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MULTIVALUED SOLUTIONS FOR SHALLOW SHELLS

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1. Introduction

The displacement vector **v** of the middle surface of a shell and the stress function vector **v*** associated with **v** through the static-geometric analogy, are the general solutions of the set of equations of the linear theory of shells [1, 2, 3 and 4].

For shells with simply connected region which are loaded with surface loading, these solutions are obtained with accuracy up to the terms of "displacement of the shell as a rigid body in space". These terms do not, however, affect the uniqueness of deformations and internal forces in the shell.

For shells with multiply connected region (the points of application of the concentrated loadings may also be regarded as internal boundary contours [5, 6]), the vectors **v** and **v*** can have additionally other multivalued terms different from the terms of the type of "displacement of the shell as a rigid body", to which there also correspond unique deformations and internal forces. The investigation, therefore of the multivaluedness of solution is here the essential part of the process of construction of that solution.

The basic relations for multivalued solutions of the linear theory of shells were given by Černykh [7]. Unlike Lurie [1], he has presented the expressions for **v** and **v*** along an arbitrary curve Γ on the middle surface of the shell in the form of a single integral formula (cf. also [9, 10]). In the paper [7] the terms multivalued with respect to the passage round the internal closed boundary contour have been separated from the complete solution and expressed by two vectorial, so-called, dislocation parameters(1) (6 scalar components) and by the multivalued scalar function $\Phi(\theta^{\alpha})$.

The general considerations contained in [6, 7, 9, 10] are of qualitative character only, because they make it possible only to state when the multivaluedness of the solutions **v** and **v*** exists. But the problem of how to construct the multivalued term of solutions [leading to construction of the function $\Phi(\theta^{\alpha})$] has not, so far, been sufficiently investigated.

The object of the present work is to investigate the multivaluedness and to indicate a manner for construction of the multivalued term of the solutions **v** and **v*** for the class of shallow shells. In the present author's works [11, 12] is given a variant of the linear theory of shallow shells, in which all vector relations are presented consequently through

(1) The multivalued displacements corresponding to unique deformations are called by some authors "distortion".

the components on the basis of the fundamental vectors of the shell projection into the reference plane Π (cf. [13]). The essential point of this conception is that the basic vector $\mathbf{e}_3$ is perpendicular to the plane Π, and the components of vectors $\mathbf{v}$ and $\mathbf{v}^*$ in the $\mathbf{e}_3$ direction have constant direction which is independent of the shallowness function $z(\theta^\alpha)$. These components are the fundamental unknowns of the so-called mixed method of solution for the set of equations of shallow shell [11, 12].

The simple geometric transformations presented in this work make it possible to show that, within the framework of simplifying assumptions of the linear theory of shallow shells [11, 13], the multivalued part of the components of $\mathbf{v}$ and $\mathbf{v}^*$ in the $\mathbf{e}_3$ direction depends only on three (from among six) dislocation parameters. It is also shown that the form of the multivalued term for these components does not depend on the function of shell shallowness, and this term may be constructed similarly as for plates or for the plane stress state of a disk with the boundary contour of the shell projection into the plane Π and with an appropriate loading [14]. These results have been compared with the known solutions for the shallow shells of revolution characterized by the two-connected region, which were obtained by REISSNER [15], LARDNER and SIMMOND [16] and the present author [8, 17]. In the works cited above, the possibility of appearance of multivalued stress functions in the solution was taken into account.

2. Geometric Relations

The fundamental denotations used here agree with those introduced by GREEN and ZERNA [13] and used in [11, 12, 6]. The following geometric quantities are associated with the surface M (Fig. 1), (cf. [6]): $\bar{\theta}^\alpha$, $\bar{\mathbf{r}}(\bar{\theta}^\alpha)$, $\bar{\mathbf{r}}^k$, $\mathbf{a}_\alpha$, $\mathbf{a}_3$, $a_{\alpha\beta}$, $b_{\alpha\beta}$, $\bar{\epsilon}_{\alpha\beta}$, $(\ )_{\|\alpha}$, $\bar{\Gamma}^\alpha_{\beta\gamma}$, $\bar{\Gamma}_i$, $\bar{\Gamma}$, $\bar{s}$, $\bar{\mathbf{t}}$, $\bar{\mathbf{v}}$, etc. With the plane Π, the following quantities are associated correspondingly: θ^α, $\mathbf{r}(\theta^\alpha)$, $\mathbf{r}^k$, $\mathbf{e}_\alpha$, $\mathbf{e}_3$, $e_{\alpha\beta}$, $\epsilon_{\alpha\beta}$, $(\ )_{|\alpha}$, $\Gamma^\alpha_{\beta\gamma}$, Γ_i, Γ, s, $\mathbf{t}$, $\mathbf{v}$, etc. If we assume the systems of coordinates in such a way that for the corresponding points $\bar{\theta}^\alpha = \theta^\alpha$, the vector $\bar{\mathbf{r}}$ can be presented in the form

$$\bar{\mathbf{r}}(\bar{\theta}^\alpha) \equiv \bar{\mathbf{r}}(\theta^\alpha) = \mathbf{r}(\theta^\alpha) + z(\theta^\alpha)\cdot \mathbf{e}_3 . \tag{2.1}$$

The approximate relations among the geometric quantities associated with M and Π have the form [11, 13]

$$\begin{aligned}
&a_{\alpha\beta} \approx e_{\alpha\beta}, \quad a^{\alpha\beta} \approx e^{\alpha\beta}, \quad b_{\alpha\beta} \approx z|_{\alpha\beta},\\
&\bar{\Gamma}^\alpha_{\beta\gamma} \approx \Gamma^\alpha_{\beta\gamma}, \quad \bar{\epsilon}_{\alpha\beta} \approx \epsilon_{\alpha\beta},\\
&\mathbf{a}_\alpha \approx \mathbf{e}_\alpha + z_{,\alpha}\mathbf{e}_3 \quad \mathbf{a}_3 \approx \mathbf{e}_3 - z|^\alpha \mathbf{e}_\alpha,\\
&\bar{\mathbf{t}} \approx \mathbf{t} + z_{,\alpha} t^\alpha \cdot \mathbf{e}_3, \quad \bar{\mathbf{v}} \approx \mathbf{v} + z_{,\alpha}\nu^\alpha \cdot \mathbf{e}_3.\\
&\bar{t}^\alpha \approx t^\alpha, \quad \bar{\nu}^\alpha \approx \nu^\alpha.
\end{aligned} \tag{2.2}$$

Let us assume that the origin of system 0 lies on the plane Π (Fig. 1). For the spatial point i, which is associated with the internal closed boundary contour $\bar{\Gamma}_i$ [6], we obtain

$$\bar{\mathbf{r}}-\mathbf{R}^i = \boldsymbol{\rho}^i+(z-z^i)\mathbf{e}_3\,,\tag{2.3}$$
$$\boldsymbol{\rho}^i = \mathbf{r}-\mathbf{r}^i, \qquad z^i = \mathbf{R}^i\cdot\mathbf{e}_3;$$

and for an arbitrary point k on the surface M

$$\bar{\mathbf{r}}-\mathbf{r}^k = \boldsymbol{\rho}^k+[z-z(\theta^\alpha_k)]\mathbf{e}_3.\tag{2.4}$$

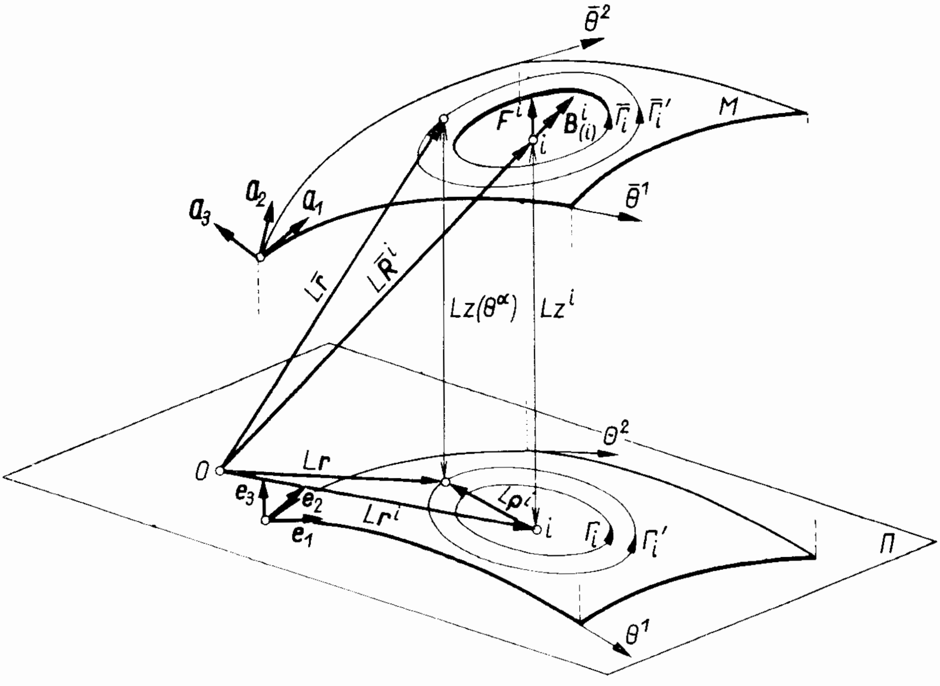

FIG. 1.

3. Multivaluedness of Displacements

The multivalued term of the displacement vector $\mathbf{v}$, after passing through the internal closed boundary contour $\bar{\Gamma}_i$ (Fig. 1) [7, 9, 10, 6], has the form

$$\overset{d_i}{\mathbf{v}} = [\mathbf{v}^i+\boldsymbol{\Omega}^i\times(\bar{\mathbf{r}}-\mathbf{R}^i)]\cdot\Phi_i(\bar{\theta}^\alpha)\,,\tag{3.1}$$

where $\Phi_i(\bar{\theta}^\alpha)$ is a scalar multivalued function of the contour $\bar{\Gamma}_i$, which after passing through the contour $\bar{\Gamma}_i$ causes a unit increment, but its derivatives in all directions are the unique functions; $\mathbf{v}^i$, $\boldsymbol{\Omega}^i$ are the so-called displacement dislocation parameters of the contour $\bar{\Gamma}_i$, which are expressed through the vector $\boldsymbol{\varkappa}_t$ of the contour curvature variation during deformation by the relations [7, 9]

$$\mathbf{v}^i = \oint_{\bar{\Gamma}_i}[(\bar{\mathbf{r}}-\mathbf{R}^i)\times\boldsymbol{\varkappa}_t+(a_{\alpha\beta}\bar{t}^\alpha\bar{t}^\beta)\bar{\mathbf{t}}]\,d\bar{s}\,,\tag{3.2}$$
$$\boldsymbol{\Omega}^i = \oint_{\bar{\Gamma}_i}\boldsymbol{\varkappa}_t\,d\bar{s}\,.$$

The displacement vector $\mathbf{v}$ and the rotation vector $\mathbf{\Omega}$ can be expressed by means of their components in the basis $\mathbf{a}_i$ and $\mathbf{e}_i$ [11, 12].

$$\mathbf{v} = v_\alpha \mathbf{a}^\alpha + v_3 \mathbf{a}^3 = u_\alpha \mathbf{e}^\alpha + u_3 \mathbf{e}^3 ,$$
(3.3)
$$\mathbf{\Omega} = \Omega_\alpha \mathbf{a}^\alpha + \Omega_3 \mathbf{a}^3 = \omega_\alpha \mathbf{e}^\alpha + \omega_3 \mathbf{e}^3 ,$$

where with accuracy to the simplifying assumptions for the shallow shells [11, 13], we have

$$v_\alpha \approx u_\alpha + z_{,\alpha} u_3 , \qquad v_3 \approx u_3 ,$$
(3.4)
$$\Omega_\alpha \approx \omega_\alpha ; \qquad \Omega_3 \approx \omega_3 - \omega_\alpha z|^\alpha .$$

Presenting the vectors $\overset{d}{\mathbf{v}}{}^i$, $\mathbf{v}^i$ and $\mathbf{\Omega}^i$ in (3.1) analogously to (3.3), and taking into account the geometric (2.2), (2.3), (2.4) and physical (3.4) relations, after transformations, we obtain

$$\overset{d}{u}{}^i_\alpha = \{u^i_\alpha + \epsilon_{\beta\alpha}[\varrho^{i\beta}\omega^i_3 - (z - z^i)\omega^{i\beta}]\} \Phi_i(\theta^\alpha),$$
(3.5)
$$\overset{d}{u}{}^i_3 = \{u^i_3 + \epsilon^{\alpha\beta}\omega^i_\alpha \varrho^i_\beta\} \Phi_i(\theta^\alpha) .$$

The component $\overset{d}{u}{}^i_3$ then depends only on the three components of the dislocation parameters u^i_3 and ω^i_α. Within the framework of simplifying assumptions of the linear theory of shallow shells [11, 13], after taking into account (2.2) and (3.4), we obtain from (3.2)

$$u^i_3 \approx \oint_{\Gamma_i} \epsilon_{\alpha\beta}\varrho^{i\alpha}\varkappa^\beta_t \, ds , \qquad \omega^i_\alpha = \mathbf{e}_\alpha \cdot \mathbf{\Omega}^i \approx \mathbf{e}_\alpha \cdot \oint_{\Gamma_i} \varkappa^\beta_t \mathbf{e}_\beta \, ds , \tag{3.6}$$

where

$$\varkappa^\beta_t = \mathbf{e}^\beta \cdot \boldsymbol{\varkappa}_t \approx -u_{3|\varrho\lambda} t^{\varrho} \cdot \delta^{\lambda\beta}_{\sigma\tau} \nu^\sigma t^\tau . \tag{3.7}$$

The shell shallowness function $z(\theta^\alpha)$ is not involved in the relations $(3.5)_2$, (3.6) and (3.7).

4. Multivaluedness of Stress Functions

The multivalued term of the stress function vector $\mathbf{v}^*$, after passing through the internal closed boundary contour $\bar{\Gamma}_i$, can be presented, in agreement with static-geometric analogy, in the form analogous to (3.1) [7, 6]

$$\overset{d}{\mathbf{v}}{}^{*i} = [\mathbf{v}^{*i} + \mathbf{\Omega}^{*i} \times (\bar{\mathbf{r}} - \mathbf{R}^i)]\Phi_i(\bar{\theta}^\alpha) , \tag{4.1}$$

where $\Phi_i(\bar{\theta}^\alpha)$ is a scalar multivalued function as that in Sec. 3; $\mathbf{v}^{*i}$, $\mathbf{\Omega}^{*i}$ are the stress function dislocation parameters of the contour $\bar{\Gamma}_i$.

The dislocation parameters $\mathbf{v}^{*i}$ and $\mathbf{\Omega}^{*i}$, after passing through the internal closed boundary contour $\bar{\Gamma}_i$, can be expressed by the following relations [6]

$$\mathbf{\Omega}^{*i} = -\frac{1}{kL}\left[\mathbf{F}^i - \overset{s}{\mathbf{F}}{}^i - L \iint_{S_{\bar{\Gamma}_i}} \mathbf{p}_i \, d\bar{S}\right] ,$$
(4.2)
$$\mathbf{v}^{*i} = -\frac{1}{kL^2}\left[\mathbf{B}^i_{(i)} - \overset{s}{\mathbf{B}}{}^i_{(i)} - L^2 \iint_{S_{\bar{\Gamma}_i}} (\bar{\mathbf{r}} - \mathbf{R}^i) \times \mathbf{p}_i \, d\bar{S}\right] ,$$

where $\mathbf{F}^i$, $\mathbf{B}^i_{(i)}$ are the total force and the total couple vectors of the boundary loadings on the contour $\overline{\Gamma}_i$ with respect to the point i; $\overset{s}{\mathbf{F}}{}^i$, $\overset{s}{\mathbf{B}}{}^i_{(i)}$ are the total force and the total couple of the internal forces on the contour $\overline{\Gamma}_i$ with respect to the same point i, which are contributed by particular integrals of the equilibrium equations for the simply connected region $S_{\overline{\Gamma}}+\Sigma S_{\overline{\Gamma}_i}$ of the shell, [6]; $\frac{1}{L}\,\mathbf{p}_i$ is a fictitious surface loading assumed for the extended region $S_{\overline{\Gamma}_i}$ of the shell when the particular integral was being determined, [6].

When the contour $\overline{\Gamma}_i$ contains in its interior the point k at which the concentrated loading with the force vector $\mathbf{P}^k$ and couple vector $\mathbf{M}^k$ is applied, the parameters $\mathbf{v}^{*k}$ and $\mathbf{\Omega}^{*k}$ have the form [6]

$$
\begin{aligned}
\mathbf{\Omega}^{*k} &= -\frac{1}{kL}\,[\mathbf{P}^k-\overset{s}{\mathbf{P}}{}^k],\\
\mathbf{v}^{*k} &= -\frac{1}{kL^2}\,[\mathbf{M}^k-\overset{s}{\mathbf{M}}{}^k],
\end{aligned}
\tag{4.3}
$$

where $\overset{s}{\mathbf{P}}{}^k$, $\overset{s}{\mathbf{M}}{}^k$ are the total force and the total couple of the internal forces on the contour $\overline{\Gamma}_i$ with respect to the point i, which are contributed by the particular integrals of the equilibrium equations.

The relations analogous to (3.3) and (3.4) hold also for $\mathbf{v}^*$ and $\mathbf{\Omega}^*$. For the components of the vector $\overset{d}{\mathbf{v}}{}^{*i}$ in the basis $\mathbf{e}_i$, the relations analogous to (3.5) (cf. [11, 12]) have the form

$$
\begin{aligned}
\overset{d}{\varphi}{}^i_\alpha &\equiv \overset{d}{u}{}^{*i}_\alpha = \{u^{*i}_\alpha+\epsilon_{\beta\alpha}[\varrho^{i\beta}\omega^{*i}_3-(z-z^i)\omega^{*i\beta}]\}\Phi_i(\theta^\alpha),\\
\overset{d}{\varphi}{}^i_3 &\equiv \overset{d}{u}{}^{*i}_3 = \{u^{*i}_3+\epsilon^{\alpha\beta}\omega^{*i}_\alpha\varrho^i_\beta\}\Phi_i(\theta^\alpha).
\end{aligned}
\tag{4.4}
$$

In this case the component $\overset{d}{\varphi}{}^i_3$ depends also only on the three components of the dislocation parameters u^{*i}_3 and ω^{*i}_α. These components can be obtained by writing the components of, e.g., the relation (4.2) in the basis $\mathbf{e}_i$

$$
\begin{aligned}
u^{*i}_3 &= -\frac{1}{kL^2}\Big\{B^i_{(i)3}-\overset{s}{B}{}^i_{(i)3}-L^2\iint\limits_{S_{\Gamma_i}}\epsilon^{\alpha\beta}\varrho^i_\alpha s_{i\beta}\,dS\Big\},\\
\omega^{*i}_\alpha &= -\frac{1}{kL}\Big\{F^i_\alpha-\overset{s}{F}{}^i_\alpha-L\iint\limits_{S_{\Gamma_i}}s_{i\alpha}\,dS\Big\}.
\end{aligned}
\tag{4.5}
$$

The shell shallowness function $z(\theta^\alpha)$ is not involved in the relations $(4.4)_2$ and (4.5).

5. Conclusions

It results from the relations $(3.5)_2$ and $(4.4)_2$ that, within the framework of the assumptions of the linear theory of shallow shells [11, 13], the following conclusions are valid:

1. The existence of the multivalued term of the displacement u_3, after passing through the internal closed boundary contour $\overline{\Gamma}_i$ of the shell, is associated with the existence of only three (from among six) components of the displacement dislocation parameters u^i_3 and ω^i_α; it does not depend on the remaining three components.

2. The existence of the multivalued term of the stress function φ_3, after passing through the internal closed boundary contour $\bar{\Gamma}_i$ of the shell or the point k of the concentrated loading application, is associated with the existence of only three (from among six) components of the stress function dislocation parameters u_3^{*i} and ω_α^{*i}; it does not depend on the remaining three components.

The shell shallowness function $z(\theta^\alpha)$ is not involved in the relations $(3.5)_2$, (3.6), (3.7) and $(4.4)_2$, (4.5), which define the form of the multivalued term of u_3 and φ_3. This means that, within the framework of the assumptions of the linear theory of shallow shells [11, 13], the form of the multivalued term of u_3 and φ_3 is identical for shells with different shapes of $z(\theta^\alpha)$ but with the same shell projection into the plane Π, as well as with the same dislocation parameters u_3^i, ω_α^i and u_3^{*i}, ω_α^{*i}. In particular, this is true for $z(\theta^\alpha) \equiv 0$, when the equations of shells are decoupled into the equations of plates and the plane elasticity. Thus we arrive at the following conclusion:

3. The form of the multivalued term of u_3 and φ_3 for shallow shell is identical with that for, respectively, plate or the plane stress state of a disk with the shape of the shell projection into the plane Π, when the same dislocation parameters u_3^i, ω_α^i or of u_3^{*i}, ω_α^{*i} are assumed correspondingly.

The above three conclusions enable us to construct effectively the multivalued terms of u_3 and φ_3 making use of the plane elasticity solutions for the multiply connected regions [14]. Applying the mixed method of solution of the shallow shell equations, in which u_3 and φ_3 are fundamental unknowns [11, 12], the problem of the multivaluedness may thus be eliminated from consideration, and the solution reduces itself to solving the unique boundary value problem.

6. Concluding Remarks

It is interesting to compare the conclusions 1 and 2 of the present work with some known multivalued solutions for shallow shells of revolution, characterized by the two-connected region. For spherical, conical and logarithmic shells the form of the multivalued term of u_3 and φ_3 may be obtained analytically direct from integration of the constitutive equations [8]. On the other hand, in the polar system of coordinates ϱ, θ, the function Φ in $(3.5)_2$ and $(4.4)_2$ may be assumed in the form $\Phi = \theta/2\pi$ due to the rotary symmetry of the shell geometry.

Reissner [15] investigated the solution of the first state of asymmetry (expansion in $\cos\theta$) of a spherical shell and obtained the multivalued part of stress function φ_3 of the type $C\cdot\theta\sin\theta$, which agrees with the relation $(4.4)_2$ written for this case. In the work [15] the spherical shell, loaded on the internal boundary by the loads reducing to a pair of vertical forces, is considered as an example of using the general relations obtained. In agreement with the conclusion 2 of the present work, such load does not cause the appearance of the multivalued term of φ_3. Indeed, after performing numerical calculations and computing the constant C, we find that it is in this case equal to zero. In the author's paper [8, 17], among other problems, the shallow shell of revolution with the meridian in the form of the logarithmic curve has been solved. It is shown in [8, 17] that for the asymmetry state with vertical surface and boundary loading, the constant C is equal to zero, which agrees with the conclusion 2. For the horizontal surface loading and boundary loading the constant C is equal to that obtained directly from $(4.4)_2$.

Lastly, LARDNER and SIMMONDS [16], by reducing the order of the equation of asymmetry state, have obtained the form of the multivalued term φ_3 which coincides with that obtained directly from (4.4).

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S t r e s z c z e n i e

WIELOZNACZNOŚĆ ROZWIĄZAŃ POWŁOK O MAŁEJ WYNIOSŁOŚCI

W pracy rozpatrzono zagadnienie wieloznaczności wektora przemieszczeń oraz wektora funkcji naprężeń w liniowej teorii powłok o małej wyniosłości o obszarze wielospójnym. Wykazano, że w ramach założeń upraszczających powłok o małej wyniosłości, wieloznaczność składowych tych wektorów w kierunku wektora prostopadłego do płaszczyzny rzutu powłoki zależy jedynie od trzech (spośród ogólnie sześciu) składowych tzw. parametrów dyslokacji. Wykazano, że postać wieloznacznego członu dla tych składowych nie zależy od kształtu powłoki i można go zbudować jak dla płyty i płaskiego stanu naprężenia tarczy o konturze brzegowym rzutu powłoki na płaszczyznę odniesienia oraz tych samych parametrach dyslokacji. Wnioski te skonfrontowano ze znanymi rozwiązaniami analitycznymi powłok obrotowych o obszarze dwuspójnym.

Резюме

МНОГОЗНАЧНОСТЬ РЕШЕНИЙ ПОЛОГИХ ОБОЛОЧЕК

Рассматривается вопрос многозначности вектора перемещений и вектора функций напряжений в линейной теории пологих оболочек многосвязной области. Указывается, что с точностью до основных упрощающих гипотез пологих оболочек, компоненты этих векторов по вектору перпендикулярному к плоскости проекции оболочки зависят только от трех (из общего числа шести) компонент так наз. параметров дислокации. Вид многозначного члена этих компонент не зависит от формы оболочки и можно его строить как для пластины и плоского напряженного состояния с граничным контуром проекции оболочки на плоскость и тех-же компонентах параметров дислокации. Эти заключения сравниваются с известными аналитическими решениями пологих оболочек вращения в двухсвязной области.

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