajectories under conditions of oil whip can take a character of so called determined chaos.

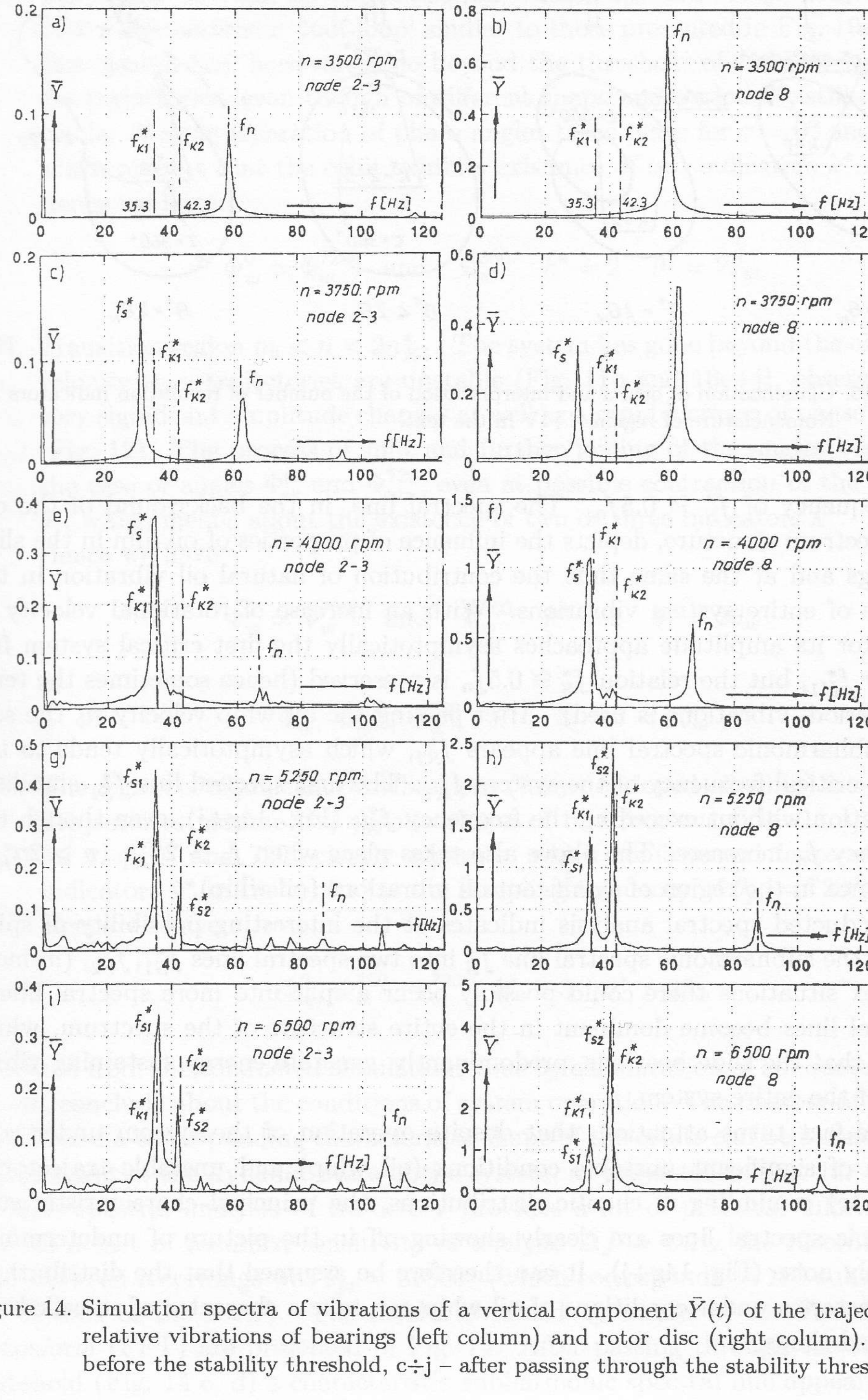


Figure 14. Simulation spectra of vibrations of a vertical component $\bar{Y}(t)$ of the trajectory of relative vibrations of bearings (left column) and rotor disc (right column). a, b – before the stability threshold, c÷j – after passing through the stability threshold.

The above conclusion can be regarded as one of the more interesting ones obtained during investigations on the physics of hydrodynamic instability in journal slide bearings.

Obtained theoretical results have been, to a large extent, experimentally verified at a purpose built large-scale rig at the IFFM PASci. Presentation of results goes beyond the volume of the present article.

7 Concluding remarks

Presented in the paper approach to describe the phenomenon of hydrodynamic instability of slide bearings have been primarily been conducted based on the Eqs. (1) and (3), hence classical equations of fluid mechanics. On the other hand the possibility of analysis of the limits of stable operation of bearings nodes and bearings-machine systems has a paramount importance with the view to exploitation and diagnostics of various types of technical objects. The link between theory (Reynolds and energy equations) and operational practice of machines (dynamics) is apparent here. The present work is only a small part of such kind of relations.

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