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Theoretical foundation for velocity measurement by means of the five holes sphere probe

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Abstract

The measurement of the local velocity belongs to the most common in the fluid dynamics and power engineering. This measurement is mostly done by the five hole sphere probe. In this paper, basing on the potential flow past the sphere, the analysis of the velocity measurement by means of the sphere probe is presented. The transformation relation between the spherical and the probe coordination i.e. yaw and pitch angle α and β is found. Solving next the system of five nonlinear equations with four unknowns the simple theoretical characteristics of the five hole probe is found and compared with the experimental one. There is a quite good agreement in the angle ranges of $-10^{o} < \alpha < 10^{o}$ and $-10^{o} < \beta < 10^{o}$. The discrepancies between theoretical and experimental characteristics outside this region are probably caused by the inaccuracies in manufacturing of the sphere probe and differences between potential past a sphere and real flow past a sphere probe.

Keywords: Velocity mesurement; Sphere probe

Nomenclature

A, B	–	constants
R	–	radius of sphere
u	–	local velocity, m/s
U	–	stream velocity, m/s
p	–	pressure, Pa
α	–	declination angle, rad
β	–	pitch angle, rad
θ	–	angle from the stagnation point, rad
r, ϕ, ψ	–	spherical coordinates

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Φ	–	velocity potential, m^2/s
ρ	–	density, kg/m^3
x, y, z	–	Cartesian coordinates

Subscripts

$i = 1, \ldots 5$	–	denotation of points on the sphere

1 Introduction

The sphere probe is one of the most used device for velocity measurement in flows. By means of such probe the measurement of the velocity amplitude and two angles of the coordinate system connected with the probe is possible. The measurement is carried out by the pressure measurements in the chosen points on the surface of sphere i.e. mostly in five points given on the two intersecting planes creating a cross on the sphere surface. The sphere probe is known as a probe of great durability and simplicity, contrary to the hot wire probe. And, if additionally, its small size, e.g. a diameter of 5 mm and the possibility of using the pressure sensors of small size and fast dynamical response on the sphere surface, are taken into account, than a measurement device with great capabilities of recording of fast varying velocity fluctuations in a small measurement space is possible to be accomplished.

In this work, basing on the potential fluid flow past a sphere and solution of the system of five nonlinear equations with four unknowns given from the pressure measurement on the sphere, is shown that it is possible to gain a simple theoretical characteristics of the five holes sphere probe. Such theoretical characteristics is then compared with the experimental one and the consistency of characteristics in the range of deflection angle $-10^{\circ} < \alpha < 10^{\circ}$ and pitch angle $-10^{\circ} < \beta < 10^{\circ}$ is stated. The discrepancies shown in the greater range of angles are related probably to the inaccuracies of manufacturing of the sphere and differences between potential and real flow past a sphere probe.

Up to now, there was almost no interest in the theoretical characteristics of the sphere probe because the disrepancies between the potential and real flow past a sphere are considered so great and without a chance for success.

2 Pressure distribution on the sphere surface as a base for velocity measurement

Theoretical foundation of the velocity measurements by means of the sphere probe may be the potential uniform flow of the ideal fluid past the sphere, on which it is possible to determine the velocity and pressure distribution on the sphere surface.

In other words, to measure the velocity by means of the sphere probe the knowledge of the pressure distribution on the sphere surface is needed, as the velocity measurement consists in the measurements of pressure in several points – mostly in five points – on the sphere surface.

The mostly used sphere probe is the five hole probe, i.e. five holes creating a isosceles cross intersecting at the straight angle on the sphere surface.

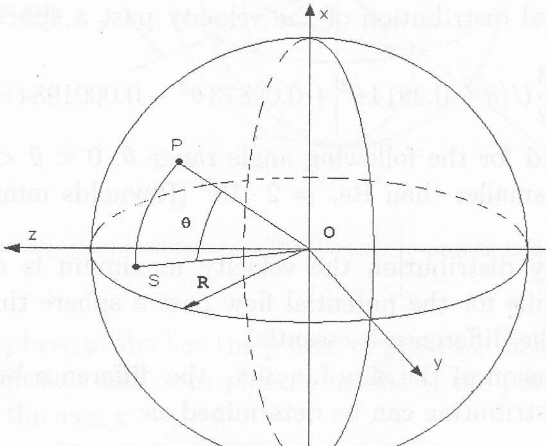

Figure 1. Axial symmetry of the flow past the sphere of radius R: S – a stagnation point, P – point where the flow is considered (velocity and pressure), θ – angle between point S and P.

The flow past a sphere belongs to the flows with axial symmetry, thus to describe it the radius R of the sphere, the momentary position of the stagnation point S and angle θ between stagnation point S and considered point P freely chosen on the sphere surface.

Examining the stationary uniform flow of fluid of density ρ and velocity U the velocity distribution past the sphere can be given by following well known formulae flow, e.g. [3]

$$u = \frac{\partial \Phi}{\partial R} = -U\cos\theta\left[1 - \left(\frac{r}{R}\right)^3\right] \qquad v = \frac{1}{R}\frac{\partial \Phi}{\partial R} = -U\sin\theta\left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^3\right], \tag{1}$$

where Φ is the velocity potential of flow.
At the sphere surface for $R = r$ it is then:

$$u = 0 \quad v = \frac{3}{2}U\sin\theta\,. \tag{2}$$

Applying Bernoulli equation we determine the interesting pressure distribution on the sphere surface following the formula (e.g.[3]):

$$p_P = p_O - \frac{1}{2}\rho U^2 \left(1 - \frac{9}{4}\sin^2\theta\right). \tag{3}$$

It is well known from the experiment [2, 11] that the real velocity distribution past the sphere differentiate from the distribution given by the solution for the ideal fluid flow.

The experimental distribution of the velocity past a sphere is given by the formula:

$$v = \frac{3}{2}U(\theta - 0.2914\,\theta^3 + 0.09873\,\theta^5 - 0.0001984\,\theta^7). \tag{4}$$

This formula is valid for the following angle range θ, $0 < \theta < 1.48$ rad and for Reynolds numbers smaller than $\mathrm{Re}_r = 2 \cdot 10^5$ (Reynolds number based on the sphere radius).

For this velocity distribution the velocity maximum is at the angle $\theta = 1.291$ rad $= 72°$, while for the potential flow past a sphere this maximum is at $\theta = \pi/2 = 90°$, so the difference is essential.

Using the expansion of the sine function, the difference between theoretical and real velocity distribution can be determined as:

$$\Delta v = \frac{3}{2}U(0.124\,\theta^3 - 0.0904\,\theta^5 + 0.2818\,\theta^7 + \ldots). \tag{5}$$

Dispite the difference we will continue our consideration treating the solution of potential flow as a certain pattern for the sphere probe.

3 Measuring probe in the flow

The problem of velocity measurement is considered in two systems of coordinates. Each of them is connected with the sphere probe which is used for velocity measurement and more correctly the pressure measurements on the probe surface. To solve the problem of velocity measurement by means of the sphere probe the transformation between these two systems should be found [12].

The first system is connected with the displacement of the velocity vector relative to the probe shown in Fig. 2. In this system two perpendicular to each other planes are derived and the axis of the probe z lies in both these planes. The plane zx is the co called declination plane and the plane zy is so called pitch plane. The projection of the velocity on these planes determines the declination angle α and pitch angle β appropriately. In this system calibration of the probe is usually carried out. In order to assure that the direction of the velocity vector coincides the probe axis the probe should be turn of corresponding angles (α, β).

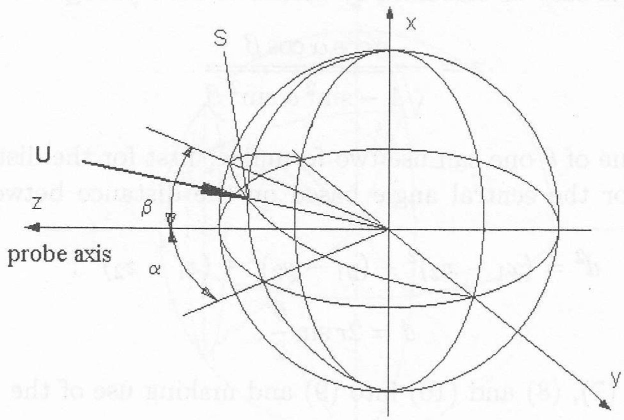

Figure 2. Coordinates system connected with the sphere probes and the velocity vector.

The five hole sphere probe has the points of pressure measurements symmetrically in the declination and pitch planes but one of the points so called central point is placed in the axis z.

In Eq. (3) for pressure distribution on the sphere the expression with the θ angle is included, so the formula tying the θ angle depending on the system containing the measuring point P and the stagnation pressure point S is to be found, Fig. 1.

To achieve that the dependence between the system connected with the probe and the cartesian system is tried to be determined and after the setting of the linear distance between these points on the sphere the angle θ between these points is calculated.

It can be shown that all the points of coordinate α or β are situated on the ellipses created by the projection of the great circle of the sphere on the plane xy and given by following formulae:

$$x^2 + \frac{y^2}{\sin^2\alpha} = r^2; \qquad y^2 + \frac{x^2}{\sin^2\beta} = r^2 . \tag{6}$$

These equations pose the simple equation system with unknowns x and y and the solution of this system is given by following equations:

$$x = \frac{r\cos\alpha\sin\beta}{\sqrt{1-\sin^2\alpha\sin^2\beta}}; \qquad y = \frac{r\cos\beta\sin\alpha}{\sqrt{1-\sin^2\alpha\sin^2\beta}} . \tag{7}$$

The coordinate z is easy to calculate by means of the Pythagoras theorem:

$$z = \frac{r \cos \alpha \cos \beta}{\sqrt{1 - \sin^2 \alpha \sin^2 \beta}} \,. \tag{8}$$

Searching the value of θ one can use two formulae: first for the distance between two points and for the central angle based on the distance between these two points:

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \,; \tag{9}$$

$$d = 2r \sin \frac{\theta}{2} \,. \tag{10}$$

Putting formulae (7), (8) and (10) into (9) and making use of the identity given below:

$$\cos \theta = 1 - 2 \sin^2 \frac{\theta}{2} \tag{11}$$

after simple, but rather long calculations the expression for the θ angle is found:

$$\cos \theta = \frac{\cos(\alpha_1 - \alpha_2)\cos(\beta_1 - \beta_2) - \sin\alpha_1 \sin\alpha_2 \sin\beta_1 \sin\beta_2}{\sqrt{1 - \sin^2 \alpha_1 \sin^2 \beta_1}\sqrt{1 - \sin^2 \alpha_2 \sin^2 \beta_2}} \,. \tag{12}$$

It can be seen that in the cartesian coordinate system the calculation of the θ angle by means of two points on the sphere surface is rather complicated, and its farther use even rather unfeasible.

That is why the use of the spherical coordinate system is much more natural and appropriate for the description of the velocity measurement by the sphere probe.

The second coordinate system is a spherical coordinate system ($r = 1$, φ, ψ) linked with the probe axis, Fig. 3. In this system are also defined the coordinates of the pressure measuring points on the sphere surface ($r = 1$, φ_i, ψ_i). Also in this coordinate system the temporary position of the stagnation point is to be searched. Thus relative to this temporary stagnation point the theoretical pressure field is found out.

Below Eqs. (13) show the commonly used spherical coordinates with a small departure not to mix the symbols, i.e. instead of the symbol θ the symbol ψ is used for colatitude coordinate leaving θ for the denomination of the coordinate of the temporary axialsymmetric flow past a sphere, i.e. a flow which is independent on the coordinate ϕ.

$$x = r \sin\psi \cos\varphi, \quad y = r \sin\psi \sin\varphi, \quad z = r \cos\psi. \tag{13}$$

In the experimental praxis the system of coordiante angles α, β is used, it is necessary to find the transformation between it and the spherical coordinates.

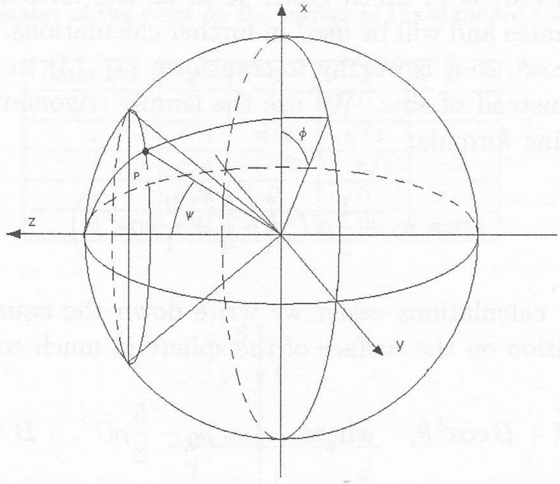

Figure 3. Spherical coordinate system (r, φ, ψ) for sphere probe, z – probe axis.

In this end in view the comparison of the cartesian coordinates (7) and (8) with the spherical one (13) is made. Simple calculations give in result two formulae joining the angles ϕ and ψ with the angles α and β appropriately:

$$\tan\varphi = \frac{\tan\alpha}{\tan\beta}; \qquad \tan^2\psi = \tan^2\alpha + \tan^2\beta \tag{14}$$

and inversely angles α and β with the angles ϕ and ψ:

$$\tan\alpha = \tan\psi\sin\varphi; \qquad \tan\beta = \tan\psi\cos\varphi. \tag{15}$$

Because in the formula for pressure distribution on the sphere surface the angle θ appears, the expression connecting angle θ with angles ϕ_1 and ψ_1 as well as angles ϕ_2 and ψ_2 of the two points on which the angle θ is spanned.

Again, putting Eqs. (6), (7) and (8) into (9) and again using the identity (11) after series of calculations with application of (15) the following formula is obtained:

$$\cos\theta = \sin\psi_1 \sin\psi_2 \cos(\varphi_1 - \varphi_2) + \cos\psi_1 \cos\psi_2 \,. \tag{16}$$

It is easy to note that for $\varphi_1 - \varphi_2 = 90^\circ$ the equation (16) has a form of Pythagoras theorem for the spherical triangle based on the angle θ (section) and two angles ψ_1 and ψ_2 [10]. The spherical triangle is based on the angle θ and its vertex lies in the point, where z axis intersects the surface of the sphere.

For $\varphi_1 = \varphi_2$ Eq. (16) is the cosine of the angle difference ψ_1 and ψ_2, and for $\varphi_1 - \varphi_2 = 180^\circ$ is the cosine of the angle total ψ_1 and ψ_2, as then the angle θ lies on the circumference of the great circle going through the z axis. Thus these relationships confirm Eq. (16).

This formula (16) is of much easier form as the formula (12) derivated in cartesian coordinates and will be used in further calculations. We received simple formula for the $\cos\theta$, so it is worthy to transform Eq. (3) to the form containing cosine function instead of sine. We use the simple trigonometric unity relation to get the following formula:

$$p = p_0 + \frac{1}{2}\rho U^2 \left(-\frac{5}{4} + \frac{9}{4}\cos^2\theta \right) . \tag{17}$$

To make further calculations easier we write down the equation describing the pressure distribution on the surface of the sphere in much compact way:

$$p = A + B\cos^2\theta, \quad \text{where} \quad A = p_0 - \frac{5}{8}\rho U^2, \quad B = \frac{9}{8}\rho U^2 . \tag{18}$$

In further calculations we will use a little different notation as before: subscript will denote the coordinates of the point on the surface of the sphere where the value of pressure is measured or calculated. Values without subscript refer to the stagnation point.
Thus the pressure in the i-th point is given by the formula:

$$p_i = A + B\cos^2\theta_i \tag{19}$$

where:

$$\cos\theta_i = \sin\psi\sin\psi_i\cos(\varphi - \varphi_i) + \cos\psi\cos\psi_i . \tag{20}$$

Thus pressure in i-th point of coordinates φ_i and ψ_i is given by following formula:

$$p_i = A + B\left(\sin\psi\sin\psi_i\cos(\varphi - \varphi_i) + \cos\psi\cos\psi_i\right)^2 . \tag{21}$$

In this equation there are four unknowns: A, B, ψ i φ. Thus, to measure the velocity vector in a flow it is sufficient to measure the pressure only in four points on the sphere surface, i.e. φ_i ψ_i, $i = 1...4$ [12]. In this way the system of four nonlinear equations with four unknowns is obtained, for which the condition for unique solution should be investigated [8]. To find them it is necessary to solve the system of four independent equations, thus to put to the Eq. (21) four sets of different coordinates of points, hence to carry out the pressure measurements in four different points on the sphere.

The problem of determination of the velocity by means of the pressure measurements in four points will be undertaken in the near future and some results are still given in [12].

Table 1. Coordinates of the point on the surface of the standard five hole probe.

Point No.	ψ	ϕ	α	β
1	0	–	0	0
2	$\pi/4$	$\pi/2$	$\pi/4$	0
3	$\pi/4$	$-\pi/2$	$-\pi/4$	0
4	$\pi/4$	0	0	$\pi/4$
5	$\pi/4$	π	0	$-\pi/4$

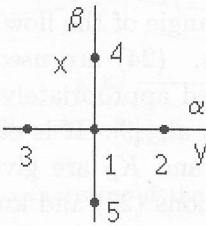

Figure 4. The arrangement of points of the standard five holes sphere probe.

4 Five hole sphere probe

The coordinates of the five points of the standard sphere probe are shown in Tab. 1 together with the configuration scheme given in the Fig. 4. This numbering of five probe points is most frequently used e.g. [1].
The equations for the five points of pressure measurements according to the general equations for the pressure on the sphere surface are given in Eq. (22).

$$p_i = A + B\left(\sin\psi\sin\psi_i\cos\left(\varphi-\varphi_i\right)+\cos\psi\cos\psi_i\right)^2 \qquad i = 1...5\,. \tag{22}$$

After setting the appropriate values of the angles from the Tab. 1. we get the following system of nonlinear equations:

$$\begin{aligned} p_1 &= A + B\cos^2\psi; \\ p_2 &= A + \tfrac{B}{2}\left(\sin\psi\sin\varphi+\cos\psi\right)^2; \\ p_3 &= A + \tfrac{B}{2}\left(-\sin\psi\sin\varphi+\cos\psi\right)^2; \\ p_4 &= A + \tfrac{B}{2}\left(\sin\psi\cos\varphi+\cos\psi\right)^2; \\ p_5 &= A + \tfrac{B}{2}\left(-\sin\psi\cos\varphi+\cos\psi\right)^2. \end{aligned} \tag{23}$$

This system can be easy solved by elimination of the constants A and B making use of the Eq. (15) and receiving following expression for the declination α and pitch β angles [12]:

$$2\tan 2\alpha = \frac{p_2 - p_3}{p_1 - \frac{p_2+p_3}{2}} \qquad 2\tan 2\beta = \frac{p_4 - p_5}{p_1 - \frac{p_4+p_5}{2}}\,. \tag{24}$$

In principle we use here the two subsystems i.e. Eqs. 1, 2 and 3 for calculating of the angle α and Eqs. 1, 4 and 5 for calculating the angle β because these points lie in threes on the same plane and the points 2 and 3 as well as the points 4 and 5 are appropriately symmetrical relative to the point 1. Japikse [5] quoting the work of Pien [6] mentioned only that placing three holes in one plane of the sphere allows to determine the angle of the flow in this plane.

The rights sides of the Eqs. (24) are used by the calibration of the five hole sphere probe and designated appropriately by coefficients K_α and K_β, e.g. Poensgen [7], as well Smolny et al. [9]. It is easy to see that for the ideal five hole probe the coefficients K_α and K_β are given by elementary trigonometric functions. Next, applying equations (23) and knowing the values of angles α i β it is possible to calculate the values of A and B by means of following formulae:

$$B = \frac{p_2 - p_3}{2\sin\psi\cos\psi\sin\varphi} = \frac{p_4 - p_5}{2\sin\psi\cos\psi\cos\varphi}, \tag{25}$$

$$A = p_1 - B\cos^2\psi = p_1 - \frac{p_2 - p_3}{2\tan\alpha} = p_1 - \frac{p_4 - p_5}{2\tan\beta} \tag{26}$$

and from these coefficients it is painless to find the dynamics and static pressure in the flow.

Summarizing the results of the Section 4 it can be said that the ideal standard five hole probe do not need utterly calibration to measure the velocity vector in the ideal fluid flow. It is worth to emphasize again that in the case of standard five hole probe one has to do with five nonlinear equations with four unknowns from which two subsystem of three equations can be obtained and used to determine the velocity angle in the plane, where the three holes lie.

In Fig. 5 the experimental calibration of the five hole sphere probe is shown (Smolny et a., 1994), and in Fig. 6 the theoretical characteristics of the five hole sphere probe is shown according to the formulae written above. Comparing these two calibration charactericstics some inaccuracies in carrying out of the probe are seen such as skewness of the holes plane relative to the basic plane and a lack of placement of the probe in axis z.

The unequality of the increase of the K_α and K_β coefficient depending, on the linear rise of angles, takes note in the theoretical characteristics plotted in Fig. 6 as a consequence of their dependence on the tangens of the duplicated pitch and deflection angle 2α or 2β appropriately. This inequality of the K_α and K_β is also seen in the experimental characteristics of the probe given in Fig. 5.

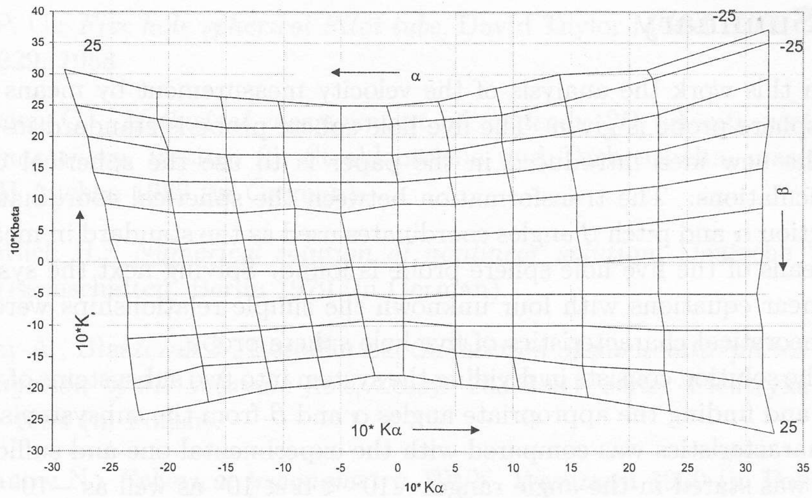

Figure 5. Experimental calibration characteristics of the five hole sphere probe according to Smolny et al. 1994.

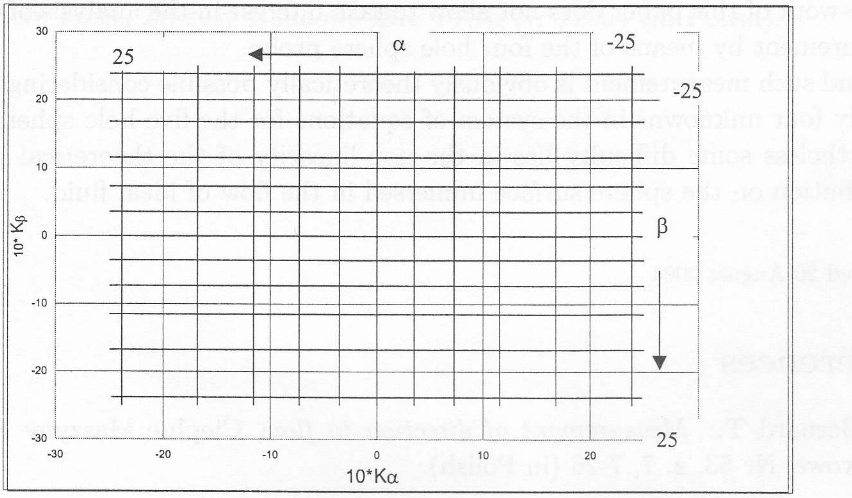

Figure 6. Theoretical calibration characteristics of the five hole probe.

Comparing both characteristics it can be said that they are sufficiently accurate in the following range of angles $-10^{\circ} < \alpha < 10^{\circ}$ as well as $-10^{\circ} < \beta < 10^{\circ}$. Furthermore, it is to be noticed that the probe from the Smolny et al. paper [9] is actually not of spherical shape, thus its shape differs significantly from the ideal sphere shape. Summarising, it seems that the manufacturing of the sphere probes can be considerably improved to bring nearer the experimental and theoretical characteristics.

5 Summary

In this work the analysis of the velocity measurement by means of the five hole sphere probe is given. The five hole sphere probe is standard in this field.

The new idea introduced in the paper is to use the spherical coordinates in calculations. The transformation between the spherical coordinates and the deflection α and pitch β angles coordinates used as the standard in measurements by means of the five hole sphere probe is found. Solving next the system of five nonlinear equations with four unknown the simple relationships were found for the theoretical characteristics of five hole sphere probe.

The solution consists in dividing the system into two subsystems of three equations and finding the appropriate angles α and β from the subsystems. Theoretical characteristics was compared with the experimental one and sufficient agreement was stated in the angle ranges $-10^{\circ} < \alpha < 10^{\circ}$ as well as $-10^{\circ} < \beta < 10^{\circ}$. The discrepancies in the broader angle range i.e. greater than $\pm 10^{\circ}$ can be put down to the inaccuracies of manufacturing of the sphere probe and the differences in the potential flow past the sphere and the real flow past the sphere probe. The frame-work of this paper does not allow to take interest in the analysis of velocity measurement by means of the four hole sphere probe.

And such measurement is obviously theoretically possible considering the fact of only four unknowns in the system of equations for the five hole sphere probe. Nevertheless some difficulty lies in the non-linearity of the theoretical pressure distribution on the sphere surface immersed in the flow of ideal fluid.

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