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Phenomenon of hydraulic jump formed on liquid layer by single-phase impinging liquid jet

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Abstract

In the paper theoretical analysis of the phenomenon of impingement of a circular liquid jet upon a plate has been carried out. In the considered case, spreading of the liquid on a plate is caused by the inertia and gravity forces. For supercritical liquid flow, in certain conditions a hydraulic jump is formed. The circular hydraulic jump exhibits behaviour quite different from that commonly observed in planar jets. In the paper a new theoretical model of the phenomena of hydraulic jump has been formulated. Theoretical treatment have been compared with own and other available experimental data.

Keywords: Hydraulic jump; Impinging jet

Nomenclature

d – nozzle diameter
g – acceleration of gravity
h_{st} – energy losses
h – film thickness
p – pressure
r_h – radius of the hydraulic jump
R – radius of curvature (see Fig.2)
u – film velocity
u_d – liquid velocity in the nozzle
ν – kinematic viscosity
σ – surface tension

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ρ – liquid density
Ma – Mach number
$\mathrm{Fr}_h = \frac{u_d d^2}{8 r_h \sqrt{g h_1^3}}$ – Froude number
$\mathrm{Fr}_d = \frac{u_d}{\sqrt{gd}}$ – Froude number
$\mathrm{We} = d\sqrt{\frac{\rho\, g}{\sigma}}$ – Weber number

Subscripts

1 – before the hydraulic jump
2 – behind the hydraulic jump

1 Introduction

Liquid flow down the surface of the wall occurs in water-air mixture cooled heat exchangers or condensers as well as in a variety of chemical engineering installations. The process of liquid flow in water-air mixture cooled heat exchangers is different from that in condensers. In both cases motion of the liquid layer is governed by different forces. In the case of gravity driven flow, the trajectory of the liquid film is determined by the gradient of wall steepness. The inertia forces are neglected there. However, in the case of impingement of a liquid jet on the surface, the inertia forces must be taken into account as they have a great effect on the flow of forming liquid film. Both these cases were investigated theoretically and experimentally [1, 2]. In the case of jets impinging upon a surface, the inertia forces must be taken into account as they have a great effect on the flow of a formed liquid layer. The process of flow of a liquid film on a horizontal surface after the impingement was investigated theoretically and experimentally in [2, 3]. The phenomenon under consideration consist in forming a circular-symmetrical area where the thickness of the liquid film is at least an order of magnitude lower than that outside the area. The center of symmetry of the area is located in a spot, where the jet hits the surface. This area may be easily formed and its parameters can be precisely controlled. In the area where the liquid layer is thin, a very high local heat transfer between the liquid layer and the solid surface occurs. Prediction and control of jump location is important in thermal design In the paper mathematical model of the phenomenon proposed by J. Mikielewicz and D. Mikielewicz [6] which allows to predict thickness of the liquid layer before and beyond the hydraulic jump. Besides the model allows to evaluate diameter where the hydraulic jump occurs.

2 Analysis

Let us consider liquid jet impinging upon the horizontal plate as shown in Fig. 1. Liquid on the plate is spreading due to inertia and gravity forces.

A hydraulic jump is occurring when the flow suddenly changes from supercritical (Fr > 1) to subcritical (Fr < 1) flow, which is accompanied by a sudden increase in liquid height. Critical conditions of the flow correspond to Fr = 1, when liquid mass velocity and the velocity of disturbance propagation over the shallow water surface are equal. The hydraulic jump is analogous to shock wave in gas flow when the flow changes from supersonic (Ma > 1) to subsonic (Ma < 1) flow. In the paper the case is discussed when the inlet Froude number Fr_d, is greater than unity. In such a case it is possible that a round hydraulic jump occurs at some distance r from center of impingement.

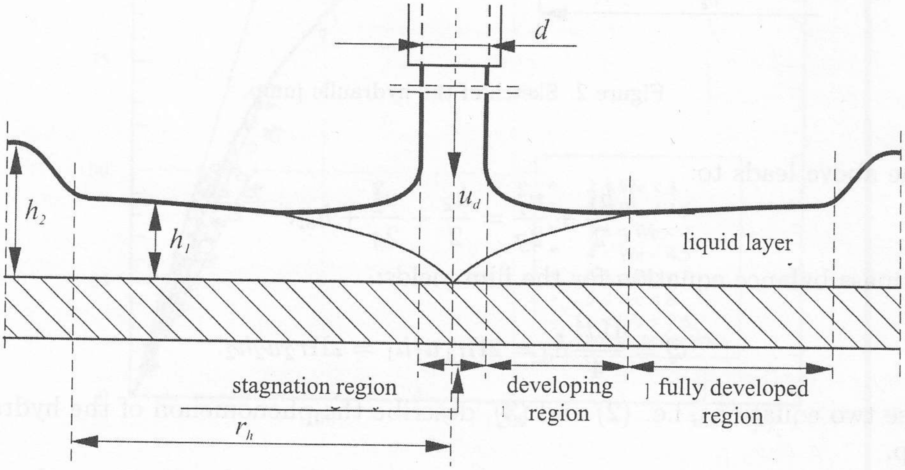

Figure 1. Impingement of a liquid jet upon a flat surface.

Direct adaptation of the momentum balance theory to impinging jet jumps was first made by Watson [4] in 1964. Subsequent investigators modified his theory. However, some investigators have reported rough agreement of their experimental data with the standard model of hydraulic jump. Previous studies have generally concluded the failure of the standard jump momentum balance. The purpose of the present paper is to improve the standard model by including into the model of energy losses due to sudden expansion of the flow after the jump. The standard approach to solution of the described problem is to consider a momentum balance for inviscid flow. In the present approach we use Bernoulli equation for average streamline instead of momentum equation (Fig. 2) In order to extend the standard approach, in [6] included are the energy losses caused by sudden expansion in extended Bernoulli equation for viscid flow.

$$\frac{p_1}{\rho g} + H_1 + \frac{u_1^2}{2g} = \frac{p_2}{\rho g} + H_2 + \frac{u_2^2}{2g} + h_{st}. \tag{1}$$

For the considered case: $p_1 = p_2$ and, $H_1 = h_1/2$ and $H_2 = h_1/2$. Substitution

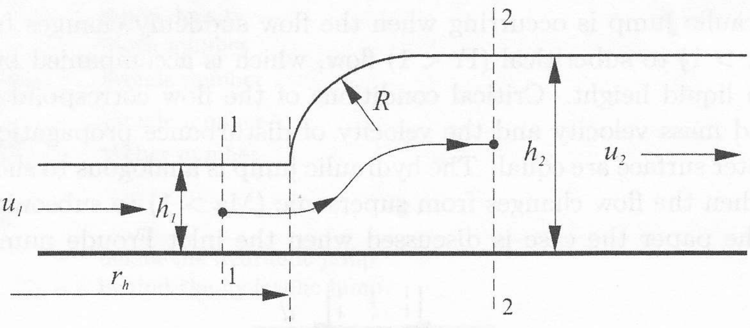

Figure 2. Sketch of the hydraulic jump.

of the above leads to:

$$\frac{h_1}{2}+\frac{u_1^2}{2g}=\frac{h_2}{2}+\frac{u_2^2}{2g}+h_{st}. \tag{2}$$

The mass balance equation for the film yields:

$$Q=\frac{\Pi d^2}{4}u_d=2\Pi r_1 u_1 h_1=2\Pi r_2 u_2 h_2. \tag{3}$$

These two equations, i.e. (2) and (3), describe the phenomenon of the hydraulic jump.

The losses of mechanical energy occurring during hydraulic jump, shown in (1) and (2), can be defined as:

$$h_{st}=\frac{(u_1-u_2)^2}{2g}k \tag{4}$$

in which k is a coefficient of the energy losses taken from the experiment. Substituting (3) and (4) into (2), and after some algebraic transformations one obtains:

$$\begin{aligned}\frac{h_2}{h_1}&=\frac{1}{2}\frac{u_1^2}{gh_1}\left[1-k+\sqrt{(1-k)^2+4\frac{(1+k)gh_1}{u_1^2}}\right]\\&=\frac{1}{2}\mathrm{Fr}_h^2\left[1-k+\sqrt{(1-k)^2+4\frac{(1+k)}{\mathrm{Fr}_h^2}}\right]\end{aligned} \tag{5}$$

what is the main point of the proposition [6]. For $k=1$ one obtains:

$$\frac{h_2}{h_1}=\sqrt{2}\frac{u_1}{\sqrt{gh_1}}=\sqrt{2}\,\mathrm{Fr}_h \tag{6}$$

instead of the standard formula [4][5], in which energy losses are neglected.

$$\frac{h_2}{h_1}=\frac{1}{2}\left(-1+\sqrt{1+8\,\mathrm{Fr}_h^2}\right). \tag{7}$$

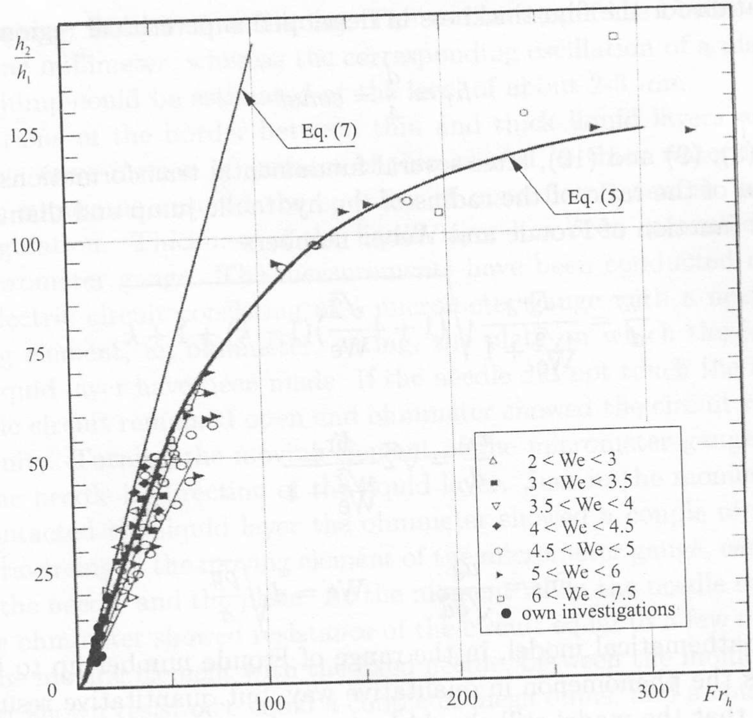

Figure 3. Comparison of theoretical equations (5) and (7) with experimental data.

Graphically relations (5) and (7), which represent the dependence between the ratio of the thickness of the liquid layer after and before the hydraulic jump h_2/h_1, as a function of the Froude number, for $k = 1.015$, are shown in Fig. 3. In the same figure results of the experimental investigation published in [7] supplemented by authors' experimental investigations are also shown.

Balancing the hydrostatic pressure force behind the jump and the surface tension force along the surface of the jumps one obtains:

$$(h_2 - h_1)\rho g = \frac{\sigma}{R} \tag{8}$$

where R is the radius of curvature of the surface roller. This radius of curvature at the jump is about one half of the jump height. Hence,

$$(h_2 - h_1) = \sqrt{\frac{2\sigma}{\rho g}}. \tag{9}$$

In the case of laminar flow, the upstream height, which depends on radius coordinate r, can be calculated according to [6]. Assuming $u_d = u_1$ we have good approximation for the film thickness in developed supercritical region as [6]:

$$h_1 = \frac{d}{4} = const \tag{10}$$

Relations (8), (9) and (10), after several fundamental transformations, allow determination of the ratio of the radius of the hydraulic jump and diameter of the nozzle as a function of Froude and Weber numbers

$$\frac{r_h}{d} = \frac{Fr_d}{\frac{4\sqrt{2}}{\mathrm{We}}+1}\sqrt{(1+4\frac{\sqrt{2}}{\mathrm{We}})(1-k)+1+k}\,. \tag{11}$$

For $k = 1$

$$\frac{r_h}{d} = \sqrt{2}\frac{\mathrm{Fr}_d}{\frac{4\sqrt{2}}{\mathrm{We}}+1} \tag{12}$$

where :

$$\mathrm{Fr}_d = \frac{u_d}{\sqrt{gd}}, \qquad \mathrm{We} = d\sqrt{\frac{\rho g}{\sigma}}$$

Presented mathematical model, in the range of Froude number up to $\mathrm{Fr}_d = 15$, characterises the phenomenon in qualitative way, but quantitative results of the model show that the model still should be improved. The results of calculations using Eq. (11), for $k = 1.015$, have been shown graphically in Fig. 5.

It is likely that in order to improve the mathematical model of the analysed phenomenon, the gravity forces in the region of the hydraulic jump, caused by vortex movement in the thicker layer of the liquid layer ought to be taken into account.

3 Experimental apparatus

Experimental apparatus, shown schematically in Fig. 4, consists of a closed water loop consisting of a water tank, pump, and flow-meter, as well as a gauze filter, control valve, nozzle and hoses.

The plates, on which the jet impinges, were brass, aluminium, glass and organic glass and they were interchangeable. During experiments, the impinging jet was directed to the centre of the plate, but the assembly was equipped with a special device which allowed the jet to be directed to an arbitrary point on the plate at an arbitrary angle and optional nozzle-to-plate distance. The measurement of the diameter of the hydraulic jump formed by an impinging two-phase jet jump was made by means of a projected shadow of net, permitting a non-invasive measurement of the parameters of the hydraulic jump. The border between the area of a very thin film and much higher film (i.e. before and beyond the hydraulic jump), was very sharp and oscillating. The oscillations of the border were equal to about one millimeter, whereas the corresponding oscillation of a diameter of a hydraulic jump could be estimated at the level of about 2-3 mm.

Oscillations of the border between thin and thick liquid layers were caused by intensive wave motion of surfaces of thin-as well as thick water layers. The measuring apparatus ensured the stability of mass flow rates, as well as their precise regulation. Thickness of the liquid layer has been measured by means of the micrometer gauge. The measurements have been conducted in this way, that an electric circuit consisting of a micrometer gauge with a needle fixed to its moving element, an ohmmeter, wiring, the plate on which the jet impinges and the liquid layer have been made. If the needle did not touch the liquid layer, the electric circuit remained open and ohmmeter showed the circuit resistance to equal infinity. Turning the moving element of the micrometer gauge caused the shift of the needle in direction of the liquid layer. Just in the moment when the needle contacted the liquid layer the ohmmeter showed a couple of mega ohms. Further traversing of the moving element of the micrometer gauge, caused contact between the needle and the plate. At the moment when the needle contacted the plate, the ohmmeter showed resistance of the circuit equal to a few of ohms. The shift of the moving element with the fixed needle, between the moment when the ohmmeter shown resistance equal a couple of mega ohms, and a couple of ohms, recorded on the scale of the micrometer gauge, corresponded to a thickness of the liquid layer. Described method of measuring of the thickness of the liquid layer, applied by many authors, including [6], despite its simplicity, seams to be useful because practically does not depend on chemical constitution of the water. Repeatability of the measurements of the thickness of the liquid layer, confirmed usefulness of the applied method.

Based on the authors own and taken from the literature [7-9] experimental data, the experimental formula characterising the dependence between the ratio of the radius of the hydraulic jump to diameter of the nozzle as a function the Froude number in the form:

$$\frac{r_h}{d} = 4.9\ln(\mathrm{Fr}_d) + 0.5 \tag{13}$$

has been developed.

The formula (13), graphically depicted in Fig. 5 satisfies, in the wide range of Froude numbers the results of experimental investigations, whereas in the range of low Froude numbers it approximately satisfies the equation (11).

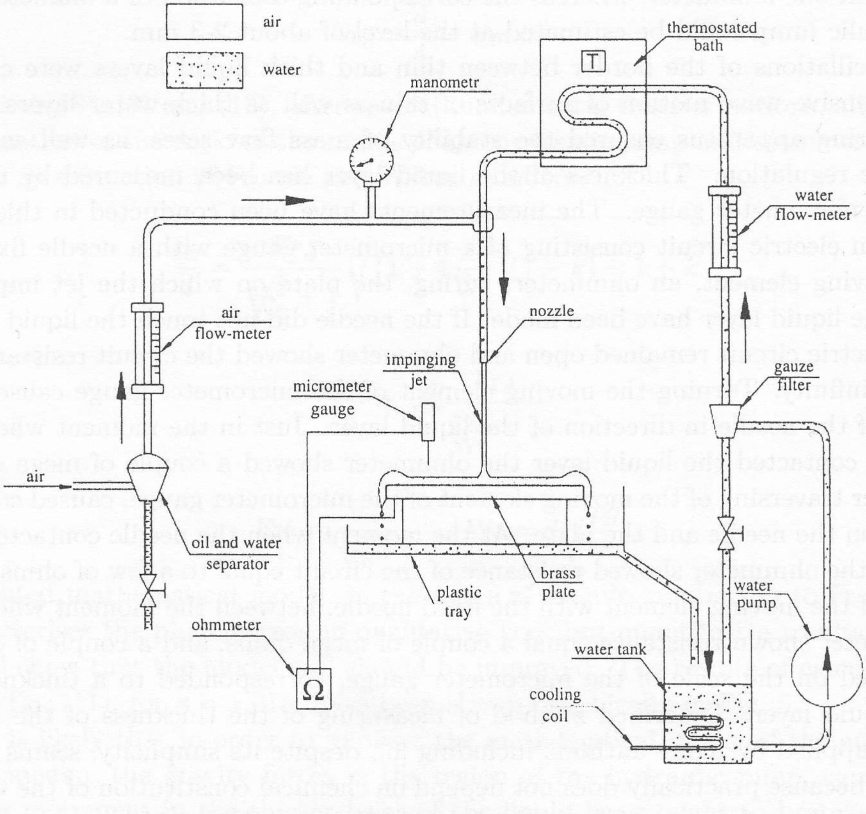

Figure 4. Schematic diagram of the experimental set-up

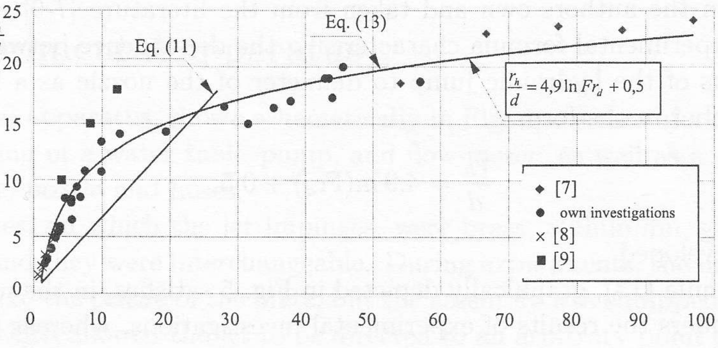

Figure 5. Results of the experimental investigations dependence between the radius of the hydraulic jump r_h, to diameter of the nozzle d, ratio as a function of Froude number and graphical run of the theoretical equations (11) and experimental (13).

4 Conclusions

It seems that proposed in [6] theoretical model characterising the dependence between thickness of liquid layer after and before the hydraulic jump ratio $\frac{h_2}{h_1}$, as a function of Froude number Fr_h, describes the phenomenon of the hydraulic jump satisfactory.

In the presented model, characterising the dependence radius of hydraulic jump to diameter of the nozzle as a function of Froude number another physical quantity e.g inertia forces caused by vortex movement, could be included. Proposed experimental relation (13) seems to characterises the phenomenon of hydraulic jump satisfactory in the wide range of Froude numbers as it is based on substantial amount of the experimental data carried out independently by the different authors.

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