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Towards to efficiency maximum within a pressure increasing process

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Abstract

The article presents a mathematical description of compression process in a pump and an original concept to increase efficiency in this process.

Keywords: Efficiency; Loss: Kinematic flow; Pumps; Incompressible fluid

1 Introduction

A typical rotor machine for pressure increasing process for incompressible fluid is rotordynamic pump. Designing the system of vane pump is more often supported by higher order numerical methods 2D-model or 3D-model. Those models give many detailed informations about field of flow parameters by stage what allows improving the system of flow thereby eliminate (for example) separation or minimalization of dissipation loss. It has influence on increasing efficiency in compression process of incompressible fluid in nowadays designed pump. However, from the obtaining of dissipation loss isn't the only factor which decisived in efficiency in pressure rise process of incompressible fluid. As we can see in analysis below efficiency in pressure increase of incompressible fluid is also dependent to kinematic flow parameters like angles of flow deviations in channels. Achieving maximum efficiency in pressure rise process of incompressible fluid in pump is determined by appropriate correlation between dissipation loss and flow

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kinematics. It turns out that different dissipation loss levels are corresponds to different kinematics implementing maximum efficiency in compression process.

Presented analysis is based on well known 1D-model of flow by stage of centrifugar pump [3]. Novelty of proposed optimalization comparing to available literature sources consists analysis method for expression for efficiency of compressing process. The results of this analysis can constitute a fundamental accuracy assessment of matching rotor channels with reversing channels in pumps providing maximum efficiency in compression process for given dissipation loss levels. It can be a tool, which let us evaluate experimental research data of pump stage in respect of getting maximum accumulation efficiency for provided profiles of rotor and reversing channel of pump.

2 Basic relation

To find a relation between efficiency in pressure rising process and characteristics of pump blade system it is necessary to take into account the basic laws of physics which are used according to flow-model [2, 3]. Down below this model is 1D-model understood like a scheme on Fig.1. The picture presents abstract trajectory of fluid element in multistage pump. Fluid element starting from the position '1' and after a while it is in the position '2'. Observer related with this element placed in 'L' – intermediate position is noticing the changes of parameters in time. In this meaning we are discrubing the changes of parameters in Lagrangian coordinates.

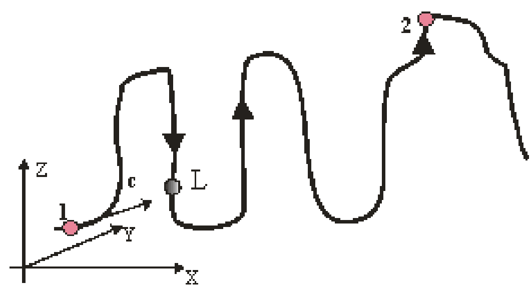

Figure 1. Typical trajectory of fluid element in blade system of multistage pump.

In chosen sectional view along trajectory of fluid element we will use this form of mass conservation equation:

$$m = \rho\, S_1\, c_{1n} = \rho\, S_2\, c_{2n} = const\,, \tag{1}$$

where: m – mass rate of flow, kg/s, ρ – density, kg/s, S_1, S_2 – channel face area, m^2, c_{1n}, c_{2n} – normal velocity to channel face area, m/s.

After eliminate ρ from Eq. (1), another form of this equation for incompressible fluid is

$$V = S_1 c_{1n} = S_2 c_{2n} = const\,. \tag{2}$$

Energy resource carried by fluid element consists of four types and is expressed by the sum of four energy types:

$$E = \frac{c^2}{2} + \Pi + \frac{p}{\rho} + e\,, \tag{3}$$

where

- kinetic energy $\frac{c^2}{2}$ of fluid element movement in (X,Y,Z)-system,
- potential energy $\Pi = \pm gz - \frac{u^2}{2}$ which is sum of potential energy of gravitational field (gz) and potential energy of centrifugal force ($-\frac{u^2}{2}$),
- pressure energy $\frac{p}{\rho}$, where ρ is density,
- internal energy $e = c_p T$ determined by level of temperature, where c_p is specific heat.

Sum of those four types of energy could stay constant in stationary blade to coordinate system or change in moving blade. Transition of fluid element from stationary blade to moving blade and in opposite side is associated with changes the sum of four energy types.

Process of changing parameters runs according to Gibbs relation in form

$$T\frac{ds}{dt} = \frac{de}{dt} + p\frac{d}{dt}\Big(\frac{1}{\rho}\Big)\,, \tag{4}$$

where: $\frac{d}{dt}$ – time differential symbol by observer 'L', T – temperature, s – entropy, e – internal energy, p – pressure. Without any discussions about a limits of applicability this equation we can assume that for this technical case – pressure rising process of fluid – applicability this equation is justified.

Changes in time of total fluid energy seen by observer moving together with element is determined by two factors according to relations [3]:

$$\frac{dE}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} + O(\mu)\,. \tag{5}$$

The first factor is nonstationary pressures field along trajectory of fluid element. The second factor is friction on the blade channel walls which is dependent to viscosity coefficient μ.

Lets enter definition facilitated discribing pressure rising process – enthalpy definition

$$h \overset{def}{=} e + \frac{p}{\rho}\,, \tag{6}$$

allows us to record Gibbs relation in form

$$T\frac{ds}{dt} = \frac{dh}{dt} - \frac{1}{\rho}\frac{dp}{dt}\,, \tag{7}$$

which after including entropy gain expression as result of positive entropy sources

$$\rho\,\frac{ds}{dt} = s_\mu^+\,, \tag{8}$$

leads to relation

$$\frac{dp}{dt} = \rho\frac{dh}{dt} - Ts_\mu^+. \tag{9}$$

If $\rho = const$ relation (9) allows integration along trajectory of fluid element and after leads to expression

$$p_2 - p_1 = \rho\,(h_2 - h_1) - \int_1^2 T\,s_\mu^+ dt\,. \tag{10}$$

This relation is a fundamental for definition of efficiency in pressure rising process.

If $\rho = const$ we could eliminate integral from Eq. (10) because

$$T\frac{ds}{dt} = \frac{de}{dt} \qquad \rho\frac{ds}{dt} = s_\mu^+\,, \tag{11}$$

hence

$$\int_1^2 T\,s_\mu^+ = \rho(e_2 - e_1) > 0\,. \tag{12}$$

Relation (10) could be expressed

$$p_2 - p_1 = \rho(h_2 - h_1) - \rho(e_2 - e_1) \quad | : \rho(h_2 - h_1)\,, \tag{13}$$

which allows to insert the definition of efficiency of pressure rising process (after dividing by $\rho(h_2 - h_1)$)

$$\eta_{1,2} \stackrel{def}{=} \frac{p_2 - p_1}{\rho(h_2 - h_1)} = 1 - \frac{\rho(e_2 - e_1)}{\rho(h_2 - h_1)}\,. \tag{14}$$

We transform Eq. (14) by eliminating ρ from $\frac{\rho(e_2 - e_1)}{\rho(h_2 - h_1)}$ to get efficiency expression in form

$$\eta_{1,2} = 1 - \frac{e_2 - e_1}{h_2 - h_1} = 1 - \frac{e_2 - e_1}{\frac{p_2}{\rho} + e_2 - \frac{p_1}{\rho} - e_1} = \frac{p_2 - p_1}{p_2 - p_1 + \rho(e_2 - e_1)}\,. \tag{15}$$

Right side of relation (15) suggested that internal energy gain from dissipation process, defined by positive entropy sources s (12), leads to decrease this efficiency. To adapt efficiently (15) expression to a special technique situation like a pump it is necessary to enter to the relation (15) parameters which determinated kinematics of flow through blade channel of pump.

3 Presentation of compression process in a pump

One of the most difficulty to describe flow through blade channel of pump is to express internal energy gain as result of dissipation process. For this analysis we will defined loss coefficient as:

- for pump rotor defined by kinetic energy after rotor with velocity signed as w_2

$$\varsigma_R \stackrel{def}{=} \frac{\Delta e_{1,2}}{\frac{w_2^2}{2}}\,, \tag{16}$$

- for diffuser (reversing channel) after rotor similarly where velocity is signed as c_2

$$\varsigma_D \stackrel{def}{=} \frac{\Delta e_{2,3}}{\frac{c_2^2}{2}}\,. \tag{17}$$

In this way internal energy gains in effciency definition (15) are determinated by: rotor

$$\Delta e_{1,2} = \varsigma_R \frac{w_2^2}{2}\,, \tag{18}$$

for diffuser

$$\Delta e_{2,3} = \varsigma_D \frac{c_2^2}{2}\,. \tag{19}$$

Second parameter describing diffuser (reversing channel) is coefficient of orbital momentum reduction defined as

$$\varphi \stackrel{def}{=} \frac{orbital\, momentum_{outlet}}{orbital\, momentum_{inlet}}\,. \tag{20}$$

Value of this coefficient can range within [0,1]. Total orbital momentum reductions can be accepted only if it doesn't cause to much dissipation.

Compression process in diffuser could be presented on 'total energy - internal energy gain', E–Δe, diagram (Fig. 2).

From energy balance expression in form

$$E = \frac{c_1^2}{2} + \frac{p_1}{\rho} + e_1 = \frac{c_2^2}{2} + \frac{p_2}{\rho} + e_2 = const\,, \tag{21}$$

we can get the expression

$$\frac{c_1^2}{2} + \frac{p_1}{\rho} = \frac{c_2^2}{2} + \frac{p_2}{\rho} + (e_2 - e_1)\,, \tag{22}$$

when $\Delta e_{1,2} > 0$, which. allows to present this process in E–Δe diagram (Fig. 2.)

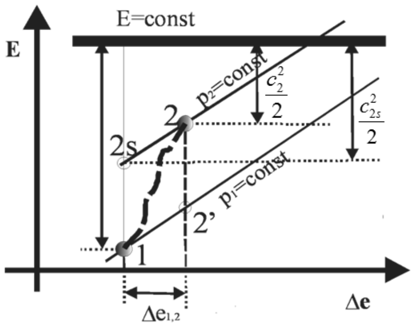

Figure 2. Presentation of compression process on a diagram E–Δe.

Let's enter designation for velocity triangles (Fig. 3) while inlet to rotor pump (index 1), while outlet from rotor and inlet to diffuser (reversing channel) (index 2), while outlet from diffuser (index 3):

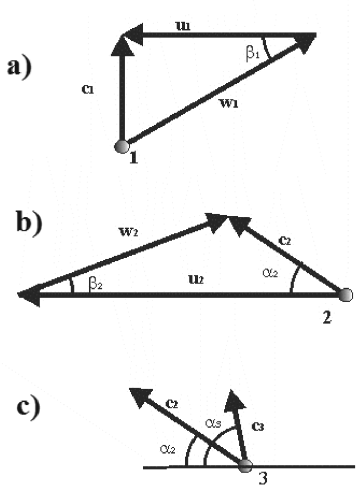

Figure 3. Designation of velocity vectors.

- Inlet to rotor (Fig. 3a): c_1 – velocity in absolute system (laboratory 'E'),
  w_1 – velocity in convection system (rotor 'L'),
  u_1 – convection velocity.
- Outlet from rotor (inlet to diffuser, reversing channel RC) – (Fig. 3b):
  c_2 – velocity in absolute system (laboratory 'E')

w_2 – velocity in convection system (rotor ‘L’),

u_2 – convection velocity.

- Outlet from diffuser (Fig. 3c):

  c_3 – velocity in absolute system (laboratory ‘E’).

Pressure rising process in a pump in coordinate system E–Δe is presented in Fig. 4.

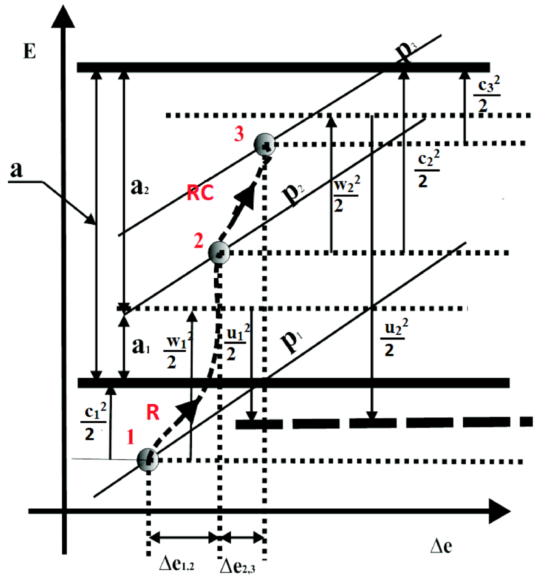

Figure 4. Compression process in a rotor of pump (R) and reversing channel (RC).

We could distinguish the following energetic levels

$$E_1 = \frac{p_1}{\rho} + \frac{c_1^2}{2} + e_1 \,, \qquad E_2 = \frac{p_2}{\rho} + \frac{c_2^2}{2} + e_2 \,, \qquad E_3 = E_2 = \frac{p_3}{\rho} + \frac{c_3^2}{2} + e_3 \,, \tag{23}$$

$$E_w = \frac{p_1}{\rho} + e_1 + \frac{w_1^2}{2} - \frac{u_1^2}{2} = \frac{p_2}{\rho} + e_2 + \frac{w_2^2}{2} - \frac{u_2^2}{2} \,. \tag{24}$$

Simple calculation shows us a unitary work leaded to pressure rising

$$a = \underbrace{\frac{w_1^2}{2} - \frac{c_1^2}{2}}_{a_1} + \underbrace{\frac{u_2^2}{2} - \frac{u_1^2}{2} + \frac{c_2^2}{2} - \frac{w_2^2}{2}}_{a_2} \,, \tag{25}$$

which can be taken into account the energy balance

$$E_2 = E_1 + a \,, \tag{26}$$

leading to the expression

$$\frac{p_2}{\rho}+\frac{c_2^2}{2}+e_2=\frac{p_1}{\rho}+\frac{c_1^2}{2}+e_1+\frac{w_1^2}{2}-\frac{c_1^2}{2}+\frac{u_2^2}{2}-\frac{u_1^2}{2}+\frac{c_2^2}{2}-\frac{w_2^2}{2}\,, \tag{27}$$

and the expression of differential pressure

$$\frac{p_2}{\rho}-\frac{p_1}{\rho}=\frac{w_1^2}{2}+\frac{u_2^2}{2}-\frac{u_1^2}{2}-\frac{w_2^2}{2}-(e_2-e_1)\,. \tag{28}$$

Relations for velocity triangle

$$w_1^2=c_1^2+u_1^2\,, \tag{29}$$

$$w_2^2=c_2^2+u_2^2-2c_2u_2\,\cos\alpha_2,$$

allow to have an expression for differential pressure in form

$$\frac{p_2-p_1}{\rho}=\frac{c_1^2}{2}-\frac{c_2^2}{2}+c_2u_2\,\cos\alpha_2-\frac{\varsigma_R}{2}\big(c_2^2+u_2^2-2c_2u_2\,\cos\alpha_2\big). \tag{30}$$

Now we can present efficiency in pressure rising process as

$$\eta_R=\frac{\frac{p_2-p_1}{\rho}}{\frac{p_2-p_1}{\rho}+(e_2-e_1)}=\frac{c_1^2-c_2^2+2c_2u_2\,\cos\alpha_2-\varsigma_R(c_2^2+u_2^2-2c_2u_2\,\cos\alpha_2)}{c_1^2-c_2^2+2c_2u_2\,\cos\alpha_2}. \tag{31}$$

For reversing channel diffuser we have relation

$$\frac{p_2}{\rho}+\frac{c_2^2}{2}+e_2=\frac{p_3}{\rho}+\frac{c_3^2}{2}+e_3\,, \tag{32}$$

$$\frac{p_3-p_2}{\rho}=\frac{c_2^2}{2}-\frac{c_3^2}{2}-(e_3-e_2)\leftarrow e_3-e_2=\varsigma_D\frac{c_2^2}{2}\,, \tag{33}$$

$$\frac{p_3-p_2}{\rho}=(1-\varsigma_D)\frac{c_2^2}{2}-\frac{c_3^2}{2}\,. \tag{34}$$

Using the relations (mass conservation equation and definition of arbital angular momentum reduction) Eq. (20)

$$\rho S_2c_2\,\sin\alpha_2=\rho S_3c_3\,\sin\alpha_3\,, \tag{35}$$

$$\varphi r_2c_2\,\cos\alpha_2=r_3c_3\,\cos\alpha_3\,.$$

We can eliminate c_3 and α_3 from Eq. (34) gaining

$$\frac{p_3 - p_2}{\rho} = (1 - \varsigma_D)\frac{c_2^2}{2} - (\varphi^2 r^2 \cos^2 \alpha_2 + s^2 \sin^2 \alpha_2)\frac{c_2^2}{2}\,, \tag{36}$$

where there were included geometric (overall) parameters

$$s = \frac{S_2}{S_1} = \frac{S_2}{S_3}\,, \qquad r = \frac{r_2}{r_1} = \frac{r_2}{r_3}\,. \tag{37}$$

Now it is possible to designate total efficiency of rotor and reversing channel

$$\eta_{RD} = \frac{\frac{p_3-p_1}{\rho}}{\frac{p_3-p_1}{\rho} + \Delta e_{1,3}} = \frac{\frac{p_2-p_1}{\rho} + \frac{p_3-p_2}{\rho}}{\frac{p_2-p_1}{\rho} + \frac{p_3-p_2}{\rho} + \Delta e_{1,2} + \Delta e_{2,3}}\,, \tag{38}$$

through coefficient characterizing rotor or relation rotor-reversing channel

$$\eta_R = 1 - \varsigma_R \frac{1 + \nu^2 - 2\nu \cos\alpha_2}{s^2 \sin^2 \alpha_2 + 2\nu \cos\alpha_2 - 1}\,, \tag{39}$$

$$\eta_{R,D} = 1 - \frac{\varsigma_R(1 + \nu^2 - 2\nu \cos\alpha_2) + \varsigma_D}{2\nu \cos\alpha_2 - \varphi^2 r^2 \cos^2 \alpha_2}\,, \tag{40}$$

where velocity coefficient is defined as

$$\nu \overset{def}{=} \frac{u_2}{c_2}\,. \tag{41}$$

Equation (40) is fundamental to searching efficiency in compression process conditions.

4 Formulating task for efficiency in compression process

Searching geometrical parameters of pump providing maximal efficiency defined by Eqs. (39) and (40) is possible in few variants. Let's consider the first variant I (Eq. (42)) where we have rotor loss coefficient, ς_R, reversing channel loss coefficient, ς_D, and coefficient of orbital angular momentum reduction, φ. Then the task needs only to find extremum of function of variables α_2, and ν

$$\eta_{RD_{max}} = \eta_{RD}(\alpha_{2opt}, \nu_{opt})\,. \tag{42}$$

We could consider variant II (Eq. (43)) if we have rotor loss coefficient, ς_W, reversing channel loss coefficient, ς_d, and outlet angle from rotor β_2. We can need extremum of function of variables ν and φ

$$\eta_{RD_{max}} = \eta_{RD}(\nu_{opt}, \varphi_{opt})\,. \tag{43}$$

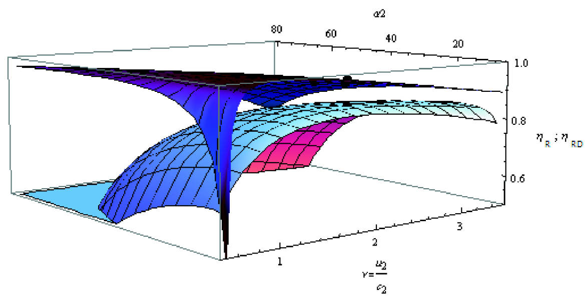

Figure 5. Two efficiency surfaces according to (42) stretched over α_2, ν.

The numerical example below wrote in Wolfram Mathematica software, illustrating variant 1 (Eq. (42)), has been presented in Appendix 1.

5 Results

For maximum efficiency presented in Fig. 5 there is included program allowing designate kinematic of flow in velocity triangles form in maximal efficiency point η_{RD}. The top surface concerns about rotor efficiency, lower surface concerns about relation rotor-reversing channel. Visible points on surfaces are illustrating maximal efficiency η_{RD}.

In Appendix 2 the numerical example of results of kinematic has been presented. In Fig. 6 the velocity triangles for maximum efficiency η_{RD} has been estimated.

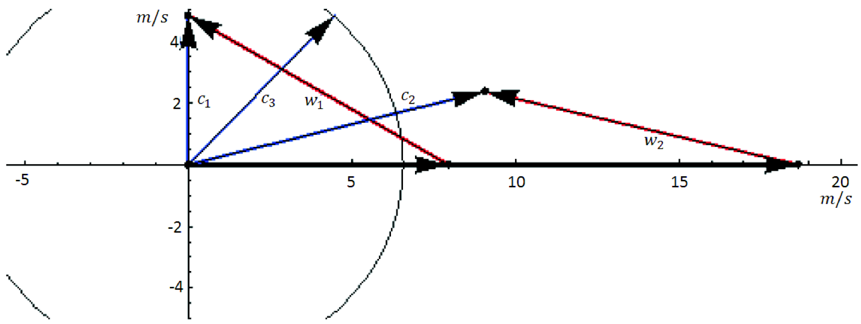

Figure 6. Velocity triangles for maximal efficiency η_{RD}.

6 Conclusions

A significant request consequential from presented analysis is necessary to take into account the dissipation level in rotor channel and reversing channel (the value of loss coefficients)while designing pump kinematic to get the maximal efficiency in compression process. It's very often this fact is ignored [1]. The analysis also presents the way of crossing from basic flow equation to relation form of technical meaning.

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References

[1] Troskolański A. T.: *Hydraulic Machines and Devices (Basic Concepts).* PWN Warszawa-Poznań 1974 (in Polish).

[2] Puzyrewski R., Sawicki J.: *Base of Fluids Mechanics and Hydraulics*, 3rd Edn. 2013 (in Polish).

[3] Puzyrewski R.: *Basic Theory of Rotor Machines in Terms of One-Dimensional*. Ossolineum, Wrocław, 1992 (in Polish).

Appendix.1: The numerical example proceeding in Wolfram Mathematica software

(*PROGRAM OF MAXIMALIZATION OF EFFICIENCY IN COMPRESSION PROCESS OF INCOMPRESSIBLE FLUID*)

(*DATA*)
n=2850;
m=14;
ρ=1000;
r1z=0.0353;
r1w=0.018
s=2.03342; (*s=s2/s1*)

(*Quality of channels*)
dzw=0.1;(*$\Delta e_{12} = \varsigma_W \frac{w2^2}{2}$*)
dzd=0.32313;(*$\Delta e_{23} = \varsigma_D \frac{c2^2}{2}$*)
φ=.21;(*$K_3 = \varphi K_2$*)

(*THE DATA END*)

Continuation 1st of Appendix.1: The numerical example proceeding in Wolfram Mathematica software

```
r1=0.5(r1z+r1w);
s1=Pi*(r1z^2-r1w^2);
u1=(Pi*2*r1*n)/60
μ=m/(ρ*s1*u1);
r=(μ *ν)/(s* Sin[(Pi*α)/180]);
(*r=r2/r1*)
```

$$\text{EtWD} = 1 - \frac{\text{dzw} * (1+\nu^2 - 2*\nu*\text{Cos}[\frac{\text{Pi}*\alpha}{180}]) + \text{dzd}}{2*\nu*\text{Cos}[\frac{\text{Pi}*\alpha}{180}] - \varphi^2 r^2 \text{Cos}[\frac{\text{Pi}*\alpha}{180}]^2}$$

$$\text{EtWD} = 1 - \frac{\text{dzd} + \text{dzw}(1+\nu^2 - 2\nu\text{Cos}[\frac{\pi\alpha}{180}])}{2\nu\text{Cos}[\frac{\pi\alpha}{180}] - \frac{36.95753611686362 m^2\nu^2\varphi^2\text{Cot}[\frac{\pi\alpha}{180}]^2}{n^2(\text{r1w}-1.\text{r1z})^2(\text{r1w}+\text{r1z})^4 s^2\rho^2}}$$

X = Plot3D[EtWD,{ν,.5,3.5},{α,8,80},PlotRange → {0.5,1}];

$$\text{EtW} = 1 - \text{dzw} * \frac{1+\nu^2 - 2*\nu*\text{Cos}[\frac{\text{Pi}*\alpha}{180}]}{(s*\text{Sin}[\frac{\text{Pi}*\alpha}{180}])^2 + 2*\nu*\text{Cos}[\frac{\text{Pi}*\alpha}{180}] - 1};$$

Z = Plot3D[EtW,{ν,.5,3.5},{α,5,80},PlotRange → {0.5,1},ColorFunction
→ Function[{x,y,z},Hue[z]]];

FindMaximum[EtWD,{{ν,2.},{α,10.}}];

SequenceForm["EtaMaxWD = ",%[[1]],"dla ν = ",%[[2,1,2]],", α2 = ",%[[2,2,2]]]

```
eT=%%[[1]];
ni=%%%[[2,1,2]];
a2=%%%%[[2,2,2]];
niC=ni;
a2C=a2;
Y=Graphics3D[{PointSize[0.02],Point[{ni,a2,eT}]}];
```

$$\text{EtW} = 1 - \text{dzw} * \frac{1+\text{ni}^2 - 2*\text{ni}*\text{Cos}[\frac{\text{Pi}*\text{a2}}{180}]}{(s*\text{Sin}[\frac{\text{Pi}*\text{a2}}{180}])^2 + 2*\text{ni}*\text{Cos}[\frac{\text{Pi}*\text{a2}}{180}] - 1};$$

YW = Graphics3D[{PointSize[0.02],Point[{ni,a2,EtW}]}];

Show[X,Y,YW,Z,AxesLabel → {"ν = "$\frac{u_2}{c_2}$,"α2",η_W";"η_{WD}}]

Continuation 2nd of Appendix.1: The numerical example proceeding in Wolfram Mathematica software

```
r1=0.5(r1z+r1w);
s1=Pi*(r1z^2-r1w^2);
u1=(Pi*2*r1*n)/60
μ=m/(ρ*s1*u1);
r=(μ *ν)/(s* Sin[(Pi*α)/180]);
(*r=r2/r1*)
```

$$\text{EtWD} = 1 - \frac{\text{dzw} * (1 + \nu^2 - 2 * \nu * \text{Cos}[\frac{\text{Pi} * \alpha}{180}]) + \text{dzd}}{2 * \nu * \text{Cos}[\frac{\text{Pi} * \alpha}{180}] - \varphi^2 r^2 \text{Cos}[\frac{\text{Pi} * \alpha}{180}]^2}$$

$$\text{EtWD} = 1 - \frac{\text{dzd} + \text{dzw}(1 + \nu^2 - 2\nu\text{Cos}[\frac{\pi\alpha}{180}])}{2\nu\text{Cos}[\frac{\pi\alpha}{180}] - \frac{36.95753611686362 m^2 \nu^2 \varphi^2 \text{Cot}[\frac{\pi\alpha}{180}]^2}{n^2(\text{r1w} - 1.\text{r1z})^2(\text{r1w} + \text{r1z})^4 s^2 \rho^2}}$$

X = Plot3D[EtWD,{ν,.5,3.5},{α,8,80},PlotRange → {0.5,1}];

$$\text{EtW} = 1 - \text{dzw} * \frac{1 + \nu^2 - 2 * \nu * \text{Cos}[\frac{\text{Pi} * \alpha}{180}]}{(s * \text{Sin}[\frac{\text{Pi} * \alpha}{180}])^2 + 2 * \nu * \text{Cos}[\frac{\text{Pi} * \alpha}{180}] - 1};$$

Z = Plot3D[EtW,{ν,.5,3.5},{α,5,80},PlotRange → {0.5,1},ColorFunction → Function[{x,y,z},Hue[z]]];

FindMaximum[EtWD,{{ν,2.},{α,10.}}];

SequenceForm["EtaMaxWD = ",%[[1]],"dla ν = ",%[[2,1,2]],", α2 = ",%[[2,2,2]]]

Continuation 3rd of Appendix.1: The numerical example proceeding in Wolfram Mathematica software

```
SequenceForm["m=",m,"  ηW=",EtW,"  ηWD=",eT,"  dpW=",dp21,"
 dpD=",dp32," dpWD=" ,dp31,"  ν=",ni ]
SequenceForm["r2=",r2,"  h2=",h2,"  u2=",u2,"  c2=",c2,"  w2="
 ,w2,"  α2=",a2,"  β2=",b2]
SequenceForm["ν=u2/c2=",ni,"  r=r2/r1=",rw,"  μ=",m/(ρ*s1*u1),"
 c1=" ,c1,"  u1=",u1,"  β1=",b1]
SequenceForm["c3=",c3,"  α3=",a3]

m=ρ*s1*c3*Sin[(Pi*a3)/180]
φ*r2*c2*Cos[(Pi*a2)/180]

r1*c3*Cos[(Pi*a3)/180]
t=((φ*rw)/s)^2
t1=((s*rw)/(μ*ni))^2-1
t*t1
s=2Pi*r2*h2/s1

EtaMaxWD= 0.880169  dla  ν= 1.9898 ,  α2= 14.6865

m= 14.  ηW= 0.964379  ηWD= 0.880169  dpW= 132015.  dpD=
 8090.76  dpWD= 140106.  ν= 1.9898

r2= 0.0625008  h2= 0.0149998  u2= 18.6534  c2= 9.3745  w2=
 9.87548  α2= 14.6865  β2= 13.926

ν=u2/c2= 1.9898  r=r2/r1= 2.34524  μ= 0.607623  c1= 4.83287
 u1= 7.95373  β1= 31.2838
 c3= 6.58048  α3= 47.2586
```

Appendix.2: The numerical example of kinematic results

```
(*VELOCITY TRIANGLES*)
SequenceForm["alfa1=",90,"°"]
SequenceForm["c1=",c1,"m/s"]
SequenceForm["c1u=",c1*Cos[Pi*90/180],"m/s"]
SequenceForm["c1a=",c1*Sin[Pi*90/180],"m/s"]
SequenceForm["u1=",u1,"m/s"]
SequenceForm["w1=",w1,"m/s"]
SequenceForm["w1u=",w1*Cos[Pi*b1/180],"m/s"]
SequenceForm["c1a=",w1*Sin[Pi*b1/180],"m/s"]
SequenceForm["c2=",c2,"m/s"]
SequenceForm["c2u=",c2*Cos[Pi*a2/180],"m/s"]
SequenceForm["c2a=",c2*Sin[Pi*a2/180],"m/s"]
SequenceForm["alfa2=",a2,"°"]
SequenceForm["w2=",w2,"m/s"]
SequenceForm["w2u=",w2*Cos[Pi*b2/180],"m/s"]
SequenceForm["w2a=",w2*Sin[Pi*b2/180],"m/s"]
SequenceForm["beta2=",b2,"°"]
SequenceForm["c3=",c3,"m/s"]
SequenceForm["alfa3=",a3,"°"]

xc1=c1*Cos[Pi*90/180];
yc1=c1*Sin[Pi*90/180];

xu1=u1;
yu1=0;

xw1=w1*Cos[Pi*b1/180];
yw1=w1*Sin[Pi*b1/180];

xc2=c2*Cos[Pi*a2/180];
yc2=c2*Sin[Pi*a2/180];

xu2=u2;
yu2=0;

xw2=u2;
yw2=w2*Sin[Pi*b2/180];

xc3=c3*Cos[Pi*a3/180];
yc3=c3*Sin[Pi*a3/180];


f2=Plot[{0},{x,-5,20},PlotRange→{-5,5},Frame→None];

kc0=Point[{0,0}];ku1=Point[{xu1,yu1}];ku2=Point[{xu2,yu2}];
kc2=Point[{xc2,yc2}];kw2=Point[{xw2,yw2}];kc1=Point[{xc1,yc1}];

l1c=Line[{{0,0},{xc1,yc1}}];
l1cc=Show[Graphics[{Thickness[0.005],RGBColor[0,0,1],l1c}]];

l1w=Line[{{xu1,yu1},{xc1,yc1}}];
```

Continuation 1nd of Appendix.2: The numerical example of kinematic results

```
l1ww=Show[Graphics[{Thickness[0.005],RGBColor[1,0,0],l1w}]];

l1u=Line[{{0,0},{xu1,yu1}}];
l1uu=Show[Graphics[{Thickness[0.005],RGBColor[0,0,0],l1u}]];

l2c=Line[{{0,0},{xc2,yc2}}];
l2cc=Show[Graphics[{Thickness[0.005],RGBColor[0,0,1],l2c}]];

l2u=Line[{{0,0},{xu2,yu2}}];
l2uu=Show[Graphics[{Thickness[0.005],RGBColor[0,0,0],l2u}]];

l2w=Line[{{xu2,0},{xc2,yc2}}];
l2ww=Show[Graphics[{Thickness[0.005],RGBColor[1,0,0],l2w}]];

l3=Line[{{0,0},{xc3,yc3}}];
l3t=Show[Graphics[{Thickness[0.005],RGBColor[0,0,1],l3}]];

s1=Graphics[Arrow[{{0,0},{xc1,yc1}}]];
s2=Graphics[Arrow[{{0,0},{xu1,yu1}}]];
s1w=Graphics[Arrow[{{xu1,yu1},{xc1,yc1}}]];
s2u=Graphics[Arrow[{{0,0},{xu2,yu2}}]];
s2c=Graphics[Arrow[{{0,0},{xc2,yc2}}]];
s2w=Graphics[Arrow[{{xu2,0},{xc2,yc2}}]];
st3=Graphics[Arrow[{{0,0},{xc3,yc3}}]];

z1=Graphics[Text["c1",{xc1/1.3+0.5,yc1/2},{0,1}]];
z1w=Graphics[Text["w1",{xw1/2,yw1/2},{0,1}]];
z2=Graphics[Text["c2",{xc2/1.3,yc2/1},{0,1}]];
z2w=Graphics[Text["w2",{xw2/1.3,yw2/2},{0,1}]];
z3=Graphics[Text["c3",{xc3/1.5-0.5,yc3/2},{0,1}]];


t1=Show[Graphics[{PointSize[0.01],{kc0,kc1,ku1,kc2,ku2}}]];
R=Graphics[Circle[{0,0},c3]];
Show[f2,l1cc,l1ww,l1uu,l2uu,l2cc,l2ww,l3t,s1,s2,s1w,s2u,s2c,s2w
,st3,z1,z1w,z2,z2w,z3,t1,R,AspectRatio→0.36,AxesLabel→{
"(m/s)","(m/s)"}]
```

Results of velocity triangles designation

alfa1= 90 °
c1= 4.83287 m/s
c1u= 0 m/s
c1a= 4.83287 m/s
u1= 7.95373 m/s
w1= 9.3069 m/s
w1u= 7.95373 m/s
c1a= 4.83287 m/s
c2= 9.3745 m/s
c2u= 9.06822 m/s
c2a= 2.37672 m/s
alfa2= 14.6865 °
w2= 9.87548 m/s
w2u= 9.58522 m/s
w2a= 2.37672 m/s
beta2= 13.926 °
c3= 6.58048 m/s
alfa3= 47.2586 °