Analysis of variability and methodology for determining measurement errors in the example of heat recovery systems in air handling units (AHUs)

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Abstract

In the paper, a description of the analysis of parameter variability in the studied process is presented, with a focus on the selection of measurement instrument class, using the example of a ventilation system where heat recovery takes place. Additionally, a methodology for determining measurement errors in calculating the heat transfer coefficient from the heat balance equations is provided.

Keywords: Measurement Errors; Measurement Methodology; Heat Transfer Coefficient; Ventilation; Air Conditioning

1 Introduction

In measurements of thermal-fluid parameters, after conducting experimental research, the accuracy of the obtained measurements is determined. The class and range of selected instruments have an impact on the quality and accuracy of the measurements. Knowing the instrument's class and its range allows one to determine the limiting values of measurement errors. However, the choice of a particular measurement instrument is often influenced by its price. In cases where the price is the sole determining criterion for instrument purchase, the quality and accuracy of the measurements may not be satisfactory.

During the design phase of a research setup, calculations of thermal-fluid parameters should be performed. At this stage, the required accuracy of measurement equipment relative to the conducted thermal-fluid process and the requirements for the entire measurement setup can also be determined. Such an analysis enables the specification of instrument ranges and classes and can reduce statistical errors that may arise during data analysis of conducted experiments.

In the literature [1–3] the fundamentals of error analysis in measurement techniques used in engineering are presented. According to [2], measurement uncertainty can be expressed using the estimator of the standard deviation for the mean value [1]. In error analysis, it is crucial to maintain the measurement paths in the appropriate condition,

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which can introduce measurement errors [3]. In the field of thermodynamic measurements, there are numerous instances where experimental research is presented with specified measurement errors [4–6] although the methodology for arriving at the values of these errors for complex quantities is not precisely described mathematically. In this paper, an attempt is made to analyze the variability of technological parameters with regard to the precision of measurements of mathematically complex quantities already during the design phase of the research setup. Mathematical relationships necessary for determining the relative error of the heat transfer coefficient values were also specified.

2 Analysis of Variability in Measurement System Parameters and Instrument Class Selection

This chapter discusses the impact of process variability on the feasibility of obtaining accurate results in experimental research, which is contingent on the choice of measurement equipment. In the conducted analysis, it was assumed that the research subject is a ventilation unit with a heat recovery system. The experimental studies are intended to ascertain heat exchange efficiency and heat transfer coefficient values. To select appropriate measurement devices, a theoretical assessment of the influence of parameter variations, such as dry bulb temperature, volumetric flow rate, relative humidity, and static pressure, on the heat recovery system's efficiency in the ventilation unit was conducted. To effectively balance such a heat recovery system, measurements of these four parameters must be made on both the exhaust and supply sides of the ventilation unit (see Fig. 1). These measurements yield parameters such as mass flow rate, enthalpy of humid air, and heat flow. By conducting an initial analysis of parameter variability, it is possible to determine the type of measurement instrument required to ensure that the measurement of heat recovery efficiency in the ventilation unit is carried out with the necessary precision.

Fig. 1 depicts the reference conditions for the analysis conducted on the influence of individual parameters on the accuracy of the obtained efficiency. Tab. 1 displays the efficiency values obtained under the assumption that only the parameters at the outlet of the ventilation unit on the supply side, which are a result of the heat recovery system's operation, undergo measurement changes.

Tab. 1 provides an example of changes in the calculations of energy efficiency due to small variations in key parameters compared to the reference parameters in Fig. 1. At measurement point number 2, changes were assumed in static pressure at a level of 100 Pa, relative humidity at a level of 0.1%, dry bulb temperature at 0.1°C, and volumetric flow rate at 100 m^3/h. From the obtained calculations, it can be inferred that the accuracy of heat recovery efficiency measurement in the ventilation unit at a 1% level can be compromised if instruments with a wide measurement scale and small elementary division are used. This is particularly noticeable for dry bulb temperature and relative humidity. In this case, if high-precision measurements of energy efficiency are required, it would be advisable to select a temperature sensor with an accuracy of 0.01°C and a humidity transducer with high resolution measuring relative humidity with an accuracy of 0.01 [-]. When setting such high expectations for measurement accuracy, it is also essential to pay attention to the quality of the measurement path and its susceptibility to disturbances (e.g., in the case of voltage measurement signals, cables should be shielded). Maintaining the entire measurement path in the proper condition is crucial, not just the measurement transducers, as a voltage drop of 0.1 V along the path can introduce measurement errors,

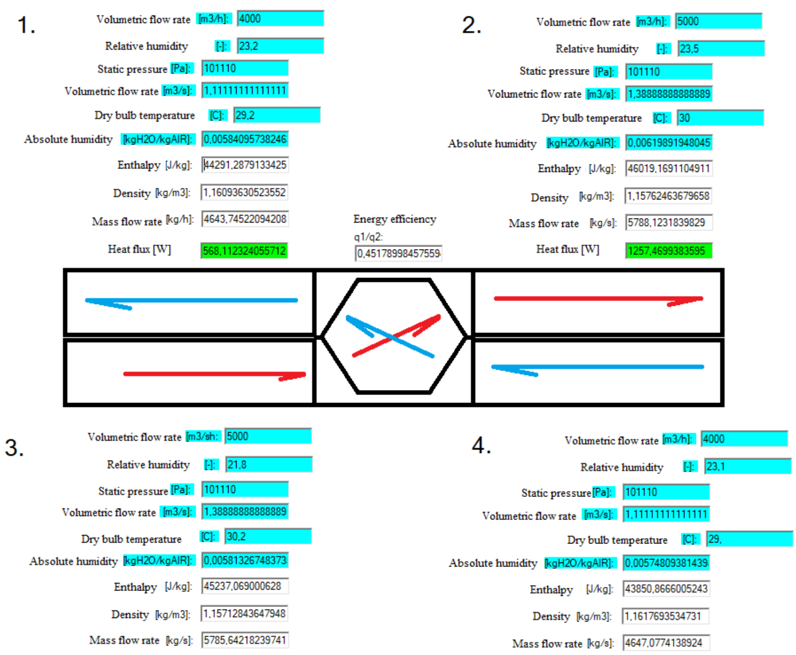

Figure 1: Reference value for impact of variability in AHU efficiency calculations.

which depending on the sensor can significantly disrupt the measurement.

In the context of thermal energy, it is also crucial to ensure high measurement repeatability and precise calibration of individual sensors under uniform test conditions on the ventilation plant.

3 Methodology of Measurement Error Calculations Using the Example of Heat Transfer Coefficient Determination

In this chapter, a methodology for determining measurement errors of the heat transfer coefficient was presented for a ventilation unit operating in conjunction with a heat pump in an intermediate system. The subject of the analysis was the condenser of the heat pump, which served as a heat supply unit for the air supplied from the heat recovery system of the exhaust air. The results of absolute error measurements were presented with respect to the determined value of the heat transfer coefficient and the logarithmic temperature difference. The condensation process involved a mixture of R404A, while water was utilized in the intermediate circuit.

An analysis of errors in the conducted measurements was performed using:

- mathematical statistics methods based on the results of measurement series. This uncertainty is quantified using the standard deviation estimator for the mean, which serves as a numerical measure of the uncertainty referred to as standard uncertainty. When having a series of measurement results, point estimation is used to determine:

Table 1: Analysis of the Impact of Variability in Measured Parameters on the Accuracy of Ventilation Unit Efficiency Measurement

	Reference Values	**Change in Static Pressure**	**Change in Dry Bulb Temperature**	**Change in Volumetric Flow Rate**	**Change in Relative Humidity**
Volumetric Flow Rate [m^3/h]	5000	5000	5000	5100	5000
Relative Humidity [-]	23.5	23.5	23.5	23.5	23.6
Static Pressure [Pa]	101110	101210	101110	101110	101110
Dry Bulb Temperature [°C]	30	30	30.1	30	30
Enthalpy of Humid Air [J/kg]	46019	46003	46212	46019	46087
Density of Humid Air [kg/m^3]	1.157	1.158	1.157	1.157	1.157
Mass Flow Rate [kg/s]	5788	5793	5786	5903	5788
Heat Flow Rate [W]	1257	1233	1568	1282	1366
Efficiency	46%	46%	36%	44%	41%

– the mean value:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \tag{1}$$

– the standard uncertainty:

$$u_A = \bar{S}_{\bar{x}} = \sqrt{\frac{1}{n\left(n-1\right)}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \tag{2}$$

- data originating from measurement instruments, with the consideration of all available information about factors that might impact the uncertainty of measurement results, along with one's own knowledge and skills acquired through practical measurement experience. When evaluating such uncertainty, only a single measurement value is employed. This value is treated as an estimator of the expected value, with

– the standard uncertainty:

$$u_B = \frac{\Delta_g}{\sqrt{3}}, \tag{3}$$

where Δ_g – the limit value of the error defined by the class indicator, most commonly expressed by the equation:

$$\Delta_g = \frac{Z \cdot K}{100}. \tag{4}$$

In intermediate measurements used to assess the total uncertainty, both of the standard uncertainties mentioned above are employed by computing the combined standard uncertainty:

$$u_ł = \sqrt{u_A^2 + u_B^2}. \tag{5}$$

The standard uncertainty for the mean value is then expressed as follows:

$$u_{\bar{y}} = \sqrt{\sum_{j=1}^{N} \left(\frac{\partial y}{\partial x_j} \cdot u_{ł_j} \right)^2}. \tag{6}$$

Below, the methodology for determining errors in measuring the heat transfer coefficient is presented. In the analysis, the overall uncertainties of the primary quantities utilized in the measurement were taken into account at the following magnitudes:

- Δm$_{H_2O}$=0.008872 kg/s – error in mass flow rate measurement,
- Δt=0.05°C – error in temperature measurement,
- Δl=0.00002 m – error in the measurement of geometric dimensions of the heat exchanger,
- $\Delta\alpha_{H_2O} = \sqrt{(0.05 \cdot \alpha_{H_2O})^2 + \left(\frac{0.05 \cdot \alpha_{H_2O}}{\sqrt{3}}\right)^2}$ – error in the measurement of the heat transfer coefficient on the water side,
- Δd$_{pz}$=0.00002 m – error in the inner diameter measurement,
- Δd$_{pw}$=0.00002 m – error in the outer diameter measurement.

Subsequently, the error in the heat transfer coefficient of the condensation process, which is described by the following equation

$$\alpha_x = \frac{1}{\frac{1}{k_x} - \frac{\frac{d_{pw}}{2}}{\lambda_{cu}} \ln \frac{\frac{d_{pz}}{2}}{\frac{d_{pw}}{2}} - \frac{\frac{d_{pw}}{2}}{\alpha_{H_2O} \cdot \frac{d_{pz}}{2}}}, \tag{7}$$

was determined as

$$\Delta\alpha_x = \sqrt{\begin{aligned} &\left(\frac{\partial \alpha_x}{\partial k_x} \Delta k_x\right)^2 + \left(\frac{\partial \alpha_x}{\partial \lambda_{cu}} \Delta \lambda_{cu}\right)^2 + \left(\frac{\partial \alpha_x}{\partial d_{pw}} \Delta d_{pw}\right)^2 + \\ &\qquad\qquad \left(\frac{\partial \alpha_x}{\partial d_{pz}} \Delta d_{pz}\right)^2 + \left(\frac{\partial \alpha_x}{\partial \alpha_{H_2O}} \Delta \alpha_{H_2O}\right)^2 \end{aligned}}. \tag{8}$$

The remaining relationships arise from the law of uncertainty propagation (based on first-order derivatives of the Taylor series) [1]

$$\frac{\partial \alpha_x}{\partial k_x} = \frac{1}{\left(\frac{1}{k_x} - \frac{d_{pw} \ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2\lambda_{cu}} - \frac{d_{pw}}{\alpha_{H_2O} \cdot d_{pz}}\right)^2 \cdot k_z^2}, \tag{9}$$

$$\frac{\partial \alpha_x}{\partial d_{pw}} = -\frac{-\frac{\ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2\lambda_{cu}} + \frac{1}{2\lambda_{cu}} - \frac{1}{\alpha_{H_2O} \cdot d_{pz}}}{\left(\frac{1}{k_x} - \frac{d_{pw} \ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2\lambda_{cu}} - \frac{d_{pw}}{\alpha_{H_2O} \cdot d_{pz}}\right)^2}, \tag{10}$$

$$\frac{\partial \alpha_x}{\partial \lambda_{cu}} = -\frac{d_{pw} \ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2 \cdot \left(\frac{1}{k_x} - \frac{d_{pw} \ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2\lambda_{cu}} - \frac{d_{pw}}{\alpha_{H_2O} \cdot d_{pz}}\right)^2 \cdot \lambda_{cu^2}}, \tag{11}$$

$$\frac{\partial \alpha_x}{\partial d_{pz}} = -\frac{-\frac{d_{pw}}{2\lambda_{cu} \cdot d_{pz}} + \frac{d_{pw}}{\alpha_{H_2O} \cdot d_{pz}^2}}{\left(\frac{1}{k_x} - \frac{d_{pw} \ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2\lambda_{cu}} - \frac{d_{pw}}{\alpha_{H_2O} \cdot d_{pz}}\right)^2}, \tag{12}$$

$$\frac{\partial \alpha_x}{\partial \alpha_{H_2O}} = -\frac{d_{pw}}{\left(\frac{1}{k_x} - \frac{d_{pw} \ln\left(\frac{d_{pz}}{d_{pw}}\right)}{2\lambda_{cu}} - \frac{d_{pw}}{\alpha_{H_2O} \cdot d_{pz}}\right)^2 \cdot \alpha_{H_2O}^2 \cdot d_{pz}}. \tag{13}$$

To determine the error of the composite variable k_x

$$k_x = \frac{Q_k}{A_x \Delta t_{\log}}, \tag{14}$$

partial derivatives were calculated for the relationship:

$$\Delta k_x = \sqrt{\left(\frac{\partial k_x}{\partial Q_k} \cdot \Delta Q_k\right)^2 + \left(\frac{\partial k_x}{\partial t_{\log}} \cdot \Delta t_{\log}\right)^2 + \left(\frac{\partial k_x}{\partial A_x} \cdot \Delta A_x\right)^2}, \tag{15}$$

where

$$\frac{\partial k_x}{\partial A_x} = -\frac{Q_k}{A_x^2 \cdot \Delta t_{\log}}, \tag{16}$$

$$\frac{\partial k_x}{\partial Q_k} = \frac{1}{A_x \cdot \Delta t_{\log}}, \tag{17}$$

$$\frac{\partial k_x}{\partial \Delta t_{\log}} = -\frac{Q_k}{A_x \cdot \Delta t_{\log}^2}, \tag{18}$$

The error in the determination of the heat exchange surface area:

$$A_x = n \cdot l \cdot d \cdot \pi \tag{19}$$

can be found as

$$\Delta A_x = \sqrt{\left(\frac{\partial A_x}{\partial n} \Delta n\right)^2 + \left(\frac{\partial A_x}{\partial l} \Delta l\right)^2 + \left(\frac{\partial A_x}{\partial d} \Delta d\right)^2}, \tag{20}$$

where

$$\frac{\partial A_x}{\partial n} = l \cdot d \cdot \pi, \tag{21}$$

$$\frac{\partial A_x}{\partial l} = n \cdot d \cdot \pi, \tag{22}$$

$$\frac{\partial A_x}{\partial d} = l \cdot n \cdot \pi. \tag{23}$$

The error in the determination of the logarithmic temperature difference Δ t_{log}:

$$\Delta t_{\log} = \frac{t_1 - t_2}{\ln \frac{t_1}{t_2}} \tag{24}$$

can be found as

$$\Delta\left(\mathit{\Delta} t_{\log}\right) = \sqrt{\left(\frac{\partial\left(\mathit{\Delta} t_{\log}\right)}{\partial t_1}\mathit{\Delta} t_1\right)^2 + \left(\frac{\partial\left(\mathit{\Delta} t_{\log}\right)}{\partial t_2}\mathit{\Delta} t_2\right)^2}, \tag{25}$$

where

$$\frac{\partial\left(\Delta t_{\log}\right)}{\partial t_1} = \frac{1}{\ln \frac{t_1}{t_2}} - \frac{t_1 - t_2}{\ln\left(\frac{t_1}{t_2}\right)^2 t_1}, \tag{26}$$

$$\frac{\partial\left(\Delta t_{\log}\right)}{\partial t_2} = \frac{-1}{\ln \frac{t_1}{t_2}} + \frac{t_1 - t_2}{\ln\left(\frac{t_1}{t_2}\right)^2 t_2}, \tag{27}$$

$$t_1 = t^{wl}_{R404A} - t^{wyl}_{H_2O}\,, \tag{28}$$

$$\mathit{\Delta} t_1 = \sqrt{\left(\frac{\partial t_1}{\partial t^{wl}_{R404A}} \cdot \mathit{\Delta} t^{wl}_{R404A}\right)^2 + \left(\frac{\partial t_1}{\partial t^{wyl}_{H_2O}} \cdot \mathit{\Delta} t^{wyl}_{H_2O}\right)^2}, \tag{29}$$

$$t_2 = t^{wyl}_{R404A} - t^{wl}_{H_2O}\,, \tag{30}$$

$$\mathit{\Delta} t_2 = \sqrt{\left(\frac{\partial t_2}{\partial t^{wyl}_{R404A}} \cdot \mathit{\Delta} t^{wyl}_{R404A}\right)^2 + \left(\frac{\partial t_2}{\partial t^{wl}_{H_2O}} \cdot \mathit{\Delta} t^{wl}_{H_2O}\right)^2}. \tag{31}$$

The heat transfer rate error for the condenser:

$$Q_k = m_{H_2O} \cdot \left(c^{wl}_p \cdot t^{wl}_{H_2O} - c^{wyl}_p \cdot t^{wyl}_{H_2O}\right), \tag{32}$$

can be calculated as

$$\Delta Q_k = \sqrt{\begin{aligned}&\left(\frac{\partial Q_k}{\partial m_{H_2O}}\mathit{\Delta} m_{H_2O}\right)^2 + \left(\frac{\partial Q_k}{\partial c^{wl}_p}\mathit{\Delta} c^{wl}_p\right)^2 + \left(\frac{\partial Q_k}{\partial t^{wl}_{H_2O}}\mathit{\Delta} t^{wl}_{H_2O}\right)^2 \\ &\qquad + \left(\frac{\partial Q_k}{\partial c^{wyl}_p}\mathit{\Delta} c^{wyl}_p\right)^2 + \left(\frac{\partial Q_k}{\partial t^{wyl}_{H_2O}}\mathit{\Delta} t^{wyl}_{H_2O}\right)^2\end{aligned}}, \tag{33}$$

where

$$\frac{\partial Q_k}{\partial m_{H_2O}} = c^{wl}_p \cdot t^{wl}_{H_2O} - c^{wyl}_p \cdot t^{wyl}_{H_2O}\,, \tag{34}$$

$$\frac{\partial Q_k}{\partial c^{wl}_p} = m_{H_2O} \cdot t^{wl}_{H_2O} \tag{35}$$

$$\frac{\partial Q_k}{\partial t_{H_2O}^{wl}} = m_{H_2O} \cdot c_p^{wl}, \tag{36}$$

$$\frac{\partial Q_k}{\partial c_p^{wyl}} = -m_{H_2O} \cdot t_{H_2O}^{wyl}, \tag{37}$$

$$\frac{\partial Q_k}{\partial t_{H_2O}^{wyl}} = m_{H_2O} \cdot c_p^{wyl}. \tag{38}$$

Below in Tab. 2 and Tab. 3 maximum errors determined by the direct and indirect methods are presented, respectively.

Table 2: Summary of Maximum Measurement Errors

Directly Measured Quantities	**Total Standard Uncertainty of Measurement**
Mass Flow Rate $\dot{m}$	$\Delta\dot{m} = \sqrt{0.008^2 \cdot \left(\frac{0.05\cdot\dot{m}}{\sqrt{3}}\right)^2}\ \left[\frac{kg}{s}\right]$
Temperature T	$\Delta T = \sqrt{0.05^2 \cdot \left(\frac{0.05}{\sqrt{3}}\right)^2} = 0.0577°\mathrm{C}$
Inner Pipe Diameter d_{pw}	$\Delta d_{pw} = 0.00002$ m
Outer Pipe Diameter d_{pz}	$\Delta d_{pz} = 0.00002$ m
Heat Transfer Coefficient for Water	$\Delta\alpha_{H_2O} = \sqrt{(0.05 \cdot \alpha_{H_2O})^2 + \left(\frac{0.05\cdot\alpha_{H_2O}}{\sqrt{3}}\right)^2}\ \left[\frac{W}{m^2K}\right]$

Table 3: Summary of Maximum Measurement Errors

Indirectly Measured Quantities	**Maximum Calculated Relative Error**
Heat Transfer Coefficient α_x	$\delta\alpha_x = 0.13$
Overall Heat Transfer Rate k_x	$\delta k_x = 0.07$
Heat flux Q_k	$\delta Q_k = 0.06$

The results of the measurements of the average heat transfer coefficient $\bar{\alpha}$ in the actual condensation zone are presented in Tab. 4, which is an excerpt from the protocol of experimental studies. The maximum deviation of the obtained results for the heat transfer coefficient from the regression line is 29%. The measurement accuracy obtained from experimental studies, which is shown in Fig. 2 and Fig. 3, is related to the measuring apparatus used and the general methodology for determining measurement errors of physically complex quantities and directly results from the class and range of the instruments used and is consistent with the methodology presented in the paper [1].

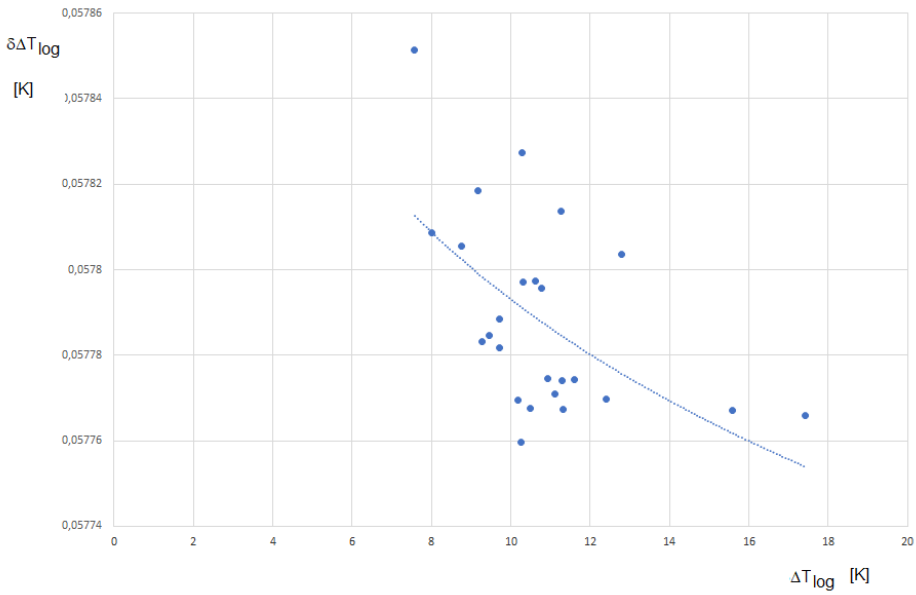

Figure 2: Absolute Measurement Errors for the Measured Logarithmic Temperature Difference.

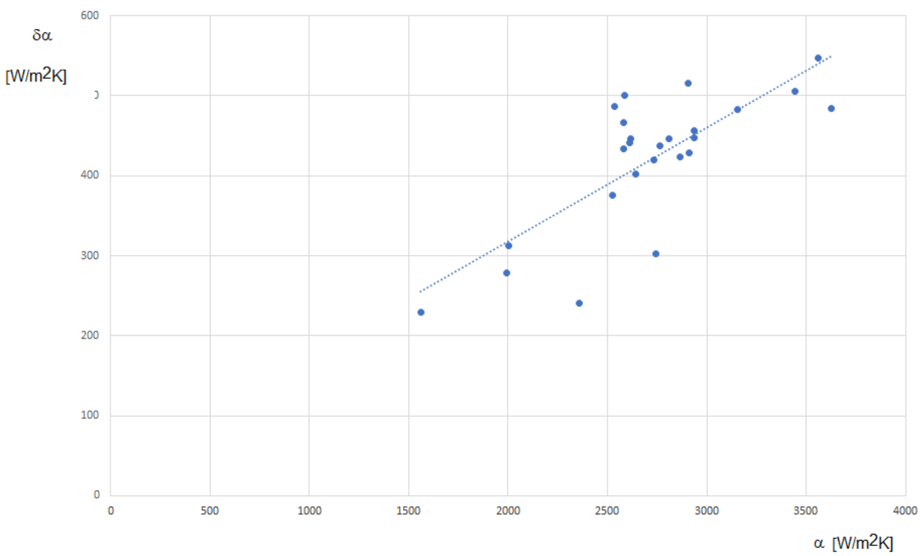

Figure 3: Absolute Measurement Errors for the Measured Heat Transfer Coefficient.

Table 4: Measurement Errors

Measurement number	$\dot{q}$ $\left[\frac{W}{m^2}\right]$	$\mathbf{w}\rho$ $\left[\frac{kg}{m^2 s}\right]$	t_s $[^\circ C]$	p_s $[MPa]$	$\bar{\alpha}_{exp}$ $\left[\frac{W}{m^2 K}\right]$	α_{reg} $\left[\frac{W}{m^2 K}\right]$	$\frac{(\bar{\alpha}_{exp}-\alpha_{reg})}{\bar{\alpha}_{exp}}$ $[-]$
1	10357	154	26.1	1.31	2074	1983	0.04
2	10436	156	25.6	1.26	2115	1957	0.07
3	14513	161	29.1	1.38	2145	1703	0.21
4	12175	211	27.0	1.35	2652	2007	0.24
5	20367	366	29.5	1.40	4134	2933	0.29
6	18259	418	32.0	1.45	3943	2898	0.26
7	22301	209	35.5	1.65	2198	1722	0.22
8	15862	209	31.0	1.51	2202	1798	0.18
9	12822	198	29.8	1.40	2168	1647	0.24
10	10479	209	28.5	1.34	2287	1880	0.18
11	11870	111	34.6	1.50	1362	1408	-0.03
12	9487	104	29.1	1.40	1295	1378	-0.06
13	12272	215	29.5	1.41	2101	1493	0.29
14	9645	209	28.2	1.35	2073	1540	0.26
15	8717	209	27.0	1.31	2095	1613	0.23
16	13158	115	31.9	1.49	1249	1400	-0.12
17	11683	104	29.6	1.40	1188	1229	-0.03
18	8098	104	26.7	1.31	1196	1252	-0.05
19	7041	104	25.3	1.27	1201	1477	-0.23

where:
$\dot{q}$ – heat flux density given off in the condensation process,
wρ – density of the mass flow rate of the refrigerant,
t_s – condensation temperature of the refrigerant
p_s – condensation pressure of the refrigerant,
$\bar{\alpha}_{exp}$ – average value of the heat transfer coefficient determined based on the experiment,
α_{reg} – average value of the heat transfer coefficient determined from trend line.

From the calculations, the maximum relative error in determining the heat conduction coefficient was obtained δk=7%. Tab. 2 and Tab. 3 present the values of errors obtained through direct and indirect measurement methods, respectively. The maximum relative error in determining the heat transfer coefficient for water was calculated to be 13%. This level of accuracy in determining the heat transfer coefficient for the condensation process was adopted based on the maximum error value obtained from the calculations. In Figs 2 and 3, you can observe the acquired values of errors in logarithmic temperature and heat transfer coefficient (which was presented in Fig. 4). The spread of measurement error results around the trendline is due to the fact that both the logarithmic temperature and heat transfer coefficient are dependent on other measured parameters, which were linked to the experimental research conducted.

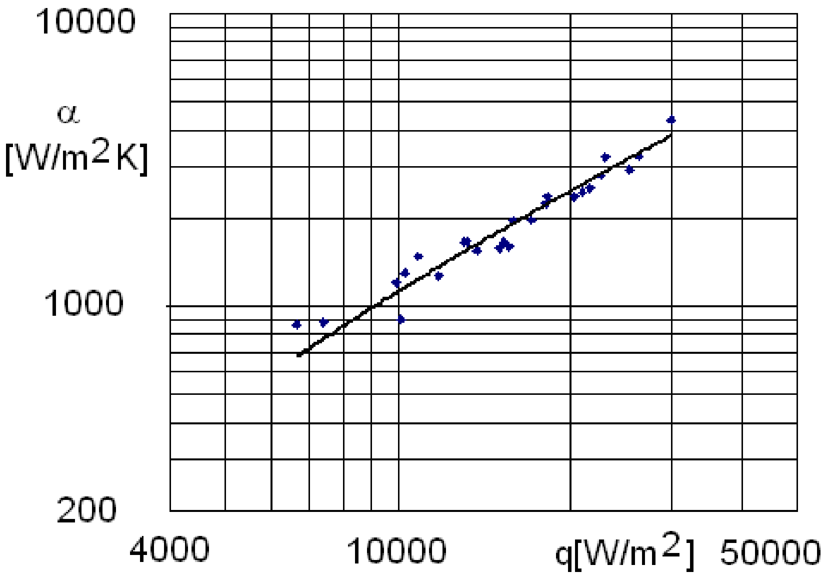

Figure 4: Results of experimental studies of the average value of the heat transfer coefficient.

4 Summary

The provided material presents an analysis of parameter variability in the examined process, taking into account the selection of the measuring instrument class. Additionally, it discusses the methodology for determining measurement errors in thermal-flow measurements practice. The introduction to the work underscores the significance of precise selection of measuring instruments when conducting thermal-flow measurements. An example of a ventilation system with heat recovery is presented as the research object, where measurement accuracy had an impact on determining heat recovery efficiency. While analyzing the variability of parameters such as temperature, flow rate, humidity, and pressure, the authors emphasized the necessity of choosing appropriate class measuring instruments. During the analysis of measurement errors, mathematical statistics methods and information from measuring instruments were utilized. Calculations of errors for the local heat transfer coefficient were also presented as an example. It was pointed out that maintaining measurement paths in the appropriate condition is crucial, as well as ensuring measurement repeatability and precise sensor calibration. In conclusion, the analysis of parameter variability and the precision of measuring instruments are crucial for obtaining reliable results in thermal-flow measurements. Simultaneously, the methodology for assessing measurement errors plays a significant role in ensuring measurement accuracy in practice.

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