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ZBIGNIEW BILICKI*

Non-equilibrium flows in machine construction and exploitation

*Institute of Fluid-Flow Machinery PAFSci., Department of Multiphase Flow,
Fiszera 14, 80-952 Gdańsk, Poland*

Abstract

The paper presents capabilities of contemporary thermodynamics in the area of flow modelling with thermodynamical non-equilibrium. A philosophy for the development of constitutive equations has been shown, which is based on Extended Irreversible Thermodynamics (EIT). Two examples of its application to flows with thermal non-equilibrium have been presented.

Keywords: Non-equilibrium flows; Extended irreversible thermodynamics

1 Introduction

Contemporary thermodynamics, that is thermodynamics of irreversible processes [1], as a discipline of widely acknowledged physics, offers an effective method of search of constitutive equations in cases, when interesting phenomena are taking place in time, τ , which are of comparable order of magnitude with the relaxation time, θ . It can be assumed, that if the Deborah number, $De = \frac{\theta}{\tau} = \theta\omega > 0.01$, then the thermodynamic system or the element of fluid are not in the thermodynamic equilibrium. In the case of an element reduced to the material point we cannot even consider the local equilibrium. This is the outmost interesting case, from the point of view of contemporary thermodynamics, as the Classical Irreversible Thermodynamics cannot be applied in such case. However, we have in hand *Extended Irreversible Thermodynamics*(EIT), *Internal Variables Thermodynamics* (IVT) and the *Rational Thermodynamics* (RT). Two first concepts of thermodynamics allows for the existence of the equation

*E-mail address: zb@imp.gda.pl

of state, a non-equilibrium equation of state. The latter, RT, replaces the equation of state and the constitutive equation with the functionals of temperature history, $T(t - \varphi)$, history of the deformation of the fluid element in the form of deformation gradient tensor $F_{ij}(t - \varphi) = \text{grad}(x_k)$, which is a set of derivatives of the functions of motion and local temperature gradient, $\frac{\partial}{\partial z_i}$, with accordance to the principle of universality. It must be strongly stressed here that RT has played a very important role in the development of other methods of contemporary thermodynamics and further is an inspiration in the development of effective and relatively simple methods of finding constitutive equations, which undoubtedly ought to depend on the history of motion. It was the RT which brought the attention of thermodynamicists to the fact that the reversible processes, occurring under conditions of thermodynamical equilibrium, are only a special case of a general class of irreversible and non-equilibrium processes. The most interesting, in the authors own opinion, is the EIT. The constitutive equations, derived according to its rules, include the history of motion and are appropriate for implementation in construction and exploitation of machinery. In the present paper presented will be two examples of implementation of EIT in practice, i.e. in the description of rapid liquid depressurisation and the hydraulic jump in liquid.

2 The EIT method

In the case of single phase media the equation of state

$$s = s(u, v), \quad (1)$$

describing the state of *local* thermodynamical equilibrium is extended to include the so called generalized thermodynamic fluxes, which render that at a point there is no equilibrium. The equation of state in EIT assumes therefore a form

$$s = s(u, v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v), \quad (2)$$

where: s is the specific entropy, u – specific internal energy, v – specific volume, q – heat flux, p^v – a difference between the hydrodynamic pressure P^h and thermodynamic pressure p , and $\hat{\mathbf{P}}^v$ is the deviator of the viscous part of the pressure tensor.

It is usually assumed that in the case of the equation of state, defined in such a way, the element ds takes a form of a total differential

$$ds = \left(\frac{\partial s}{\partial u}\right) du + \left(\frac{\partial s}{\partial v}\right) \left(\frac{\partial s}{\partial \mathbf{q}}\right) \cdot d\mathbf{q} + \left(\frac{\partial s}{\partial p^v}\right) dp^v + \left(\frac{\partial s}{\partial \hat{\mathbf{P}}^v}\right) : d\hat{\mathbf{P}}^v. \quad (3)$$

Based on (3), analogically as in the case of classical thermodynamics of reversible processes, [2], we can define an absolute non-equilibrium temperature T as well as the non-equilibrium thermodynamic pressure p .

$$T^{-1}(u, v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v) = \left(\frac{\partial s}{\partial u} \right)_{v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v} \quad (4)$$

and

$$T^{-1}p(u, v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v) = \left(\frac{\partial s}{\partial v} \right)_{u, \mathbf{q}, p^v, \hat{\mathbf{P}}^v}. \quad (5)$$

As a consequence of assumed equation of state (2) there arises a problem of definition and physical interpretation of remaining partial differentials

$$\left(\frac{\partial s}{\partial \mathbf{q}} \right) = -T^{-1}v\alpha_1(u, v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v), \quad (6)$$

$$\left(\frac{\partial s}{\partial p^v} \right) = -T^{-1}v\alpha_0(u, v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v), \quad (7)$$

$$\left(\frac{\partial s}{\partial \hat{\mathbf{P}}^v} \right) = -T^{-1}v \hat{\alpha}_2(u, v, \mathbf{q}, p^v, \hat{\mathbf{P}}^v). \quad (8)$$

In above definitions there appear quantities α_1, α_0 and $\hat{\alpha}_2$, which respectively have a vector, scalar and tensor character. The most general forms are sought for these quantities from the theorems of non-linear field description of mechanics [3]. The fundamental quantities are described by means of the following equations

$$T, p, \alpha_0 = f_{1,2,3}(u, v, p^v, I_1, I_2, I_3, I_4, I_5, I_6), \quad (9)$$

where the algebraic invariants $\mathbf{q}, \hat{\mathbf{P}}^v$ are provided in the form of relations

$$\begin{aligned} I_1 = Tr \hat{\mathbf{P}}^v &\equiv 0, & I_2 = Tr(\hat{\mathbf{P}}^v)^2 &\equiv 0, & I_3 = Tr(\hat{\mathbf{P}}^v)^3 &\equiv 0, \\ I_4 = \mathbf{q} \cdot \mathbf{q}, & & I_5 = \mathbf{q} \cdot \hat{\mathbf{P}}^v \cdot \mathbf{q}, & & I_6 = \mathbf{q} \cdot (\hat{\mathbf{P}}^v)^2 \cdot \mathbf{q}. \end{aligned} \quad (10 - 15)$$

Based on the theorem of representation, the outmost general form for the remaining two coefficients can be written in the form

$$\alpha_1 = \alpha_{10}\mathbf{q} + \alpha_{11} \hat{\mathbf{P}}^v \cdot \mathbf{q} + \alpha_{12}(\hat{\mathbf{P}}^v)^2 \cdot \mathbf{q}, \quad (16)$$

$$\hat{\alpha}_2 = \alpha_{20}\mathbf{U} + \alpha_{21} \hat{\mathbf{P}}^v + \alpha_{23}(\hat{\mathbf{q}} \mathbf{q}^{sv} + \alpha_{24}(\mathbf{q} \hat{\mathbf{P}}^v \cdot \mathbf{q})^s + \alpha_{25}(\mathbf{q}(\hat{\mathbf{P}}^v)^2 \cdot \mathbf{q})^s). \quad (17)$$

Superscripts s denote symmetrical parts of deviators of particular tensors. Appearing here coefficients α_{ij} are also the functions of invariants

$$\alpha_{ij} = g(u, v, p^v, I_1, I_2, \dots, I_6). \quad (18)$$

As can be seen from the hitherto conducted considerations, the general form of coefficients, present in Eqs. (10-15), is very complex. For the sake of simplification of the form of these equations there can be considered a linearised form of above equations. Then

$$\alpha_1 = \alpha_{10}\mathbf{q}, \quad \widehat{\alpha}_2 = \alpha_{21} \widehat{\mathbf{P}}^v, \quad \alpha_0 = \alpha_{00}p^v. \quad (19 - 21)$$

In such a version both the pressure p and temperature T assume identical interpretations as those defined by CIT. Introducing (10-21) to (3) we obtain the general Gibbs equation

$$ds = T^{-1}du + T^{-1}pdv - \\ -T^{-1}v\alpha_{00}p^v dp^v - T^{-1}v\alpha_{10}\mathbf{q} \cdot d\mathbf{q} - T^{-1}v\alpha_{21} \widehat{\mathbf{P}}^v : d\widehat{\mathbf{P}}^v, \quad (22)$$

and hence the entropy derivative with respect to time in the form

$$\dot{s} = T^{-1} \nabla \cdot \mathbf{q} - T^{-1}p^v \nabla \cdot \mathbf{w} - \\ -T^{-1} \widehat{\mathbf{P}}^v : \widehat{\mathbf{V}} - T^{-1}\alpha_{00}p^v \dot{p}^v - T^{-1}\alpha_{10}\mathbf{q} \cdot \dot{\mathbf{q}} - T^{-1}\alpha_{21} \widehat{\mathbf{P}}^v : (\widehat{\mathbf{P}}^v)^\bullet. \quad (23)$$

In order to select from (23) the term responsible for entropy production we must assume the form of the entropy rate. An interesting concept is the final formulation by Müller [4], where the first term of entropy rate is the same as in CIT

$$\mathbf{J}_s = T^{-1}\mathbf{q} + \beta_{01}p^v\mathbf{q} + \beta_{10} \widehat{\mathbf{P}}^v \cdot \mathbf{q}, \quad (24)$$

where

$$\beta = \beta(u, v). \quad (25)$$

Introducing the definition of entropy rate (24, 25) we are able to determine the entropy production rate in the form

$$\sigma = \mathbf{q} \cdot \left(\nabla T^{-1} + \beta_{10} \nabla \cdot \widehat{\mathbf{P}}^v + \beta_{01} \nabla p^v + \varepsilon_1 p^v \nabla \beta_{01} - T^{-1} \alpha_{10} \dot{\mathbf{q}} + \varepsilon_2 \widehat{\mathbf{P}}^v \cdot \nabla \beta_{10} \right) + \\ + p^v \left[-T^{-1} \nabla \cdot \mathbf{w} - T^{-1} \alpha_{00} \dot{p}^v + (1 - \varepsilon_1) \mathbf{q} \cdot \nabla \beta_{01} + \beta_{01} \nabla \cdot \mathbf{q} \right] + \\ + \widehat{\mathbf{P}}^v \cdot \left[-T^{-1} \widehat{\mathbf{V}} - T^{-1} \alpha_{21} (\widehat{\mathbf{P}}^v)^\bullet + \beta_{10} (\widehat{\nabla} \mathbf{q})^s + (1 - \varepsilon_2) (\mathbf{q} \widehat{\nabla} \beta_{10})^s \right], \quad (26)$$

where ε_1 i ε_2 denote transport coefficient dependent solely on u and v .

We can see from the entropy production equation that the entropy sources have the similar structure as in CIT, i.e. the products of rates and coupled expressions playing a role of generalized thermodynamic forces.

Confining attention to the simplest cases we neglect these terms of entropy production, which introduce the non-linearity as well as we assume a constant value of the coefficients β_{01}, β_{10} . According to the rule assumed in EIT, similarly as in

CIT, introduced dissipation rates $\mathbf{q}, p^v, \widehat{\mathbf{P}}^v$ are functions of all inter-coupled expressions in the equation of entropy production (26). Amongst these expressions are the derivatives of rates with respect to time, which provide a possibility of presenting the constitutive equations in the form of evolution equations. Simplified evolution equations, related to (26), assume a form

$$\nabla T^{-1} - T^{-1} \alpha_{10} \dot{\mathbf{q}} = \mu_{10} \mathbf{q} + (\mu_{13} + \beta_{10}) \nabla \cdot \widehat{\mathbf{P}}^v + (\mu_{14} - \beta_{01}) \nabla p^v, \quad (27)$$

$$-T^{-1} \nabla \cdot \mathbf{w} - T^{-1} \alpha_{00} \dot{p}^v = \mu_{01} p^v + (\mu_{05} - \beta_{01}) \nabla \cdot \mathbf{q}, \quad (28)$$

$$-T^{-1} \widehat{\mathbf{V}} - T^{-1} \alpha_{21} (\widehat{\mathbf{P}}^v)^{\bullet} = \mu_{21} \widehat{\mathbf{P}}^v + (\mu_{24} - \beta_{10}) (\nabla \mathbf{q})^s. \quad (29)$$

In the above equations there are the coefficients present, which values stem from the imposed constraints resulting from:

- the Second Law of classical thermodynamics $\mu_{10} \leq 0, \mu_{01} \leq 0, \mu_{21} \leq 0$;
- general form of the Second Law of thermodynamics, which postulates that all spatial derivatives of rates must disappear or must group in the form of divergence of the rates products and hence $\mu_{13} = \mu_{24}, \mu_{14} = \mu_{05}$.

Introduced into (27-29) coefficients must have a physical interpretation. From the most simplified form of the above relations, corresponding to the case where the considered thermodynamic system is influenced by the stationary rates and the spatial distribution of rates is negligible, there result the relations relevant to CIT, namely

$$\nabla T^{-1} = \mu_{10} \mathbf{q}, \quad (30)$$

$$-T^{-1} \nabla \cdot \mathbf{w} = \mu_{01} p^v, \quad (31)$$

$$-T^{-1} \widehat{\mathbf{V}} = \mu_{21} \widehat{\mathbf{P}}^v. \quad (32)$$

The coefficients appearing in (30-32) have the following physical interpretation

$$\mu_{10} = (\lambda T^2)^{-1}, \quad (33)$$

$$\mu_{01} = (\zeta T)^{-1}, \quad (34)$$

$$\mu_{21} = (2\eta T)^{-1}. \quad (35)$$

Assuming, on the other hand, the unsteadiness of dissipation rates and at the same time their spatial homogeneity we obtain from (30-32) more complex relations, however, this time in the form of evolution equations

$$\nabla T^{-1} - T^{-1}\alpha_{10}\dot{\mathbf{q}} = (\lambda T^2)^{-1}\mathbf{q}, \quad (36)$$

$$-T^{-1}\nabla \cdot \mathbf{w} - T^{-1}\alpha_{00}\dot{p}^v = (\zeta T)^{-1}p^v, \quad (37)$$

$$-T^{-1}\widehat{\mathbf{V}} - T^{-1}\alpha_{21}(\widehat{\mathbf{P}}^v)^\bullet = (2\eta T)^{-1}\widehat{\mathbf{P}}^v, \quad (38)$$

The above equations can be compared with the so called Maxwell-Cattaneo equations, known earlier than the formal nomenclature introduced by EIT,

$$\tau_1\dot{\mathbf{q}} + \mathbf{q} = -\lambda \nabla T, \quad (39)$$

$$\tau_0\dot{p}^v + p^v = -\zeta \nabla \cdot \mathbf{w}, \quad (40)$$

$$\tau_2(\widehat{\mathbf{P}}^v)^\bullet + \widehat{\mathbf{P}}^v = -2\eta \widehat{\mathbf{V}}. \quad (41)$$

where τ_1, τ_2, τ_3 denote the relaxation times of relevant rates. Bearing in mind, that (36-38), in the specific cases reduce to the Fourier and Stokes equations we can obtain the physical interpretation of the constants appearing in (36-38)

$$\alpha_{10} = \tau_1(\lambda T)^{-1}, \quad (42)$$

$$\alpha_{00} = \tau_{00}\zeta^{-1}, \quad (43)$$

$$\alpha_{21} = \tau_2(2\eta)^{-1}. \quad (44)$$

We obtain a conclusion from the Clausius-Duhem inequality, [1], as a identity to be obeyed, that *the time constants in evolution equations are positive*. These results, obtained on the grounds of thermodynamics, are consistent with the inquiry that the momentum and energy balances supplemented by the evolution equations were the hyperbolic equations, where the Cauchy problem is well posed. These equations have the hyperbolic character if τ_1, τ_0, τ_2 are physically explained as the relaxation times, i.e. defined positively. As an example we can show that the energy balance in the form of heat conduction

$$\rho c \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q},$$

becomes the hyperbolic equation after introduction of evolution Eq. (39)

$$\tau = \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} - \frac{\lambda}{\rho c} \nabla^2 T = 0. \quad (45)$$

At the same time that equation describes properly the thermal wave phenomena determining the finite values of propagation velocities even of temperature disturbances with infinite frequencies. Additionally, it is worth turning attention to the fact, that the evolution equations can be regarded as functionals with respect to time, similarly as postulated in RT. It is enough to note that the evolution equation

$$\tau \frac{\partial \mathbf{q}}{\partial t} = -(\mathbf{q} + \lambda \nabla T),$$

can be solved when the temperature gradient is assumed. The solution has then the form

$$\mathbf{q}(t) = -\frac{\lambda}{\tau} \int_{-\infty}^t \exp\left(-\frac{t-t'}{\tau}\right) \nabla T(t') dt'. \tag{46}$$

It can be seen from the above equation, that the heat flux at each instant of time t depends on the value of temperature gradient not only at the same moment t , but also in the entire range of considered observations, denoted here as $-\infty$. It has also been assumed, that \mathbf{q} at the instant $t \rightarrow -\infty$ is zero. The Eq. (46) expresses the thermodynamic rate with dependence to the history of thermodynamic force, which in the considered case is expressed by the temperature gradient.

3 Application of EIT in modelling of two-phase media flows

Assuming that the two-phase medium of liquid and its vapour can be described by means of the so called homogeneous model [1], then the conservation equations for that medium assume the identical form as in the case of a single phase medium. The difference regards only to the equation of state and definition of barycentric velocity \mathbf{w} , described by means of the equations,

$$\rho_m \mathbf{w} = \rho_l \mathbf{w}_l + \rho_v \mathbf{w}_g. \tag{47}$$

The equation of state, assumed an the extended form

$$s = s(u, v, x, p^v, q_i, \hat{P}^v_{ij}, \dot{x}). \tag{48}$$

The Gibbs equation for that model of the two-phase medium takes a form

$$\begin{aligned} \dot{s} = & -T^{-1} \nabla \cdot \mathbf{q} - T^{-1} p^v \nabla \cdot \mathbf{w} - T^{-1} (\mu_g - \mu_l) \dot{x} - T^{-1} \hat{\mathbf{P}}^v : \hat{\mathbf{V}} + \\ & -T^{-1} \alpha_{001} p^v \dot{p}^v - T^{-1} \alpha_{10} \mathbf{q} \cdot \dot{\mathbf{q}} - T^{-1} \alpha_{21} \hat{\mathbf{P}}^v : (\hat{\mathbf{P}}^v)^\bullet - T^{-1} \alpha_{002} \dot{x} \frac{d\dot{x}}{dt}. \end{aligned} \tag{49}$$

Similarly as in the case of the single phase case we must find a function describing the entropy rate. We assume that in the form

$$\mathbf{J}^s = T^{-1}\mathbf{q} + \beta_{01}p^v\mathbf{q} + \beta_{10}\widehat{\mathbf{P}}^v \cdot \mathbf{q} + \beta_{02}\dot{x}\mathbf{q}, \quad (50)$$

where β is expressed by u, v, x . Having made such assumptions we can determine the entropy production in the form of the product $\sigma = \mathbf{J}^i X^i$

$$\begin{aligned} \sigma = & \mathbf{q} \cdot (\nabla T^{-1} - T^{-1}\alpha_{10}\dot{\mathbf{q}} + \beta_{10}\nabla \cdot \widehat{\mathbf{P}}^v + \beta_{01}\nabla p^v + \varepsilon_1 p^v \nabla \beta_{01} + \\ & + \varepsilon_2 \widehat{\mathbf{P}}^v \cdot \nabla \beta_{10} + \beta_{02}\nabla \dot{x} + \varepsilon_3 \dot{x} \nabla \beta_{02}) + \\ & + p^v [-T^{-1}\nabla \cdot \mathbf{q} - T^{-1}\alpha_{001}\dot{p}^v + (1 - \varepsilon_1)\mathbf{q} \cdot \nabla \beta_{01} + \beta_{01}\nabla \cdot \mathbf{q}] + \\ & + \dot{x} [-T^{-1}(\mu_g - \mu_l) - T^{-1}\alpha_{002}\frac{d\dot{x}}{dt} + \beta_{02}\nabla \cdot \mathbf{q} + (1 - \varepsilon_3)\mathbf{q} \cdot \nabla \beta_{02}] + \\ & + \widehat{\mathbf{P}}^v \cdot [-T^{-1}\widehat{\mathbf{V}} - T^{-1}\alpha_{21}(\widehat{\mathbf{P}}^v)^\bullet + \beta_{10}(\nabla \mathbf{q})^s + (1 - \varepsilon_2)(\mathbf{q} \nabla \beta_{10})^s]. \end{aligned} \quad (51)$$

From Eq. (51) we can determine all constitutive equations describing the two-phase medium. These have a very complex form. To illustrate the problem presented below is a simplest form of constitutive equations, which describe very interesting relations

$$\begin{aligned} \nabla T^{-1} - T^{-1}\alpha_{10}\dot{\mathbf{q}} &= \mu_{10}\mathbf{q}, \\ -T^{-1}\gamma_1(\mu_g - \mu_l) - T^{-1}\nabla \cdot \mathbf{w} - T^{-1}\alpha_{001}\dot{p}^v &= \mu_{01}p^v, \\ T^{-1}\gamma_2\nabla \cdot \mathbf{w} - T^{-1}(\mu_g - \mu_l) - T^{-1}\alpha_{002}\frac{d\dot{x}}{dt} &= \mu_{001}\dot{x}, \\ -T^{-1}\widehat{\mathbf{V}} - T^{-1}\alpha_{21}(\widehat{\mathbf{P}}^v)^\bullet &= \mu_{21}\widehat{\mathbf{P}}^v. \end{aligned} \quad (52a - d)$$

Of particular interest is the relation (52c), which consists of the second derivatives of quality with respect to space and time. A detailed discussion of that relation has been presented in [5].

4 Application of EIT in the description of rapid liquid depressurisation

Obtained constitutive Eq. (37) has been successfully implemented in calculations of pressure changes during a rapid depressurisation of liquid with temperature of 20°C, where pressure was varied from 80 bar down to a negative measured pressure (positive normal stress) of about 27.5 bar. The process was

run at a rate of about $8.6 \cdot 10^{10}$ Pa/s and was experimentally investigated by the Lienhards group [6]. It has been assumed, that the equation of state takes a form

$$s = s(u, v, x, p^h) \quad (53)$$

A one-dimensional model has been used in calculations of such process which had operational physical properties [7-8]: operational viscosity $\tilde{\eta}$ and operational thermal conductivity $\tilde{\lambda}$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial t} = -\frac{\rho w}{A} \frac{dA}{dz}, \quad (54)$$

$$\rho \frac{dw}{dt} = -\frac{\partial p^h}{\partial z} + \frac{\partial}{\partial z} \left[\frac{4}{3} \tilde{\eta} \frac{dw}{dz} \right] - \tau_w \frac{C}{A}, \quad (55)$$

$$\rho \frac{dh}{dt} - \frac{dp}{dt} = \tau_w w \frac{C}{A} + (p - p^h) \frac{\partial w}{\partial z} + \left(\frac{4}{3} \tilde{\eta} \right) \left(\frac{dw}{dz} \right)^2 + \frac{\partial}{\partial z} \left(\tilde{\lambda} \frac{\partial T_s}{\partial z} \right), \quad (56)$$

where T_s is the saturation temperature, A – channel cross-section, C – channel circumference,

$$\tau_w = \frac{1}{2} f \rho \frac{w^2}{2}, \quad (57)$$

describes the shear stress at the channel wall, f is the Fanning friction factor, $\tilde{\lambda}$ and $\tilde{\eta}$ are mentioned above operational properties. The above equations have been supplemented by two constitutive equations

$$\frac{dx}{dt} = L_{11}(\mu_l - \mu_g) + L_{12} \frac{\partial w_i}{\partial z_i}, \quad (58)$$

$$\theta_P \frac{dP^v}{dt} + p^v = L_{21}(\mu_l - \mu_g) + L_{22} \frac{\partial w_i}{\partial z_i}, \quad (59)$$

where θ_P is the pressure relaxation time, $L_{12} = L_{22} = \zeta$ is the bulk viscosity, μ_l, μ_g are chemical potentials of liquid and vapour respectively, $L_{11} = L_{21}$, are phenomenological coefficients determined on the basis of the experiment. A comparison between the theory and experiment, performed by the Lienhards group, [6], has been presented in Fig. 1.

5 Application of EIT in description of hydraulic jump in the pipeline

Hydraulic pressure in the closed circuit pipeline p^h , can be described using the conservation equations of mass, momentum and energy in the form (54-56), however, with the amendment, that it has been assumed that $p^h = p$ and the

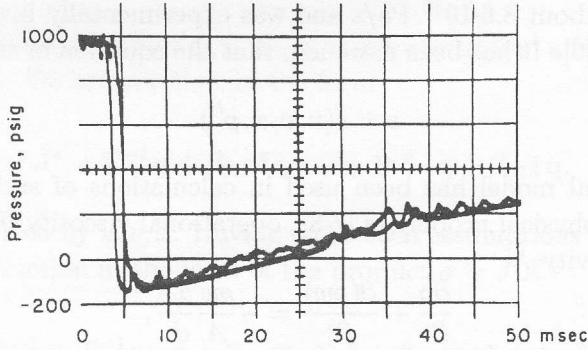


Figure 1. A comparison between theory and the experiment by the Lienhards group, [6].

equation of state, describing the fluid inside the channel, is given by the equation derived from the EIT

$$s = s(u, v, \tau_n), \quad (60)$$

a constitutive equation describing the state of stress at the wall is provided by the evolution equation

$$\tau_n = \tau_w + \theta_\tau \frac{d\tau_n}{dt}. \quad (61)$$

That equation has a form of the functional

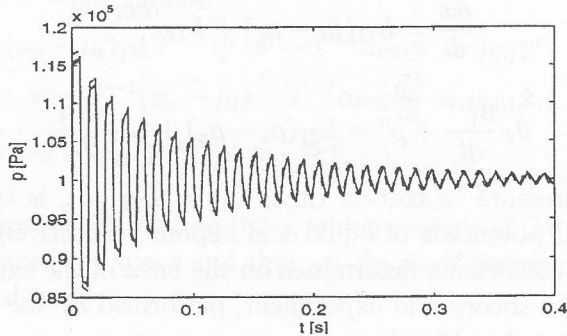


Figure 2. Air pressure in the channel after sudden closure of the valve at the outlet. Theoretical calculations are denoted by the broken line and experiment by the solid line. Calculations have been performed assuming $k = -0.05$ during the flow retardation and $k = 0$ during the flow acceleration.

$$\tau_n(t) = \frac{1}{\theta_t} \int_{-\infty}^t \exp\left(-\frac{t-t'}{\theta_\tau}\right) \tau_w(t') dt', \quad (62)$$

where $\theta_\tau = \text{const}$ and $\tau_n = 0$ when $t \rightarrow -\infty$. The stress τ_w , present in Eq. 61), is described by Eq. 57), whereas τ_n replaces τ_w in the model based on EIT. Using such model Kucieńska et al. [9] performed calculations and compared them with the experiment conducted in the tube filled with air. The tube had a length of 1.13 m and internal diameter $D = 32$ mm. Initial mass flow rate was equal to 0.04 kg/s. The hydraulic jump appeared due to a sudden closure of the valve downstream the flow. The authors of calculations suggested the following expression describing the relaxation time θ_τ

$$\theta_\tau = \frac{k\rho D}{4} \frac{de}{d\tau_n}. \quad (63)$$

6 Summary

Presented two examples of application of EIT in calculation of non-equilibrium flows in machine construction show potential capabilities, which are offered by the contemporary non-equilibrium thermodynamics. Such thermodynamics extends the modeling capabilities onto the cases of flows, where thermodynamical equilibrium is not preserved. It can be an effective tool, in particular in the case of multi-particle flows, such as for example polymers.

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