THE SZEWALSKI INSTITUTE OF FLUID-FLOW MACHINERY
POLISH ACADEMY OF SCIENCES

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

117

WYDAWNICTWO IMP PAN

GDAŃSK 2005

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ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 117, 2005, 59-79

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Gas-liquid and liquid-liquid stratified flows: exact analytical solutions and mechanistic models

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Abstract

An exact analytical solution has been obtained for fully developed laminar stratified flow in inclined pipes with a plane or curved interface. This solution is of practical significance mainly for studying liquid-liquid flows. However, it is also needed as a benchmark for testing the validity of numerical methods, and for testing closure relations for two-fluid models. Two-fluid models may yield poor predictions in inclined co-current and counter-current flows. The commonly used closure relations for the wall and interfacial shear stresses do not correctly represent the fine balance between the gravity body forces and viscous shear in inclined flows. The exact solution obtained for laminar flows is used to establish and validate new closure relations, which account for the interaction between the phases and are applicable also for turbulent stratified flows.

Keywords: Stratified flow; Curved interface; Waves; Interfacial shear; Wall shear; Two-fluid

Nomenclature

A	–	flow area, m^2
a	–	local flow area, m^2
C	–	wall shear stress parameter (Eq. 15)
c	–	coefficient in Eq. (18.2)
Ca	–	Capillary number (Eq. 20.3)
D	–	diameter, m

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Eo_D	–	Eotvős number
F	–	interaction correction factor
F_W, F_{iw}, F_{LW}	–	wave augmentation factors (Eq. 20)
Fr	–	Froude number
f	–	friction factor
g	–	gravity acceleration, m/s^2
$g_{11}, g_{12}, g_{21}, g_2$	–	geometrical correction factors (Eq. 17)
n	–	wall shear stress parameter (Eq. 15)
$\tilde{P}$	–	dimensionless driving force
p	–	pressure, Pa/m^2
q	–	flow rates ratio
Re	–	Reynolds number
S	–	perimeter, m
U	–	axial average velocity, m/s
u	–	local axial velocity, m/s
X^2	–	Martinelii parameter
Y	–	inclination parameter
x, y, z	–	Cartesian coordinates, m
β, β_I	–	inclination, radian
μ	–	viscosity, Pa-s
τ	–	shear stress, Pa/m^2
ρ	–	density, kg/m^3
γ	–	shape factor (Eq. 21)
ϕ^*	–	interface curvature
ϕ_0	–	lower phase dimensionless wetted perimeter
ϕ, ξ	–	bipolar coordinates
Φ	–	Fourier amplitude
ω	–	frequency
Θ, Λ	–	integral functions (Eq. 7)
ε	–	holdup
σ	–	surface tension, N/m

Subscripts

1	–	lower phase
$1s$	–	lower phase, superfical
2	–	upper phase
$2s$	–	upper phase, superficial
f	–	frictional
G	–	gas (Gs- superficial)
g	–	gravitational
h	–	homogenous solution
i	–	interfacial
L	–	liquid (Ls-superficial)
p	–	particular solution

Accents

$\sim$	–	dimensionless
$-$	–	average

1 Introduction

Stratified flow is considered a basic flow pattern in horizontal or slightly inclined gas-liquid and liquid-liquid systems of a finite density differential, since for some range of sufficiently low flow rates, the two phases tend to segregate. Counter-current and co-current stratified flows are encountered in the process industry in various mass transfer and direct contact heat transfer systems. Pipe lines design requires accurate prediction of the holdup and pressure gradient. Multiple holdups and pressure drops can be obtained for specified operation conditions in co-current and counter-current inclined flows, which are relevant in practical applications [1-2].

There are some significant differences between stratified flow in gas-liquid and liquid-liquid systems. In fact, compared to gas-liquid systems, liquid-liquid systems can be considered a more general case of two-fluid systems. Gas-liquid systems are characterized by low-density ratio and low viscosity ratio ($\tilde{\mu} = \mu_1/\mu_2$) between the light and heavy phases, and therefore represent a very particular extreme of two-fluid systems. In liquid-liquid systems the density difference between the phases is relatively low. However, the viscosity ratio encountered extends over a range of many orders of magnitude. The velocities of the two liquids are, in many cases, of a similar order of magnitude, and it is not evident which of the liquids dominates the interaction at the liquids' interface. Therefore, modeling of the interfacial shear stress becomes more ambiguous in liquid-liquid systems compared to gas-liquid systems. Moreover, as a result of the relatively low density difference, surface tension and wetting effects become important.

In smooth stratified flows, the Eotvős number, $\mathrm{Eo}_D = \Delta\rho g D^2/\sigma$ is an important parameter in determining the interface shape. Stratified flow with a plane interface is typical to gravity-dominated systems of large Eo_D. When surface tension forces become significant, the wetting liquid tends to climb over the tube wall resulting in a curved (convex or concave) interface (see Fig. 1). The interface shape depends on the Eo_D number, the fluids/wall contact angle and the holdup [3-4]. The possible stratified flow configurations extend from fully eccentric core of the upper phase to fully eccentric core of the lower phase. The configuration of a fully eccentric core was shown to be the basic pattern in surface tension dominated systems of $\mathrm{Eo}_D < 1$. In the stratified wavy regime, the interface curvature is dominated by the wave phenomena and the resulting secondary flows.

A configuration of a curved interface is associated with a variation of the location of the triple point (TP) and thus, with a variation in the contact area between the two fluids and the pipe wall even for a specified holdup. Depending on the physical system involved, these variations can have prominent effects on the pressure drop and transport phenomena.

Most studies on stratified flows in pipe have been carried out using mechanis-

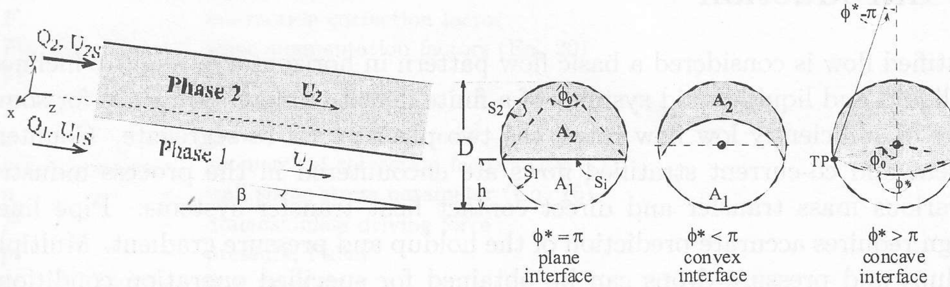

Figure 1. Schematic description of stratified flow configuration and parameters.

tic models, where various averaging techniques are used to achieve more practical models (e.g. [5-6]). But these models may only predict the integral flow characteristics, such as the axial pressure drop and the in situ holdup. The only way to obtain the velocity profiles, shear stress distribution and other local flow properties, is from a more rigorous solution of the Navier-Stokes equations.

The feasibility of obtaining exact solutions for stratified flows is restricted to laminar-laminar flows [7-8], which are frequently encountered in liquid-liquid systems. Exact solutions are also needed for validation of approximate mechanistic two-fluid models, or of numerical methods for solving stratified flows (e.g. [9-10]). Both mechanistic models and numerical methods are challenged by their capability to predict the flow characteristics in inclined pipes (e.g. [1-2]). Exact solutions for laminar stratified flows were recently used to derive new, theory-based, closure relations for the wall and interfacial shear stresses that account for the interaction between the phases [11]. These closure relations, which are valid for smooth stratified flows, were used as a platform for introducing necessary empirical corrections for the stratified wavy regime [12].

Most of the analytical solutions available in the literature for stratified laminar flows in pipe geometry are restricted to horizontal flows. The only exceptions are the solution of stratified inclined flow, which has been recently obtained, however, for the particular case of a plane interface. An analytical solution for the general case of inclined stratified flows with curved interface is presented here and is not yet available in the literature. The exact solution obtained for laminar flows is used to establish and validate new closure relations, which account for the interaction between the phases and are applicable also for turbulent stratified flows.

2 Analytical solutions

Given the location of the fluids interface, the 2-D velocity profiles in steady and fully developed axial laminar pipe flow (LPF) of stratified layers, $u_1(x,y), u_2(x,y)$ are derived from the Navier-Stokes (N-S) equations (in the z direction, see Fig. 1):

$$\mu_j \nabla^2 u_j = \frac{\partial p}{\partial z} - \rho_j g \sin\beta\,; \quad j = 1, 2\,. \tag{1}$$

The required boundary conditions follow from the no-slip condition at the pipe wall and continuity of the velocities and tangential shear stresses across the fluids' interface. Many researches have followed this approach, both analytically and numerically (e.g.[7-11]). Exact analytical solutions for Eq.(1) can be obtained in the case of constant interface curvature (represented by ϕ^* in Fig. 1). Then the bipolar coordinate system fits the flow geometry [7].

In the bipolar coordinate system (ϕ, ξ), the pipe perimeter and the interface between the fluids are iso-lines of coordinates ϕ, so that the upper section of the tube wall bounding the lighter phase is represented by ϕ_0, while the bottom of the tube, bounding the denser phase, is represented by $\phi_0 + \pi$. The interface coincides with the curve of $\phi = \phi^*$. Thus, the two-phase domains map into two infinite strips in the (ϕ, ξ) domain defined by: $-\infty < \xi < \infty$, $\phi^* > \phi > \phi_0$ for the upper phase,$-\infty < \xi < \infty$, $\phi_0 + \pi > \phi > \phi^*$ for the lower phase. The relations between ϕ_0, ϕ^* and the geometrical variables (e.g. flow areas, wetted perimeters) are given elsewhere [6].

A plane interface corresponds to a constant curvature arc, $\phi^* = \pi$. In this case the flow geometry can also be described by the thickness of the lower fluid layer, $h = 0.5(1 - \cos\phi_0)$. Analytical solutions for horizontal laminar stratified flow with a plane interface can be found in several publications in the sixties and seventies. A complete analytical solution for the more general case of a curved interface was obtained in Brauner *et al.* [7]. However, it is restricted to horizontal flows. More recently, the solution for the velocity and shear stresses profiles in the case of inclined flows with a plane interface has been also obtained [8,11]. Compared to horizontal flows, the solution for inclined systems is more complicated in case the fluids differ in their density, as the axial driving force in the two phases is not the same. The two non-dimensional N-S equations for 2-D flow of the two phases in the bipolar coordinate system are:

$$\left(\frac{\partial^2 \tilde{u}_1}{\partial \xi^2} + \frac{\partial^2 \tilde{u}_1}{\partial \phi^2}\right) = \frac{8}{\tilde{\mu}} \tilde{P}_1 \frac{\sin^2\phi_0}{(\cosh\xi - \cos\phi)^2}; \quad \left(\frac{\partial^2 \tilde{u}_2}{\partial \xi^2} + \frac{\partial^2 \tilde{u}_2}{\partial \phi^2}\right) = 8\tilde{P}_2 \frac{\sin^2\phi_0}{(\cosh\xi - \cos\phi)^2} \tag{2}$$

where $\tilde{u}_j = u_j/U_{2s}; (j = 1, 2)$ are the normalized velocity profiles with respect to the lighter phase superficial velocity, $U_{2s} = R^2\left(-dp_f/dz\right)_{2s}/(8\mu_2)$, while $\tilde{P}_1$

and $\tilde{P}_2$ represent the dimensionless driving force in the heavy and light phase respectively: $\tilde{P}_j = (dp/dz - \rho_j g \sin\beta)/(-dp_f/dz)_{2s}$. The pressure gradient used for normalizing the driving forces is the frictional pressure drop obtained in single-phase flow of the lighter phase, $(dp_f/dz)_{2s}$. Note that the inclination angle β is always taken as positive. In co-current flow, U_{1s}, U_{2s} are both positive in case of downward flow and negative for the case of upward flow. The boundary conditions required for solving equations (2) are: no-slip conditions at the tube walls and continuity of velocities and shear stresses across the free interface:

$$(\tilde{u}_1)_{\phi=\phi_0+\pi} = 0\,; \quad (\tilde{u}_2)_{\phi=\phi_0} = 0\,; \quad (\tilde{u}_1)_{\xi=\pm\infty} = 0\,; \quad (\tilde{u}_2)_{\xi=\pm\infty} = 0\,, \tag{3.1}$$

$$(\tilde{u}_1)_{\phi=\phi^*} = (\tilde{u}_2)_{\phi=\phi^*}\,; \quad \left(\mu_1 \frac{\partial \tilde{u}_1}{\partial \phi}\right)_{\phi=\phi^*} = \left(\mu_2 \frac{\partial \tilde{u}_2}{\partial \phi}\right)_{\phi=\phi^*}. \tag{3.2}$$

The general solution of Eqs. (2) is composed of the particular and homogeneous solutions, $\tilde{u}_{1p}, \tilde{u}_{2p}$ and $\tilde{u}_{1h}, \tilde{u}_{2h}$, respectively. In view of the non-homogeneous terms in Eqs. (2), the following functions, which satisfy b.c. (3.1), and (3.2), are chosen as the particular solutions:

$$\tilde{u}_{1p} = \frac{4}{\tilde{\mu}} \tilde{P}_1 \sin\phi_0 \frac{\sin(\phi-\phi_0)}{\cosh\xi - \cos\phi}\,; \quad \tilde{u}_{2p} = 4\tilde{P}_2 \sin\phi_0 \frac{\sin(\phi-\phi_0)}{\cosh\xi - \cos\phi}\,. \tag{4}$$

The homogeneous set of equations and corresponding boundary conditions (in view of the above particular solutions) are then given by:

$$\frac{\partial^2 \tilde{u}_{jh}}{\partial \xi^2} + \frac{\partial^2 \tilde{u}_{jh}}{\partial \phi^2} = 0\,; \, j = 1, 2\,; \tag{5.1}$$

$$(\tilde{u}_{1h})_{\phi=\phi_0+\pi} = 0\,; \quad (\tilde{u}_{2h})_{\phi=\phi_0} = 0\,; \quad (\tilde{u}_{1h})_{\xi=\pm\infty} = 0\,; \quad (\tilde{u}_{2h})_{\xi=\pm\infty} = 0\,; \tag{5.2}$$

$$(\tilde{u}_{1h})_{\phi=\phi^*} - (\tilde{u}_{2h})_{\phi=\phi^*} = 4 \sin\phi_0 \frac{\sin(\phi^*-\phi_0)}{\cosh\xi - \cos\phi^*}\left(\tilde{P}_2 - \frac{\tilde{P}_1}{\tilde{\mu}}\right)\,; \tag{5.3}$$

$$\left(\tilde{\mu}\frac{\partial \tilde{u}_{1h}}{\partial \phi} - \frac{\partial \tilde{u}_{2h}}{\partial \phi}\right)_{\phi=\phi^*} = 4(\tilde{P}_1 - \tilde{P}_2)\sin\phi_0 \frac{\cosh\xi \cos(\phi^*-\phi_0) - \cos\phi_0}{(\cosh\xi - \cos\phi^*)^2}\,. \tag{5.4}$$

The solution of the homogeneous set (5) can be obtained in the form the following Fourier integrals:

$$\begin{aligned} \tilde{u}_{1h} &= \int_0^\infty \Phi_1(\omega) \sinh[\omega(\phi - \pi - \phi_0)] \cos(\omega\xi) d\omega \\ \tilde{u}_{2h} &= \int_0^\infty \Phi_2(\omega) \sinh[\omega(\phi - \phi_0)] \cos(\omega\xi) d\omega\,. \end{aligned} \tag{6}$$

The r.h.s of b.c. (5.3) and (5.4) can be represented as Fourier Integrals. The solution obtained for Φ_1 and Φ_2 (Fourier transforms of $\tilde{u}_{1h}$, $\tilde{u}_{2h}$) is given by:

$$\Phi_2 = \frac{1}{\omega\Theta}\left\{\Lambda - \frac{\tilde{\mu}\omega 2\sin(\phi^* - \phi_0)\sinh[\omega(\phi^* - \pi)]}{\sin\phi^* \sinh(\omega\pi)\sinh[\omega(\phi^* - \phi_0)]}\cosh[\omega(\phi^* - \pi - \phi_0)]\right\}, \tag{7.1}$$

$$\Phi_1 = \frac{8\sin\phi_0(\tilde{P}_1/\tilde{\mu} - \tilde{P}_2)\sin(\phi^* - \phi_0) + \Phi_2 \sin\phi^* \sinh(\omega\pi)}{\sin\phi^* \sinh(\omega\pi)\sinh[\omega(\phi^* - \pi - \phi_0)]}\sinh[\omega(\phi^* - \pi)] \tag{7.2}$$

where:

$$\Lambda = \left\{\left[8\sin\phi_0(\tilde{P}_2 - \tilde{P}_1)(-\cos\phi_0 + \cos(\phi^* - \phi_0))\right]\{\omega\cosh[\omega(\phi^* - \pi)]\sin\phi^* - \cos\phi^* \sinh[\omega(\phi^* - \pi)]\} - 8\sin\phi_0\cos(\phi^* - \phi_0)(\tilde{P}_2 - \tilde{P}_1)\right\}\frac{1}{\sin^3\phi^* \sinh(\omega\pi\xi)}, \tag{7.3}$$

$$\Theta = \cosh[\omega(\phi^* - \phi_0)]\cosh[\omega(\phi^* - \pi - \phi_0)]\{\tilde{\mu}\tanh[\omega(\phi^* - \pi - \phi_0)] - \tanh[\omega(\phi^* - \phi_0)]\}/\sin[\omega(\phi^* - \phi_0)]. \tag{7.4}$$

Equations (4, 6 and 7) can be used to obtain the velocity profiles $\tilde{u}_{1,2} = \tilde{u}_{1,2h} + \tilde{u}_{1,2p}$. Integration of the velocity profiles over the corresponding flow cross-sectional area yields the flow rates of the two fluids:

$$\frac{1}{\pi}\int_{\phi^*}^{\phi_0+\pi}\int_{-\infty}^{\infty}\tilde{u}_1 J(\xi,\phi)d\xi d\phi = q = \frac{U_{1S}}{U_{2S}}; \quad \frac{1}{\pi}\int_{\phi_0}^{\phi^*}\int_{-\infty}^{\infty}\tilde{u}_2 J(\xi,\phi)d\xi d\phi = 1 \tag{8}$$

where $J(\xi,\phi) = -\sin^2\phi_0/(\cosh\xi - \cos\phi)^2$ is the transforming Jacobian from bipolar to Cartesian coordinates. For specified ϕ^*, Eqs.(8) provide the phase flow rates corresponding to a given pressure drop and holdup (or ϕ_0). The latter is determined by the following geometrical relation:

$$\varepsilon = \frac{A_1}{A} = \frac{1}{\pi}\left\{\phi_0 - \frac{1}{2}\sin(2\phi_0) - \frac{\sin^2\phi_0}{\sin^2\phi^*}\left[\phi^* - \pi - \frac{1}{2}\sin(2\phi^*)\right]\right\}. \tag{9}$$

From the practical point of view, the interest is in obtaining a solution for the local flow characteristics (velocity and shear stresses profiles) and integral flow characteristics (pressure drop and holdup) corresponding to specified flow rates of the two fluids. This is considered to be the 'inverse problem'. The solution of the inverse problem is more complicated, since the location of the interface is a priori unknown and must be determined in an iterative manner. It is much more complicated in inclined systems, as both $\tilde{P}_1$ and $\tilde{P}_2$ relate to the (unknown) frictional pressure drop and holdup. The difference between the two, however, is equal to the inclination parameter, Y:

$$\tilde{P}_1 - \tilde{P}_2 = \frac{(\rho_1 - \rho_2)g\sin\beta}{(-dp_f/dz)_{2s}} = Y. \tag{10}$$

Equations (8) can be manipulated to obtain explicit expressions for the unknown $\tilde{P}_1, \tilde{P}_2$. When these are used in Eq. (10), an implicit equation for the unknown ϕ_0 is obtained, once ϕ^* is specified. This is a complicated equation, which includes triple integrals (over ω and the flow area(ξ, ϕ)), and requires numerical computations. The so-obtained solutions for the holdup (determined by ϕ_0, ϕ^*) and the corresponding dimensionless pressure gradient $\Pi_f = (dp_f/dz)/(dp_f/dz)_{2s}$, are dependent on three dimensionless parameters $\tilde{\mu}, q$ and Y. Note that in laminar flows, the Martinelli parameter, which represents the ratio of the superficial frictional pressure drop in the two phases, is $X^2 = (dp_f/dz)_{1s}/(dp_f/dz)_{2s} = \tilde{\mu}q$, and can replace either q or $\tilde{\mu}$. As the characteristic interface curvature is dependent on the system Eo_D and the fluid/wall contact angle [3-4], the complete solution for the general case of laminar stratified flows with smooth interface is dependent on five dimensionless parameters: $q, \tilde{\mu}, Y, \mathrm{Eo}_D, \alpha$.

3 Two-fluid models

The exact solutions of the Navier-Stokes equations are restricted to laminar flows and involve extensive computations. In many practical situations, one of the phases (or both) is turbulent. Therefore, averaged one-dimensional two-fluid models are widely used for the prediction of the pressure drop and in-situ holdup (e.g. [5-6]). In the two-fluid model, the momentum and mass conservation equations are expressed in terms of the average phases' velocities. For steady and fully developed stratified flow, the momentum conservation equations are (see[1] for details):

$$-A_{1,2}\frac{dp}{dz} + \tau_{1,2}S_{1,2} \pm \tau_i S_i + \rho_{1,2}A_{1,2}g\sin\beta = 0 \text{ (upper sign for the lighter phase)}. \tag{11}$$

Eliminating the pressure drop yields:

$$\tau_1\frac{S_1}{A_1} - \tau_2\frac{S_2}{A_2} - \tau_i S_i\left(\frac{1}{A_1} + \frac{1}{A_2}\right) + (\rho_1 - \rho_2)\, g\sin\beta = 0\,. \tag{12}$$

This equation can be solved for the holdup provided closure relations are available for the wall and interfacial shear stresses, as well as for the interface shape (i.e. interface curvature). The commonly used approach is to assume a plane interface and to model the shear stresses based on single phase models/correlations for the friction factors in laminar or turbulent flows. The interfacial shear is taken as proportional to the velocity difference, and the interfacial friction factor is modeled based on the wall friction factor of the faster layer.

A crucial issue in applying the two-fluid model to gas-liquid systems is frequently the modeling of a correction factor, which accounts for the augmentation of the interfacial shear due to the wavy liquid interface. However, in the general case of liquid-liquid systems, inclined flows and counter-current flows, where velocities of the two phases are of comparable values, the main issue concerns the decision as to which of the fluids actually dominates the interfacial interaction, hence f_i. Also, commonly used wall shear expressions are problematic for inclined flows, since they are incapable of representing reversed wall shear in cases of backflow of one of the phases in the near wall region (e.g. downward flow of the liquid phase near the wall in co-current upward gas-liquid flows). The poor predictions obtained by the commonly used closure relations were demonstrated and discussed in Ullmann *et al.* [1-2]. These are a result of using single-phase based closure relations for the wall and interfacial friction factors and ignoring the interaction between the phases flowing in the same pipe.

3.1 Smooth-stratified flow

New theory-based closure relations were recently formulated, which account for the interaction between the phases (MTF model, [11-12]). These were obtained based on the exact analytical solution for laminar flow between two infinite plates, which suggests the following structure for the wall and interfacial shear stresses:

$$\tau_1 = -\frac{1}{2}\rho_1 f_1 |U_1| U_1 \, |F_1|^{n_1} \, sign(F_1)\,; \quad U_1 = \frac{U_{1s}}{\varepsilon}\,; \tag{13}$$

$$\tau_2 = -\frac{1}{2}\rho_2 f_2 |U_2| U_2 \, |F_2|^{n_2} \, sign(F_2)\,; \quad U_2 = \frac{U_{2s}}{1-\varepsilon}\,. \tag{14}$$

The friction factors, f_1 and f_2, are based on the Reynolds number of the corresponding layer, each flowing as a single phase in its own channel. In case of hydrodynamic-smooth wall surface, the Blasius-type power law expressions for the wall shear stresses can be used (rough-wall expressions are provided in [12]):

$$f_j = \frac{C_j}{(Re_j)^{n_j}}\,; \quad Re_j = \frac{\rho_j\,|U_j|\,D_j}{\mu_j}\,; \quad D_j = \frac{4A_j}{(S_j + S_i)}\,; \quad j = 1, 2\,. \tag{15}$$

Given the flow regime in the two phases, the constants, $C_{1,2}$ and $n_{1,2}$ are prescribed (e.g. laminar: $C = 16$, $n = 1$, turbulent: $C = 0.046$, $n = 0.2$), and the single-phase-based friction factors f_1, f_2 can be calculated. The factors F_1 and F_2 represent theory-based corrections of the single-phase based expressions for the wall shear stresses due the interaction between the two fluids flowing in the same channel (the conventional TF closure relations assume F_1, $F_2 \equiv 1$). These are given by:

$$F_1 = \frac{1+\frac{U_2}{U_1}\left[g_{11}X^2\left(\frac{1-\varepsilon}{\varepsilon}\right)^2-(2\varepsilon)^{1-n_2}\,g_{12}\right]}{1+\frac{U_2}{U_1}X^2\left(\frac{1-\varepsilon}{\varepsilon}\right)^2};$$
$$F_2 = \frac{1+\frac{U_1}{U_2}\left[g_{22}\frac{1}{X^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2-(2\,(1-\varepsilon))^{1-n_1}\,g_{21}\right]}{1+\frac{U_1}{U_2}\frac{1}{X^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2}. \tag{16}$$

The g_{ij} in Eqs.(16) are functions of the dimensionless wetted perimeters $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_i$, ($\tilde{S}=S/D$) and for the pipe geometry are given by:

$$g_{11}=\frac{\tilde{S}_1}{\tilde{S}_1+\tilde{S}_i}\ ;\ g_{22}=\frac{\tilde{S}_2}{\tilde{S}_2+\tilde{S}_i};\ g_{12}=\frac{4}{\pi+2}\frac{\tilde{S}_2}{\tilde{S}_2+\tilde{S}_1}\ ;\quad g_{21}=\frac{4}{\pi+2}\frac{\tilde{S}_1}{\tilde{S}_2+\tilde{S}_1}. \tag{17}$$

For the interfacial shear, the generalized MTF closure relations are:

$$\tau_i=\begin{cases}-\frac{1}{2}\rho_1 f_1|U_1|(c_{i2}U_2-U_1)\,|F_{i1}|^{n_1}\ ; & |F_{i1}|^{n_1}>|F_{i2}|^{n_2}\\ -\frac{1}{2}\rho_2 f_2|U_2|(U_2-c_{i1}U_1)\,|F_{i2}|^{n_2}\ ; & |F_{i1}|^{n_1}\le|F_{i2}|^{n_2}\end{cases} \tag{18.1}$$

with:

$$F_{i1}=\frac{1}{1+\frac{U_2}{U_1}X^2\left(\frac{1-\varepsilon}{\varepsilon}\right)^2};\ F_{i2}=\frac{1}{1+\frac{U_1}{U_2}\frac{1}{X^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2};\ c_{i1}=\left|\frac{2q}{1+q}\right|^{1-n_2};\ c_{i2}=\left|\frac{2}{1+q}\right|^{1-n_1}. \tag{18.2}$$

The first form in Eq. (18.1) corresponds to the case where the interfacial shear is dominated by the flow of the heavy phase, whereas the second form corresponds to dominance by the light phase. F_{i1}, F_{i2} represent correction factors due to the interaction between the flows in the two layers. The F_i interaction factors, Eq.(18.2), and the criterion used in Eq. (18.1) for switching between the two alternative expressions for τ_i, suggest a matching between the solutions obtained with the two expressions for the interfacial shear. The MTF closure relations for the interfacial shear thus alleviate the discontinuity and other ill-effects encountered in the predictions of TF models (see Ullmann et. al [1-2]).

3.2 Wavy stratified flow

The MTF closure relations, which are valid for smooth-stratified flow in horizontal or inclined pipes, were used in Ullmann et al [12] as a platform for introducing necessary empirical corrections required in the stratified wavy flow regime.

In wavy stratified flows, the interface curvature is dominated by the secondary flows, which develop in both phases. Based on the experimental data reported in the literature for the wetted wall perimeter and holdup in wavy gas-liquid flow, the following correlation has been recently obtained for ϕ^*[12]:

$$\phi^* = \pi + 2\, tg^{-1} Z\,; \quad Z = 57.2 \left| \frac{\tau_G}{(\rho_L - \rho_G)\, g \cos \beta D} \right|^{0.5} \left| \frac{U_L}{U_G} \right|^{0.5} \tilde{S}_i . \qquad (19)$$

The values predicted for ϕ^* by this correlation are compared with experimental data in Fig. 2. The validity of Eq. (19) is demonstrated by the favorable agreement with an additional independent data set obtained in large diameter tube of $D = 5''$ [16].

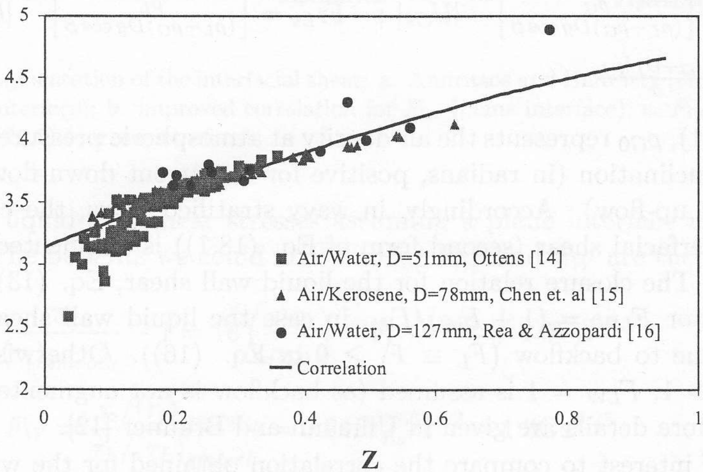

Figure 2. Experimental correlation for the interface curvature in wavy stratified gas-liquid flow, Eq. (19): comparison with data.

The wall shear stress in the gas phase, $\tau_G \equiv \tau_2$, is generally well-predicted by the closure relation used in the stratified smooth regime (e.g. [13]). However, the interfacial waves are known to have a pronounced effect on the interfacial shear and on the liquid wall shear stresses. Therefore, experimental data of simultaneous measurements of the holdup and pressure drop, combined with Eq. (19) for the interface curvature, can be used to deduce from the two momentum equations (Eqs. 11) the corresponding values of the 'experimental' liquid wall shear stress,$\tau_1 \equiv (\tau_L)_{\text{exp}}$, and interfacial shear, $(\tau_i)_{\text{exp}}$. To this aim, the database described in Ullmann and Brauner [12] was used. The data correspond to the region of large amplitude 2D and 3D waves. The so-obtained $(\tau_L)_{\text{exp}}$ and $(\tau_i)_{\text{exp}}$ were compared with the corresponding values predicted by the closure relations of smooth stratified flow (Eq. (13) and (18)). The ratios between the 'experimental' values and those predicted for smooth interface were used to derive correlations for the empirical correction factors on the friction factors that account for the wave augmentation effects. Based on data bank of simultaneous measurement of the holdup and pressure drop, the wave effects on the interfacial and liquid wall shear stresses have been correlated by (see [12]):

$$F_{iw} = \frac{\tau_i}{(\tau_i)_{smooth}} - 1 = 10.25 \left(\frac{\rho_G}{\rho_{G0}}\right)^{0.33} \mathrm{Fr}_{Gs}^{0.65} \varepsilon^{0.33} + 0.1\mathrm{Ca}^{1.2} - 15\beta_I\,; \quad (20.1)$$

$$F_W = \frac{\tau_i/(\tau_i)_{smooth}}{\tau_L/(\tau_L)_{smooth}} = 5.15\mathrm{Fr}_{Ls}^{0.75} \varepsilon^{-0.6} + 0.25\mathrm{Ca}^{0.7}\,; \quad (20.2)$$

$$\mathrm{Fr}_{Gs} = \left[\frac{\rho_G}{(\rho_L - \rho_G) Dg\cos\beta}\right]^{0.5} |U_{Gs}|\,; \quad \mathrm{Fr}_{Ls} = \left[\frac{\rho_L}{(\rho_L - \rho_G) Dg\cos\beta}\right]^{0.5} |U_{Ls}|\,;$$
$$\mathrm{Ca} = \frac{|U_G - U_L|\mu_L}{\sigma}\,. \quad (20.3)$$

In Eq. (20.1), ρ_{G0} represents the air density at atmospheric pressure (=1.2 kg/m^3), β_I is the inclination (in radians, positive for co-current down-flow, negative for concurrent up-flow). Accordingly, in wavy stratified flows, the closure relation for the interfacial shear (second form of Eq. (18.1)) is augmented by the factor $(1 + F_{iw})$. The closure relation for the liquid wall shear, Eq. (13) is augmented by the factor $F_{LW} = (1 + F_{iw})/F_W$, in case the liquid wall shear stress is not reversed due to backflow ($F_L \equiv F_1 \geq 0$ in Eq. (16)). Otherwise, for $F_L < 0$ and $F_{LW} > 1$, $F_{LW} = 1$ is assumed (as backflow is not augmented due to wave effects). More details are given in Ullmann and Brauner [12].

It is of interest to compare the correlation obtained for the wave augmentation of the interfacial shear with another, widely used correlation obtained by Andritsos and Hanratty [17]. This correlation was developed assuming a plane (rather than curved) interface. The data of $(F_{iw})_{\exp} = (\tau_i)_{\exp}/(\tau_i)_{smooth} - 1$ is compared in Fig. 3a with that correlation:

$$F_{iw} = 15\left(\frac{h}{D}\right)^{0.5}\left(\frac{U_{Gs}}{U_{Gs,t}} - 1\right)\,; \quad U_{Gs,t} = 5\left(\frac{1.2}{\rho_G}\right)\ [\mathrm{m/s}]\,. \quad (21)$$

In this correlation $U_{Gs,t}$ is the critical gas superficial velocity for transition from stratified-smooth to stratified-wavy (SS/SW) as suggested by Andritsos and Hanratty [17]. Following their assumption of a plane interface, one obtains a large scatter in $(F_{iw})_{\exp}$ compared to their original correlation. Moreover, negative values of F_{iw} may result from Eq.(21) due to miss-prediction of the critical gas-velocity for the SS/SW transition, which should be discarded.

A question may arise regarding the necessity of accounting separately for the effect of the waves on the interface curvature. To this aim the data bank in [12] was used to obtain correlations for the wave effects on the augmentation of the

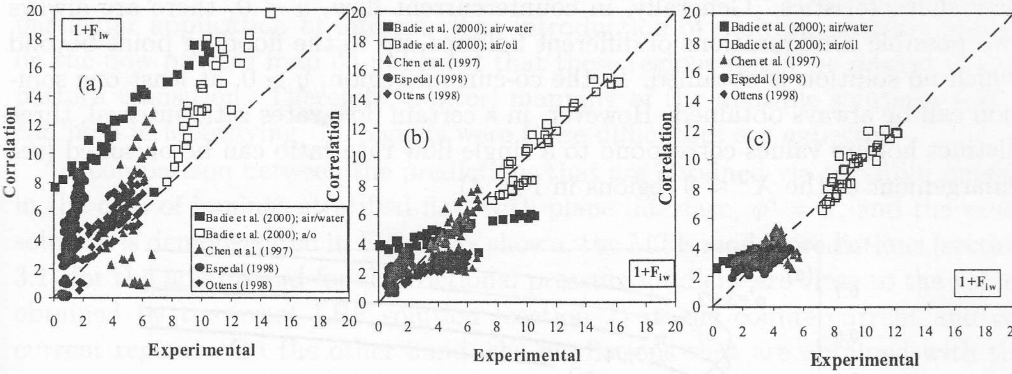

Figure 3. Wave augmentation of the interfacial shear: a. Andritsos and Hanratty [17] correlation (plane interface); b. improved correlation for F_{iw} (plane interface); c. F_{iw} calculated by Eq. (20.1) (curved interface).

interfacial and liquid-wall shear stresses assuming a plane interface also in the wavy regime. The best fits we could arrive at for F_{iW} and F_W are the following:

$$F_{iw} = \frac{\tau_i}{(\tau_i)_{smooth}} - 1 = 18\left(\frac{\rho_G}{\rho_{G0}}\right)^{0.7} \mathrm{Fr}_{Gs}^{2.5}\varepsilon^{0.33} + 1.0\mathrm{Ca}^{0.5}\,; \qquad (22.1)$$

$$F_W = \frac{\tau_i/(\tau_i)_{smooth}}{\tau_L/(\tau_L)_{smooth}} = 0.24\mathrm{Fr}_{Ls}^{1.66}\varepsilon^{-2} + 1.3\mathrm{Ca}^{0.25}\,. \qquad (22.2)$$

A comparison of the data with the F_{iw}values predicted by Eq. (22.1) is shown in Fig. 3b, while those predicted by Eq. (20.1) are shown in Fig. 3c. The latter was derived assuming a curved interface, where the curvature was calculated by Eq. (19). The scatter in Fig. 3b is evidently larger, suggesting that accounting for the wave effect on the curving of the interface allows a better representation of the wave effect on the interfacial shear for a wide variety of gas-liquid systems (see also Fig. 8 below).

4 Results and discussion

The pipe inclination is connected to a number of interesting two-phase flow phenomena. These include the possibility of obtaining multiple solutions of different holdups for specified operational conditions and partial backflow of either of the fluids due to the gravity force acting opposite to the main flow direction. Results obtained via the exact solution for laminar pipe flow (LPF) are demonstrated in Fig. 4 for different interfacial curvatures. The figure shows that the interface curvature has a pronounced effect on the holdup and thus on the stratified flow characteristics. Generally, in countercurrent flow, $q < 0$, there are always two possible configurations of different holdups, up to the flooding point beyond which no solution is obtained. In the co-current region, $q > 0$, at least one solution can be always obtained. However, in a certain flow-rates ratio interval, three distinct holdup values correspond to a single flow rate ratio can be obtained (see enlargement of the $X^2 \approx 0$ regions in Fig. 4).

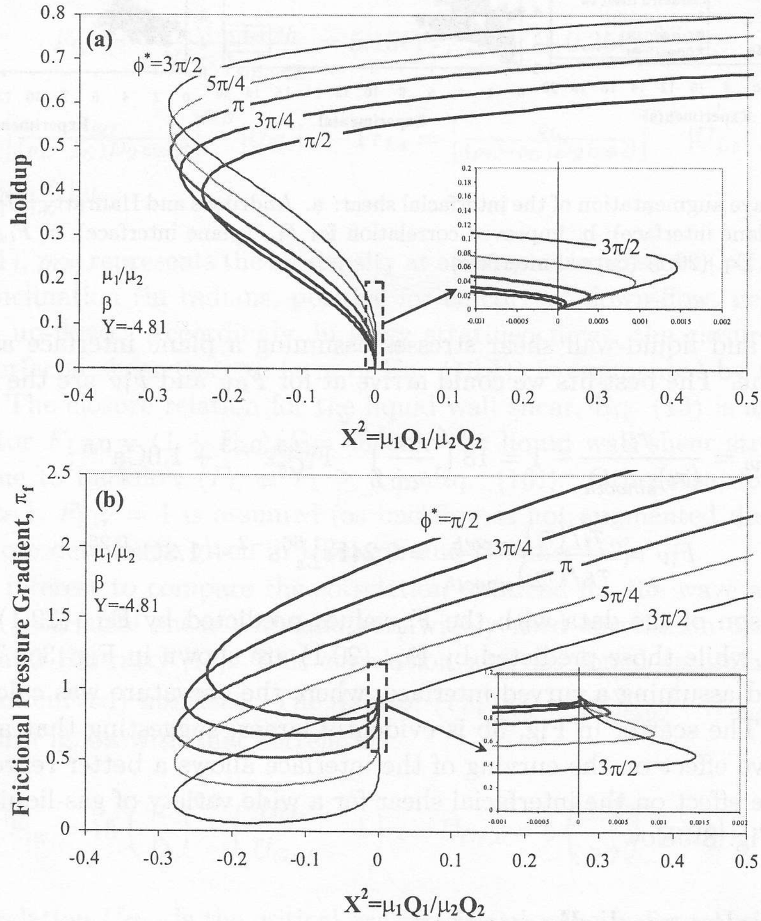

Figure 4. Effects of the flow rates and interface curvature on the holdup in laminar flow (a) and on the frictional pressure gradient (b) – exact solution.

Obviously, multi-solutions for holdup are associated with multi-solutions for the pressure drop. The possibility of obtaining multi-holdups was verified experimentally for the case of countercurrent liquid-liquid flows [1] and for a case of co-current liquid-liquid flows [2]. It is worth emphasizing, that in the case of multiple solutions, computational software usually yields only one of the solutions for specified operational conditions, which is not necessarily the relevant one for the particular application of interest. The introduction of multiple solution regions on the flow pattern map [2] indicated that these regions might be related to flow pattern transition. Therefore, a-priori mapping of the multiple solutions region can help in identifying the regions were these difficulties are expected.

A comparison between the predictions that are obtained via two-fluid models in the case of laminar stratified flow with plane interface, $\phi^* = \pi$, and the exact solution is demonstrated in Fig. 5. As shown, the MTF model predictions (section 3.1) for the holdup and for the frictional pressure gradient are close to the values obtained by the exact LPF solution (section 2) in the countercurrent and co-current regions. On the other hand, the predictions that are obtained with the commonly used two-fluid (TF) closure relations, which ignore the interactions between the two layers (assuming $F_1 = 1$ and $F_2 = 1$ in Eqs. (13) and (14), respectively) are poor and problematic. They remain poor independently on whether the interfacial shear is modeled based on the flow of the heavier phase $f_i = f_1$, or based on the flow of the light phase, $f_i = f_2$.

The inclusion of the $F-$interaction factors become more significant in inclined flows, where the gravity body force is important. For example, in Fig. 6, which corresponds to upward inclined flow, it is demonstrated that the F_1-interaction factor can attain also negative values. The velocity profile (shown on the r.h.s of Fig. 6), which is obtained by the LPF exact solution, indicates that negative F_1 is corresponding to conditions of backflow of the heavy phase in the near-wall region. Hence, the MTF closure relation for the wall shear stress correctly predicts a reversed wall shear stress compared to the direction that would have been predicted by the average fluid velocity, U_1.

The MTF closure relations can be used for the calculations of undeveloped conditions and slow transients in stratified flows via 1-D two-fluid models. The two-fluid transient momentum equations are then written in terms of the local and instantaneous holdup (flow area, a_1, a_2) and average velocity in each of the phases, $\bar{u}_1, \bar{u}_2$ (e.g. [18]):

$$\begin{aligned}\frac{\partial}{\partial t}(\rho_1 a_1 \bar{u}_1)+\frac{\partial}{\partial z}(\rho_1 a_1 \gamma_1 \bar{u}_1^2) &= -\tau_1 S_1+\tau_i S_i+\rho_1 a_1 g \sin\beta-\frac{\partial}{\partial z}(a_1 p_1)+p_{i1}\frac{\partial a_1}{\partial z}\,;\\ \frac{\partial}{\partial t}(\rho_2 a_2 \bar{u}_2)+\frac{\partial}{\partial z}(\rho_2 a_2 \gamma_2 \bar{u}_2^2) &= -\tau_2 S_2-\tau_i S_i+\rho_2 a_2 g \sin\beta-\frac{\partial}{\partial z}(a_2 p_2)+p_{i2}\frac{\partial a_2}{\partial z}\,.\end{aligned} \tag{23}$$

To represent the wall shear stresses (τ_1, τ_2) and the interfacial shear stress (τ_i), the MTF closure relations can be used in the conventional manner, namely, using the same closures expressed in terms of local/instantaneous values of the holdup and average phases velocities. However, the application of Eqs. (23) to transient/undeveloped flows requires also correction factors on the inertia terms, which are denoted as the velocity profile 'shape factors'. The definition of these

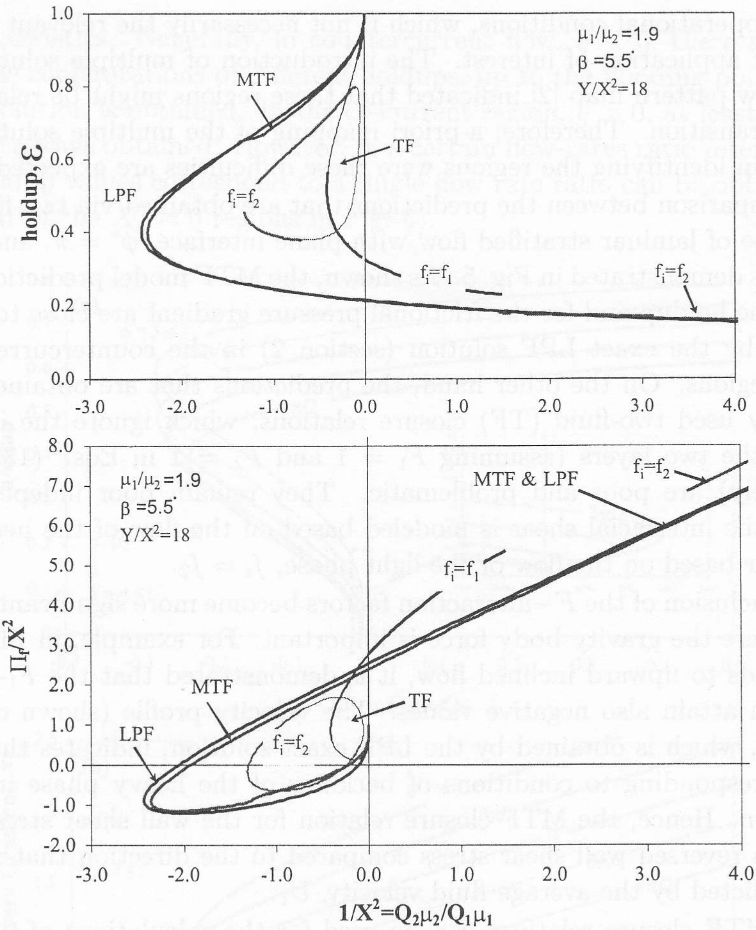

Figure 5. Comparison of the LPF solution with the predictions of TF and MTF models for the holdup and the dimensionless frictional pressure gradient.

correction factors (that evolves from the averaging procedure) is given by:

$$\gamma_1 = \frac{\int_{a_1} u_1^2 da_1}{\bar{u}_1^2 a_1}\,; \quad \gamma_2 = \frac{\int_{a_2} u_2^2 da_2}{\bar{u}_2^2 a_2}\,. \tag{24}$$

As the velocity profiles are not resolved in the framework of 1-D model, plug flow is usually assumed in both layers ($u_{1,2} = \bar{u}_{1,2}$), whereby $\gamma_{1,2} = 1$. The velocity profiles, which are obtained by the exact solution, can be used to calculate the shape factors and to test the validity of this assumption. The results of the heavy phase shape factor corresponding to the conditions of Fig. 4 are shown in Fig. 7. Obviously, the shape factor can attain very large values. The large values correspond to conditions of backflow, where part of the fluid is flowing opposite

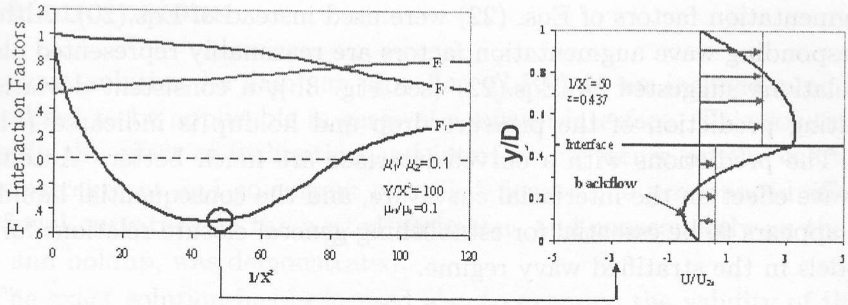

Figure 6. Variation of the MTF F-interaction factors with the light phase flow rate in the case of co-current up-flow and the LPF solution for the velocity profile at the pipe center line for $1/X^2 = 50$.

to the flow direction. Note that the two non-zero holdups that are obtained at $q_1 = 0$ (upper and middle solutions for the holdup in Fig. 4a) correspond to a complete circulation of the heavy phase in the pipe, whereby $\bar{U}_1 = U_1 = 0$ and $\gamma_1 \to \infty$.

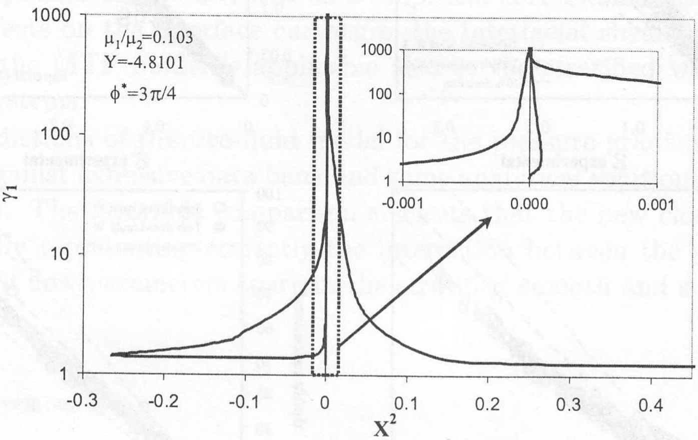

Figure 7. Effect of the flow rates on the heavier phase shape factor in an inclined tube.

The validity of the MTF closure relations for modelling turbulent stratified flows in the smooth and wavy regime was shown in Ullmann and Brauner [12]. To demonstrate the performance of MTF model in the large wave regime (section 3.2), the predicted pressure drop and holdup are compared with Espedal [19] data for inclined upward and downward flows (on the r.h.s of Fig. 8). This figure also shows the implications of assuming a plane interface (rather than curved) in the wavy regime on the MTF model predictions (l.h.s of Fig. 8). In this case, the wave augmentation factors of Eqs. (22) were used instead of Eqs.(20). Although the corresponding wave augmentation factors are reasonably represented also by the correlations suggested by Eqs.(22) (see Fig. 3b), a consistent deviation of the resulting prediction of the pressure drop and holdup is indicated (l.h.s of Fig. 8). The predictions with a curved interface are much better. Accounting for the wave effect on the interfacial curvature, and the consequential liquid wall wetting, appears to be essential for establishing general closure relations for two-fluid models in the stratified wavy regime.

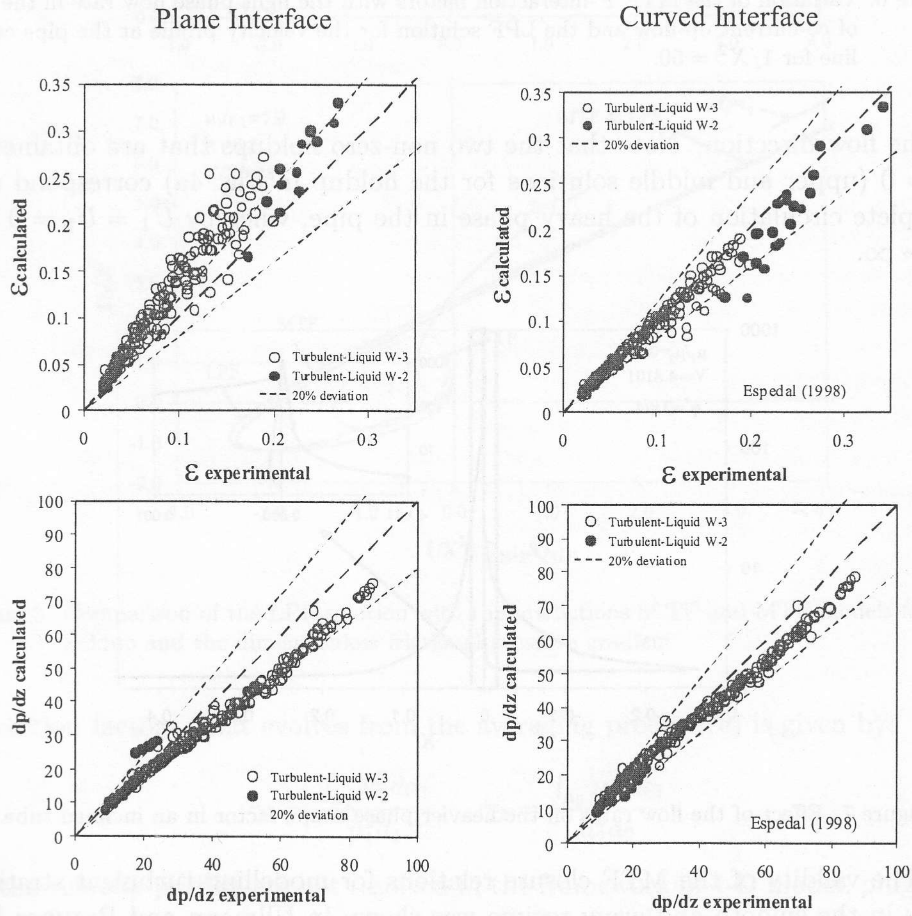

Figure 8. Comparison of the MTF model predictions with Espedal [19] data for the holdup and the pressure gradient in the stratified large wave regime (W2 – two-dimensional (2D) waves, W3-3D waves).

5 Conclusions

The exact solution for laminar pipe flow (LPF) in an inclined pipe has been generalized to be applicable in cases of a curved interface. This solution is used to study the effect of inclination and interface curvature on the characteristics of countercurrent and co-current stratified flows. The pronounced effect of the interfacial curvature on the wetted perimeter, and consequently on the pressure drop and holdup, was demonstrated.

The exact solution has been used also to examine the validity of the predictions of the holdup and pressure drop obtained by two-fluid (TF) models. It was found that the common practice of using single-phase-flow-based closure relations for the shear stresses in TF models is problematic in horizontal flows, and fails in predicting the holdup and pressure drop in co-current and counter-current inclined flows. New theory-based closure relations for the wall and interfacial shear stresses (MTF model), which account for the interaction between the phases, were suggested. These are formulated in terms of the single-phase based expressions, which are augmented by the two-phase interaction factors.

The MTF closure relations are applicable also to turbulent flows in either or both of the phases. Combined with new empirical correlations, which represent the wave effects on the interface curvature, the interfacial shear, and the liquid wall shear, the MTF model is applicable also to the stratified wavy regime of gas-liquid systems.

The predictions of the two-fluid model for the pressure gradient and holdup are tested against extensive data bank and some analytical solutions for stratified flows [11-12]. The favorable comparison suggests that the new closure relations are essentially representing correctly the interaction between the phases over a wide range of flow parameters space in the stratified smooth and stratified wavy regimes.

Received 2 November 2005

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