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2D inviscid flutter of the mistuned Fourth and First Standard Configuration

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Abstract

The trend in aviation engines with high specific power and, correspondingly, high aerodynamic loads leads to the problem of aeroelastic behaviour of blades not only in compressors and fans, but also in turbines. Investigations of aeroelastic behaviour of the blades in dependence of structural mistuning are presented.

A numerical calculation method for unsteady aerodynamic characteristics of oscillating blade cascades under the action of unstable loads is based on solution of the coupled fluid-structure problem, in which the aerodynamic and structural dynamic equations are integrated simultaneously in time, thus providing the correct formulation of a coupled problem, as the interblade phase angle, at which a stability (instability) would occur, is a part of solution.

The ideal gas flow around multiple stage passages (with periodicity on the whole annulus) is described by the unsteady 2D Euler equations in conservative form, which are integrated by using the explicit monotonous second order accurate Godunov-Kolgan finite-volume scheme on the moving grid.

The blade is modelled by a very simple two degrees of freedom discrete model. In this model cascade performs the torsional and the bending oscillations under the given law. The aeroelastic behaviour of the blades in the unsteady aerodynamic flow is calculated for the mistuned blades assemblies of the Fourth and First Standard Configurations. The computational domain of the unsteady flow can not be restricted to the single blade passage. The results in the time domain analysis show the beneficial influence of the mistuning of the bending mode in comparison to the torsional mode. The dynamic properties of the mistuned systems are dependent on the way of coupling of the blades, whether it is either aerodynamic or mechanical.

Keywords: Inviscid flutter; Cascade; Mistuning

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Nomenclature

IBPA	– interblade angle, deg	p	– pressure, MPa
h	– blade bending amplitude, m^3/s	t	– temperature, ^oC
L	– blade length, m		

1 Introduction

The flutter of turbine blades is one of the most serious problems in the design of the gas-turbine engines, since it limits the range of safe regimes and worsens the motor characteristics. A number of theoretical, computational and experimental studies has been carried out in order to investigate the physical mechanism of flutter and to develop a method for its prediction and suppression. Most calculations of the flutter and forced response in turbomachines are based on ideally tuned modes in the blading. But in a real construction small technological deviations in blade geometry lead to a scatter of the blade eigenfrequencies. Structural mistuning in the flutter calculation has been discussed in the literature for a long time [1-14]. Development of calculation techniques allows new approach to this problem, based on the solution of the coupled fluid-structure problem, in which the aerodynamic and structural dynamic equations are integrated simultaneously in time, thus providing the correct formulation of a coupled problem, as the interblade phase angle, at which stability (instability) would occur, is a part of solution. A literature review is beyond the scope of this paper, but a survey of aeroelasticity methods can be found in [15-17] and [18-22].

In the present paper the aeroelastic behaviour of the blades in 2D unsteady aerodynamic flow is calculated for the mistuned blades assembly of the Fourth and First Standard Configurations. The computational domain of the unsteady flow can not be restricted to the single blade passage. The results show the beneficial influence of mistuning of bending modes. The mechanical properties of mistuned systems in the forced vibration and flutter analysis are presented since 1969 (see [22]). The conclusions of such studies have sometimes been inconsistent. For example, it is desirable to identify in advance the blades that are likely to experience the largest vibration amplitudes. Some workers have observed that such blades are most likely to be those having extreme detuning, while others disagree, [22]. From the results presented in this paper it can be seen that the dynamic properties of the mistuned systems are dependant on the way of coupling of the blades, whether it is aerodynamic or mechanical.

2 Aerodynamic models

The aerodynamic 2D flutter model developed by Kovalyov [21] and Rzadkowski and Kovalyov [20] is capable of representing 2D inviscid flows over a wide Mach number range from low subsonic to supersonic, including transonic flows.

The 2D transonic flow of an ideal gas through a multipassage blade row is considered. In a general case the flow is assumed to be an aperiodic function from blade to blade (in the pitchwise direction), so the calculated domain includes all blades of the whole assembly. The aerodynamic model fully accounts for the blade thickness, camber and the angle-of-attack effects. The unsteady flow of the ideal gas is described by the 2D Euler equations

$$\frac{\partial}{\partial t}\mathbf{U} + \frac{\partial}{\partial x}\mathbf{F}_1 + \frac{\partial}{\partial y}\mathbf{F}_2 = 0\,, \tag{1}$$

where

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ E \end{bmatrix} \quad \mathbf{F}_1 = \begin{bmatrix} \rho v_1 \\ p + \rho v_1^2 \\ \rho v_1 v_2 \\ (p+e)v_1 \end{bmatrix} \quad \mathbf{F}_2 = \begin{bmatrix} \rho v_2 \\ \rho v_1 v_2 \\ p + \rho v_2^2 \\ (p+e)v_2 \end{bmatrix}$$

Here p and ρ are the pressure and density; v_1 and v_2 are velocity components,

$$e = \rho(\varepsilon + \frac{v_1^2 + v_2^2}{2}) \tag{2}$$

is the total energy of volume unit; ε is the internal energy of mass unit. The above system of equations is closed by the state equation for perfect gas

$$p = \rho\varepsilon(\chi - 1), \tag{3}$$

where χ denotes the ratio of the fluid specific heats. The differential equation (1) is integrated on a moving H-type grid with the use of explicit monotonous second-order accuracy Godunov's-Kolgan type difference scheme.

This model can be used as a basis for prediction of both the unsteady aerodynamic forces acting on turbine blades and the time-averaged flow field in a turbine stage under different regime conditions and geometrical characteristics. The steady and unsteady flow calculation for a cascade require implementation of different boundary conditions which determine the solution. These boundary conditions are the kinematic flow condition (vanishing normal velocity at the blade's surface), inlet and outlet boundary conditions and periodic boundary conditions at pitchwise boundaries. The boundary conditions formulation is based on the one-dimensional theory of the characteristics. According to this theory some gas-dynamic parameters on the boundary of calculated domain, in the unsteady flow, characterize the influence of the external domain (these parameters are imposed as the boundary conditions) and some parameters characterize perturbation coming from the internal domain (these ones are defined from characteristic relations). The number of boundary conditions depends on the flow conditions. When the axial velocity is subsonic, the following boundary conditions are imposed.

At the inlet of calculation domain – the total temperature T_o, the total pressure P_o and the entropy

$$\frac{p}{\rho^k} = S_0 \ ,$$

the flow angle β_1

$$v_1 = -v_2 \coth \beta_1 \ ,$$

the enthalphy

$$\frac{k}{k-1}\frac{p}{\rho} + \frac{v_1^2 + v_2^2}{2} = i_0 \ ,$$

and the left Riemann invariant

$$v_1 - \frac{2}{k-1}\sqrt{k\frac{p}{\rho}} = V_1 - \frac{2}{k-1}\sqrt{k\frac{P}{R}} \ ,$$

at the exit, in the case of subsonic flow – the static pressure P_2, the right Riemann invariant is assumed to be the constant.

In the case of supersonic inflow velocity components V_1, V_2 must be assumed. If the outflow is supersonic, the continuation condition is applied to all parameters at the downstream boundary.

3 Structural model

In order to compare the numerical results with the experimental 2D results a very simple discrete model with two degrees of freedom of the cascade is presented. In this model all blades perform the torsional and the bending oscillations under the given law

$$h_z(x,t) = h_{z0}(x) \cdot \sin[2\pi\nu_h t + (j-1)\delta],$$
$$h_y(x,t) = h_{y0}(x) \cdot \sin[2\pi\nu_h t + (j-1)\delta],$$
$$\varphi(x,t) = \varphi 0(x) \cdot \sin[2\pi\nu_\varphi t + (j-1)\delta],$$

where x, y, z are the Cartesian coordinate system fixed rigidly with j^{th} blade (origin of coordinates is coinciding with the bending centre in the case of torsional oscillations); $h_z(x,t)$ and $h_y(x,t)$ define the bending oscillations, $\varphi(x,t)$ the torsional oscillations; $h_{z0}, h_{y0}, \varphi_0$ are the amplitudes of vibrations; ν_h is the bending oscillation frequency; ν_φ is the torsional oscillation frequency, δ is IBPA. The equations of motion of the palisade

$$\begin{aligned} \ddot{h}_{zi}(t) + \omega_{hi}^2 h_{zi}(t) &= f_{zi}(t), \\ \ddot{h}_{yi}(t) + \omega_{hi}^2 h_{yi}(t) &= f_{yi}(t), \\ \ddot{\varphi}_{zi}(t) + \omega_{\varphi i}^2 \varphi_i(t) &= m_i(t) \end{aligned} \tag{7}$$

were solved for bending and torsion (see [22], p. 95). For example in the time interval $(t_1, t_1 + \Delta t)$

$$h_{zi} = (C_2 \sin \omega_h \Delta t + C_1 \sin \omega_h \Delta t) + f_{zi}/\omega_h^2, \tag{8}$$

where

$$C_1(t_1) = h_{zio}(t_1) - f_{zi}(t_1)/\omega_h^2, \;\; C_2(t_1) = (dh_{zio}(t_1)/dt - f_{zi}(t_1)/\omega_h^2)/\omega_h,$$

$h_{zio}(t_1)$ – displacement at the beginning of the period Δt, $\omega_h = 2\pi\nu_h$.

In this model the blades are free at the root cross-section. The initial conditions are: the steady flowfield and assumed unsteady forces applied to the blades. Boundary conditions from the structural and aerodynamic domains are exchanged at each step and the aeroelastic mesh is moved to follow the structural motion (the partially integrated method).

4 Numerical results

The influence of mistuning on the flutter parameters was carried out for the Fourth and First Standard Configurations (see Bölcs and Fransson [19]).

The Fourth Standard Configuration represents the annular cascade which consists of 20 vibrating prismatic blades. This configuration is of interest mainly because it represents a typical section of modern, free standing turbine blades. This type of airfoil has relatively high blade thickness and camber, and operates under high subsonic flow conditions. It normally exhibits flutter instabilities in the first bending mode. The length of the blade $L = 0.04$ m, chord $2c = 0.0774$ m, the stagger angle equal to 56.6^o, with the pitch-to-chord ratio 0.76 (see Fig. 1). The cascade geometry and profile coordinates are given in Bölcs and Fransson [19].

It is assumed that small technological deviations in a blade geometry lead to a scatter of the blade eigenfrequencies, but has no significant influence on the unsteady aerodynamic parameters of the flow. So in the numerical calculations only natural frequencies of the blades were changed, the geometry of the channels were unchanged. First calculations were performed for 8 interblade passages of 8 blades of the Fourth Standard Configuration. Mach number for calculation

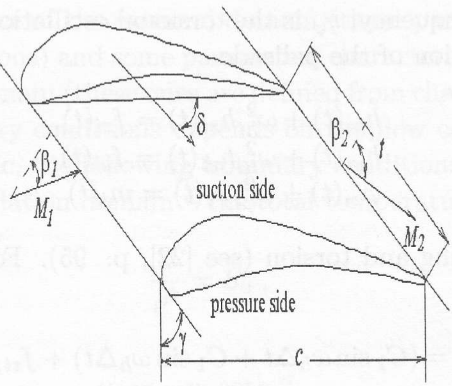

Figure 1. Fourth Standard Configuration.

regime was assumed to be 0.90. Periodical conditions have been assumed to be valid for each group of 8 blades.

The aerodynamic problem was solved for the kinematic bending oscillations of blade cascades with frequency equal to 150 Hz, amplitude 0.0033 m and interblade phase lag equal to -90^o (the flutter condition [19]). Next the coupled fluid-structure problem was solved. Three arrangements of blades in the cascade are considered:

- Tuned blades (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0) first arrangement (I);
- 1% mistune blades (1.00, 1.01, 1.00, 1.01, 0.99, 1.00, 0.99, 1.01), second arrangement (II);
- 10% mistuned blades (1.00, 1.10, 1.00, 1.10, 0.90, 1.00, 0.90, 1.10), third arrangement (III).

The coefficient 1.01 means that the natural frequency of these blades is changed by 1.01 (1%) in comparison to the tuned blade with the coefficient 1.0.

Figures 2-7 presents 1^{st}, 2^{nd} and 5^{th} blades response for arrangements I, II, III respectively. Figures 3, 5, 7 show considered blades response for the last two vibration periods. It is seen that the considered mistuning changes the interblade phase angle and blades amplitude. The higher the value of mistuning, the bigger the change of the interblade phase angle and the amplitude are.

Next, aerodynamic problem was solved for the kinematic and coupled fluid-structure bending oscillations of 20 blades with frequency equal to 150 Hz, amplitude 0.0033 m and interblade phase lag equal to -90^o (the flutter condition [19]). Only one blade is detuned by -10% and is placed at position N. Figures 8-15 present the blade amplitudes of the different blades in the cascade.

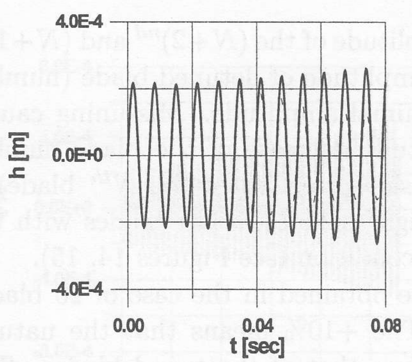

Figure 2. 1^{st} blade amplitude for different cascade mistuning, —— tuned, —— mistuned 1%, - - - mistuned 10%.

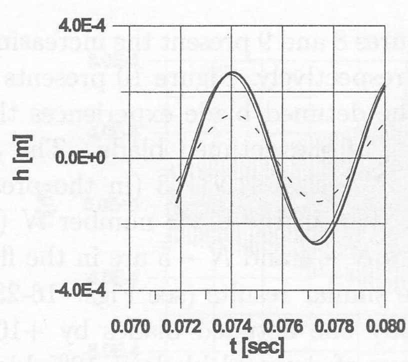

Figure 3. 1^{st} blade amplitude for different cascade mistuning in the last calculation period, —— tuned, —— mistuned 1%, - - - mistuned 10%.

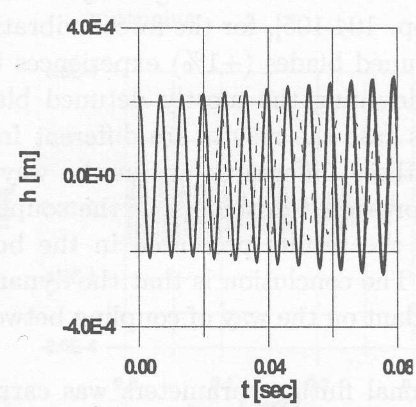

Figure 4. 2^{nd} blade amplitude for different cascade mistuning, —— tuned, —— mistuned 1%, - - - mistuned 10%.

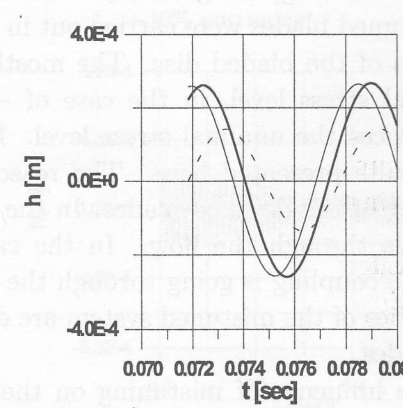

Figure 5. 2^{nd} blade amplitude for different cascade mistuning in the last calculation period, —— tuned, —— mistuned 1%, - - - mistuned 10%.

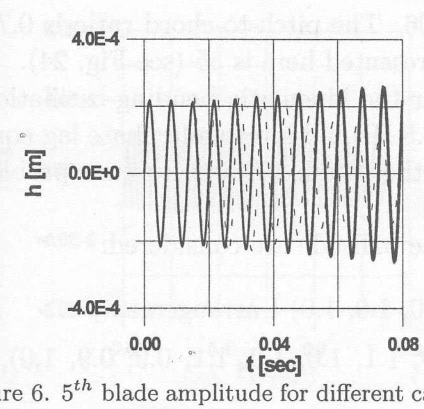

Figure 6. 5^{th} blade amplitude for different cascade mistuning, —— tuned, —— mistuned 1%, - - - mistuned 10%.

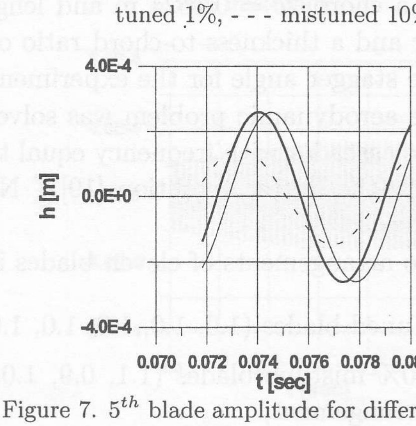

Figure 7. 5^{th} blade amplitude for different cascade mistuning in the last calculation period, —— tuned, —— mistuned 1%, - - - mistuned 10%.

Figures 8 and 9 present the increasing amplitude of the $(N+2)^{nd}$ and $(N+1)$ blades respectively. Figure 10 presents the amplitude of detuned blade (number N). The detuned blade experiences the minimal amplitude. Mistuning causes damping of the detuned blade. The amplitude decrease of the blade number $N-1$, $N-2$ and $N-3$ (in the pressure side direction of the N^{th} blade) is smaller than in the blade number N (see Figures 11-13). The blades with the numbers $N-4$ and $N-5$ are in the flutter condition (see Figures 14, 15).

The similar results (see Figs. 16-23) were obtained in the case of 20 blades with only one detuned blades by +10%. The +10% means that the natural frequency of detuned blade is 10% higher than that of the tuned blades. The blades number N, $N-1$, $N-2$, $N-3$ are damped. The damping of the blade for a mistuning +10% is bigger (see Figs. 11, 19, 12, 20), than in the case of −10% mistuning, but generally the trends are similar. Mistuning analysis of the one detuned blades were carried out in [22, pp. 104-105], for the forced vibration analysis of the bladed disc. The mostly detuned blades (+1%) experiences the maximal stress level, in the case of −1% detuning the mostly detuned blade experiences the minimal stress level. In this case the results are different from the results presented here. The reason of these differences are in the way of coupling of the detuned blades. In the case presented in this paper the coupling proceeds through the flow. In the case of the results presented in the book [22], the coupling is going through the disk. The conclusion is that the dynamic properties of the mistuned system are dependant on the way of coupling between the blades.

The influence of mistuning on the torsional flutter parameters was carried out for First Standard Configuration (see Blcs and Fransson [19]). The First Standard configuration consists of eleven vibrating NACA 65-series blades, each having a chord $2c = 0.1624$ m and length $L = 0.254$ m, with a 10 circular arc camber and a thickness-to-chord ratio of 0.06. The pitch-to-chord ratio is 0.75 and the stagger angle for the experiments presented here is 55 (see Fig. 24).

The aerodynamic problem was solved for the kinematic bending oscillations of blade cascades with frequency equal to 15.5 Hz and interblade phase lag equal to 98.1^o (the flutter condition [19]). Next the coupled fluid-structure problem was solved.

Two arrangements of eleven blades in the palisade are considered:

- Tuned blades (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0) I arrangement;
- 10% mistune blades (1.1, 0.9, 1.0, 0.9, 1.1, 1.0, 1.0, 1.1, 0.9, 0.9, 1.0) II arrangement;

Figures 25-36 presents all blades response for arrangements I and II respectively. The influence of mistuning on the palisade is not so beneficial as in the

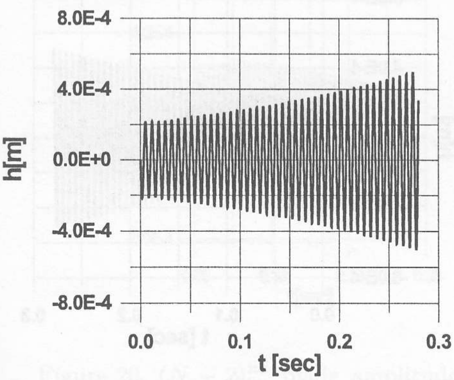

Figure 8. $(N+2)^{nd}$ blade amplitude for -10% mistuning.

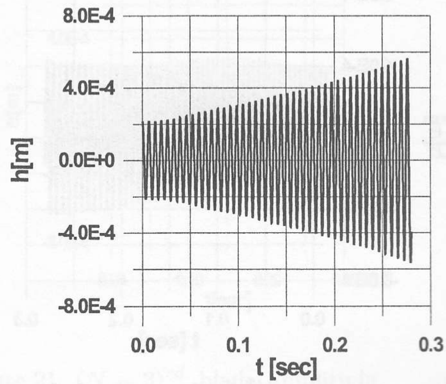

Figure 9. $(N+1)^{th}$ blade amplitude for -10% mistuning.

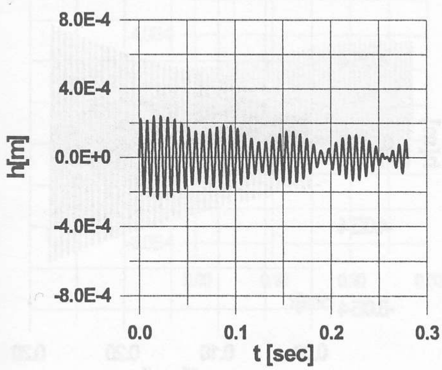

Figure 10. N^{th} blade amplitude for -10% mistuning.

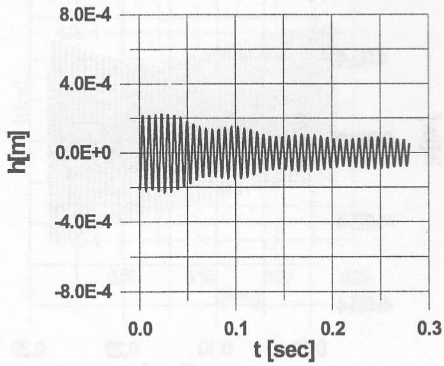

Figure 11. $(N-1)^{st}$ blade amplitude for -10% mistuning.

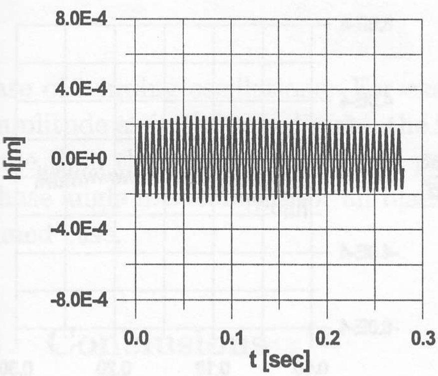

Figure 12. $(N-2)^{nd}$ blade amplitude for -10% mistuning.

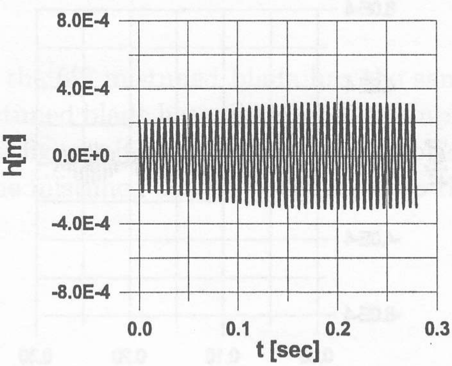

Figure 13. $(N-3)^{rd}$ blade amplitude for -10% mistuning.

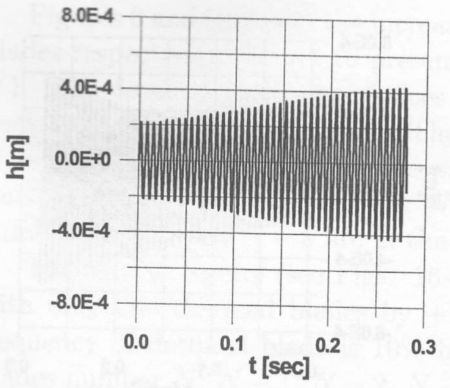

Figure 14. $(N-4)^{th}$ blade amplitude for -10% mistuning.

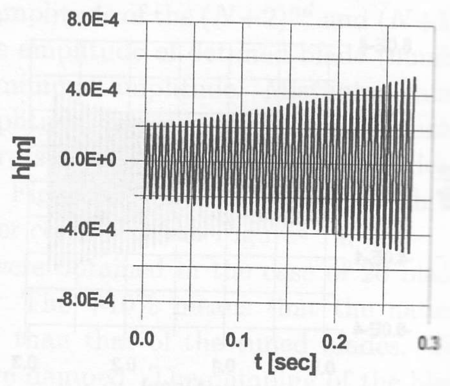

Figure 15. $(N-5)^{th}$ blade amplitude for -10% mistuning.

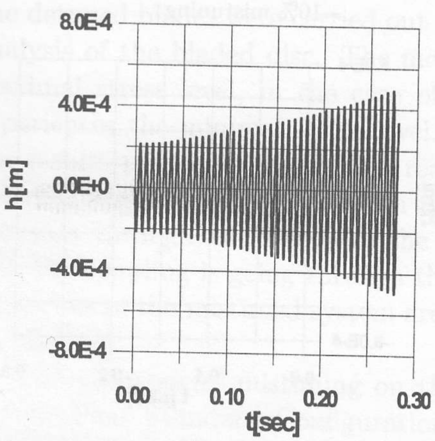

Figure 16. $(N+2)^{nd}$ blade amplitude for $+10\%$ mistuning.

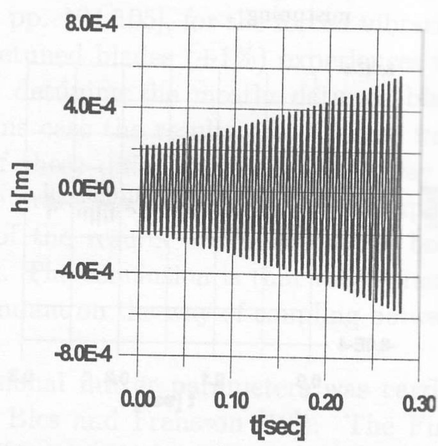

Figure 17. $(N+1)^{th}$ blade amplitude for $+10\%$ mistuning.

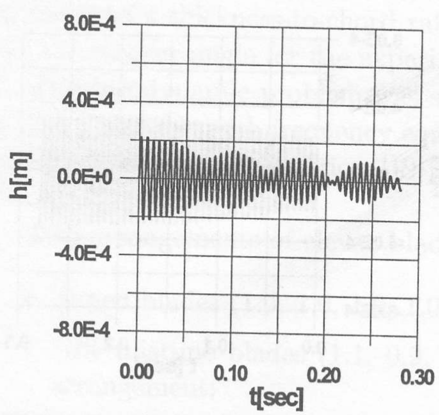

Figure 18. N^{th} blade amplitude for $+10\%$ mistuning.

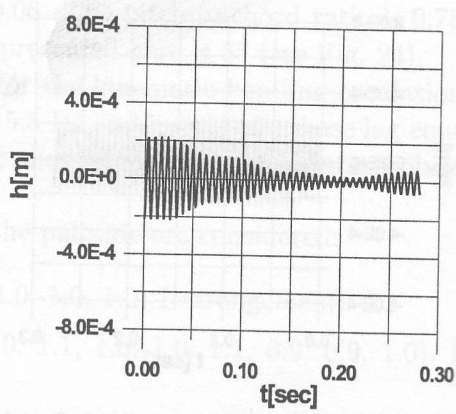

Figure 19. $(N-1)^{th}$ blade amplitude for $+10\%$ mistuning.

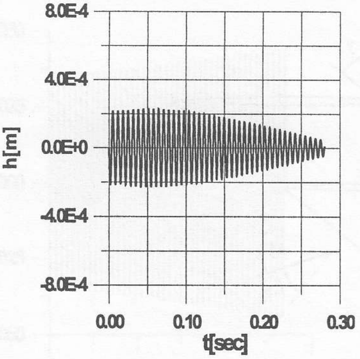

Figure 20. $(N-2)^{nd}$ blade amplitude for +10% mistuning.

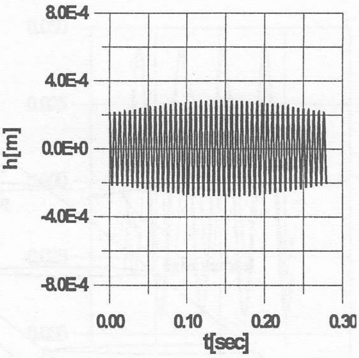

Figure 21. $(N-3)^{rd}$ blade amplitude for −10% mistuning.

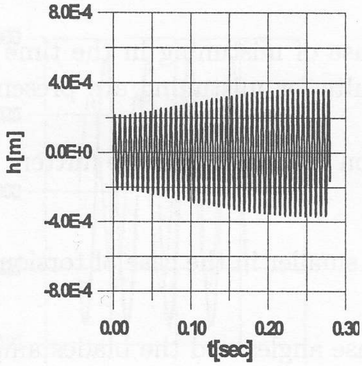

Figure 22. $(N-4)^{th}$ blade amplitude for +10% mistuning.

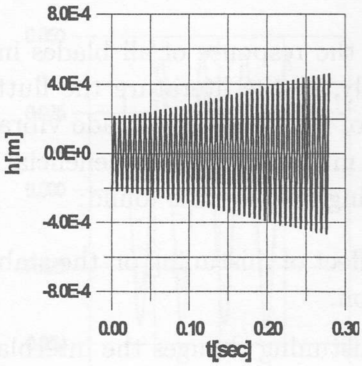

Figure 23. $(N-5)^{th}$ blade amplitude for +10% mistuning.

case of bending oscillations. For example, the 6^{th} mistuned blade has the same amplitude as in the tuned state, the 9^{th} mistuned blade has a little smaller amplitude than the tuned blade. From presented figures it is seen that the interblade phase angle has changed for all blades in the mistuned case in comparison to the tuned case.

5 Conclusions

In this paper the influence of mistuning on the response of the blades in the flutter condition was shown. The calculations were done for all blades in the cascade. The time integration method presented here gives the possibility to

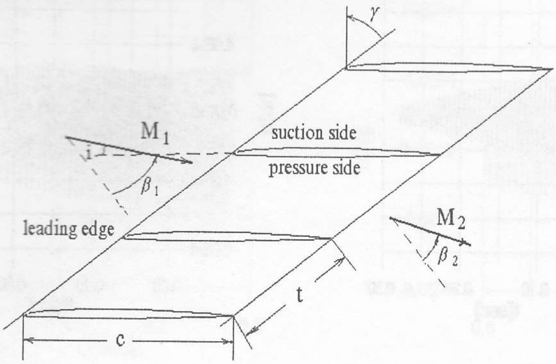

Figure 24. First Standard Configuration.

analyse the response of all blades in the case of mistuning in the time domain. Generally, in the literature the flutter results for mistuning are presented as a change of frequency of cascade vibration.

The mistuning has the beneficial effect on the bending blade flutter. The new interesting results were found:

- Effect of mistuning on the stability is smaller in the case of torsional vibration.
- Mistuning changes the interblade phase angles and the blades amplitude.
- In the case of palisade with one detuned blade, the amplitudes of blade close to detuned blade form the pressure side are decreased.

The mechanical properties of mistuned systems for a forced vibration and flutter have been presented since 1969 (see [22]). The conclusions of such studies have sometimes been inconsistent. For example, it is desirable to identify in advance the blades that are likely to experience the largest vibration amplitudes. Some workers have observed that such blades are most likely to be those having extreme detuning, while others disagree with that, [22]. From the results presented in this paper it is seen that the dynamic properties of the mistuned systems are dependent on the way of coupling of the blades, whether it is aerodynamical or mechanical.

As real turbomachinery blading will always be mistuned, these results show the necessity to include the mistuning effects in flutter analysis. The influence of the disk and the flow on the dynamic stability of the mistuned cascade have to be done as a next step.

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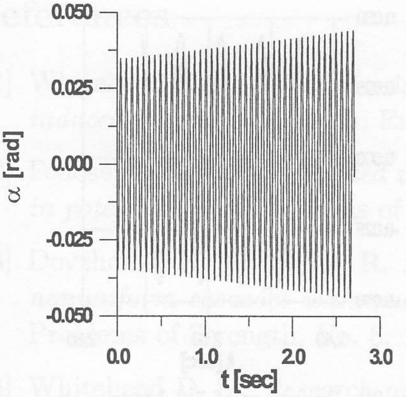

Figure 25. Response of 1st blade for tuned cascade.

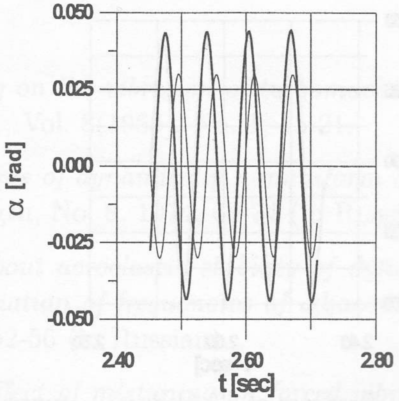

Figure 26. Response of 1st blade, —— tuned, —— mistuned.

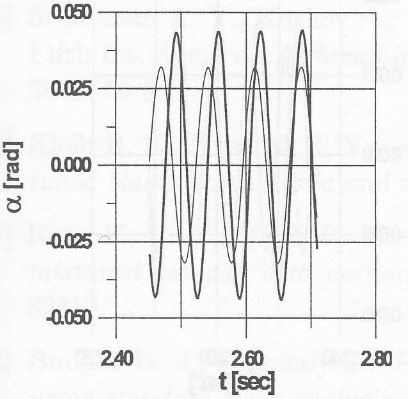

Figure 27. Response of 2nd blade, —— tuned, —— mistuned.

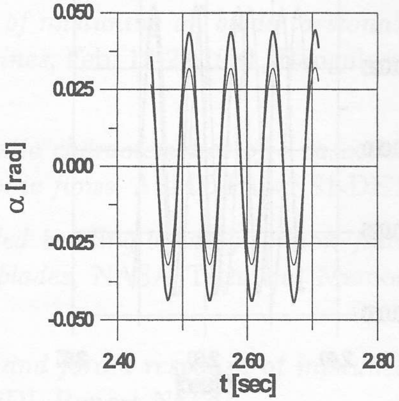

Figure 28. Response of 3rd blade, —— tuned, —— mistuned.

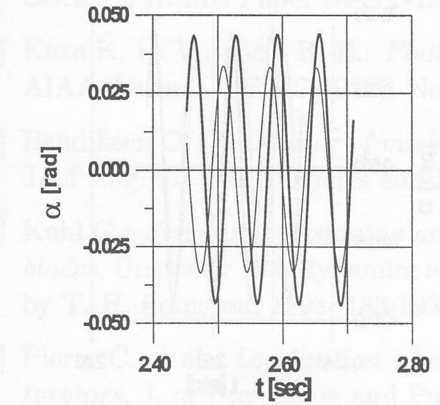

Figure 29. Response of 4th blade, —— tuned, —— mistuned.

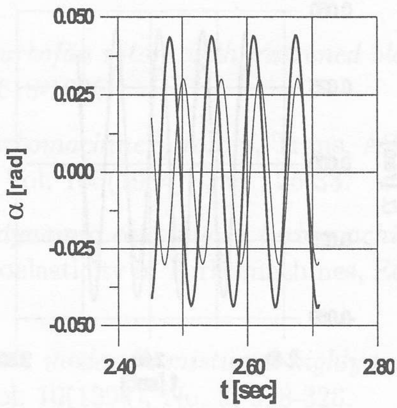

Figure 30. Response of 5th blade, —— tuned, —— mistuned.

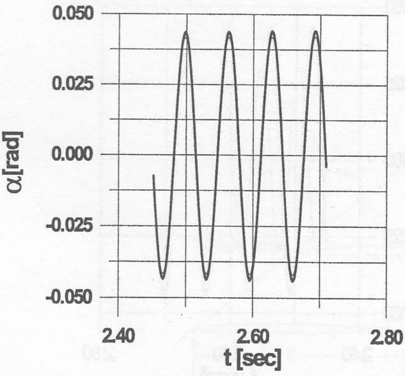

Figure 31. Response of 6^{th} blade, —— tuned, —— mistuned.

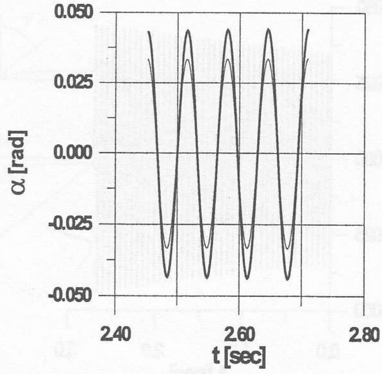

Figure 32. Response of 7^{th} blade, —— tuned, —— mistuned.

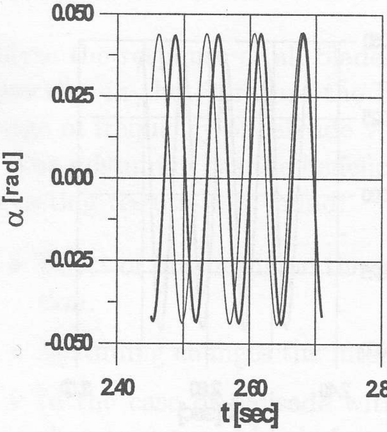

Figure 33. Response of 9^{th} blade, —— tuned, —— mistuned.

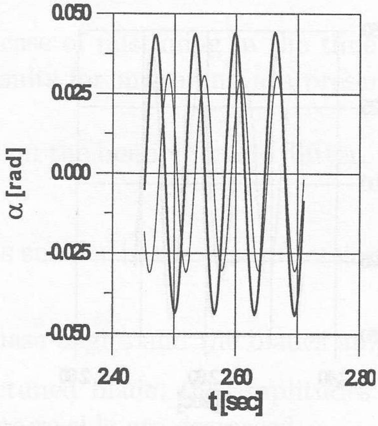

Figure 34. Response of 8^{th} blade, —— tuned, —— mistuned.

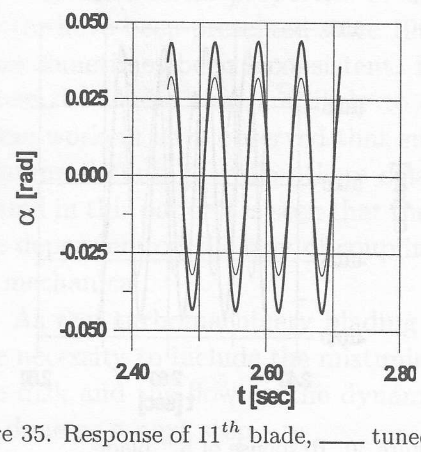

Figure 35. Response of 11^{th} blade, —— tuned, —— mistuned.

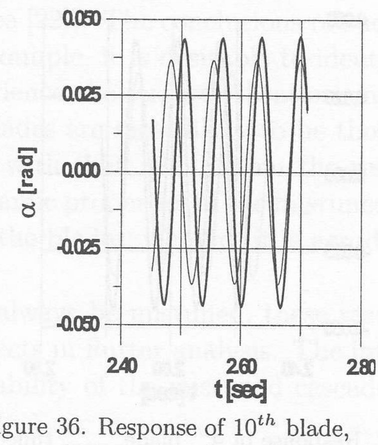

Figure 36. Response of 10^{th} blade, —— tuned, —— mistuned.

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