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APPLIED MECHANICS

On the Elasticity Tensors of Deformed Isotropic Solids

by

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Summary. From the material form of constitutive equation of the isotropic elastic solid, using absolute tensor analysis, the exact explicit formulae for the elasticity tensors up to the second order for the solid in arbitrarily and infinitesimally deformed reference configuration are derived.

1. Introduction

A form of the strain energy function for elastic solids depends on the choice of reference configuration [1, 2]. The response of a hyper-elastic material to deformation from an arbitrary deformed reference configuration is different from the response from the unstressed natural state. This fact, sometimes referred to as a deformational anisotropy [3, 6], can be taken into consideration by specifying the elasticity tensors in a reference configuration. These elasticity tensors are, in general, different from those defined in the natural state [2, 4].

In this note exact formulae for the elasticity tensors up to the second order are derived for an isotropic elastic solid in arbitrarily and infinitesimally deformed reference configuration. Appropriate formulae, relative to the deformed reference configuration, are derived from a material form of the strain energy function, using the constitutive relation involving the second Piola—Kirchhoff stress tensor [2]. The absolute tensor calculus [2, 7, 8] and derivatives of tensor functions of symmetric argument [5,13] are employed to this end. For an infinitesimally deformed reference configuration the elasticity tensors have been expressed in terms of experimental values of elastic constants of the first and second order in the natural state. There is therefore no need to know an explicit form of the strain energy function of the solid.

The derived relations for the elasticity tensors can be useful in problems of wave propagation, vibration and stability of initially deformed elastic solids, as well as when studying the second order effects, superposition of deformations, etc.

2. Notations and basic relations

The absolute euclidean tensor analysis [2, 7—10] is used throughout the paper, while the basic notations are those of [2, 5].

Let us denote the second order tensors by $\mathbf{A}, \mathbf{B}, \ldots, \mathbf{H}, \mathbf{S}, \mathbf{T} \in \mathcal{T}_2$ and the metric tensor of $\mathcal{T}_2$ by $\mathbf{1} \in \mathcal{T}_2$. The higher order tensors we denote by $\mathbf{K}, \mathbf{L} \in \mathcal{T}_4$ and $\mathbf{M}$,

$\mathbf{N} \in \mathcal{T}_6$. It is evident that tensors of the types $\mathbf{1}\otimes\mathbf{1}$, $\mathbf{1}\otimes\mathbf{E}$, $\mathbf{B}\otimes\mathbf{B} \in \mathcal{T}_4$, while $\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1}$, $\mathbf{B}\otimes\mathbf{B}\otimes\mathbf{B}$, $\mathbf{G}\otimes\mathbf{K} \in \mathcal{T}_6$.

For tensors $\mathbf{P} \in \mathcal{T}_p$, $\mathbf{Q} \in \mathcal{T}_q$, $p \geqslant q$, the operation [8, 10] $\overset{\mu,\nu}{\mathbf{P}^T}$ denotes a transposition (μ, ν), $\underset{\mu,\nu}{\operatorname{tr}} \mathbf{P} \in \mathcal{T}_{p-2}$ — a contraction (μ, ν), $1 \leqslant \nu < \mu \leqslant p$, while $\mathbf{PQ} \equiv \underset{p,p+1}{\operatorname{tr}} (\mathbf{P}\otimes\mathbf{Q}) \in \mathcal{T}_{p+q+2}$ — a simple dot operation, $\mathbf{P}\cdot\mathbf{Q} \equiv \underset{p-q+1,p+1}{\operatorname{tr}} \dots \underset{p,p+q}{\operatorname{tr}} (\mathbf{P}\otimes\mathbf{Q}) \in \mathcal{T}_{p-q}$ — a full dot operation.

The derivatives of a tensor function $f: \mathcal{T}_2 \to \mathcal{T}_p$, $\mathbf{P} = f(\mathbf{A})$ are denoted by $f_{,\mathbf{A}}$, $f_{,\mathbf{AA}}, \dots$ and their values at $\mathbf{A}_0 \in \mathcal{T}_2$ by $f_{,\mathbf{A}}(\mathbf{A}_0) \in \mathcal{T}_{p+2}, f_{,\mathbf{AA}}(\mathbf{A}_0) \in \mathcal{T}_{p+4}, \dots$. When $\mathbf{A} = g(\mathbf{B})$, $g: \mathcal{T}_2 \to \mathcal{T}_2$, the derivative of $h = f \circ g$ is defined as

$$h_{,\mathbf{B}} = \underset{p+1,p+3}{\operatorname{tr}} \; \underset{p+2,p+4}{\operatorname{tr}} (f_{,\mathbf{A}} \otimes g_{,\mathbf{B}}) \tag{1}$$

The derivatives of all tensor functions appearing in the paper, especially those of symmetric argument $\mathbf{A} \in {}^s\mathcal{T}_2$, have been given in [5, 8, 13].

Let us consider the body $\mathcal{B}$ in the three different configurations [2]: $\boldsymbol{\varkappa}_0$ — the natural state, unstressed; $\boldsymbol{\varkappa}$ — a reference configuration, arbitrarily deformed; $\boldsymbol{\gamma}$ — the actual configuration. For deformations $\chi_0 = \boldsymbol{\varkappa} \circ \boldsymbol{\varkappa}_0^{-1}$, $\chi = \boldsymbol{\gamma} \circ \boldsymbol{\varkappa}^{-1}$ and $\chi^* = \boldsymbol{\gamma} \circ \boldsymbol{\varkappa}_0^{-1}$, with deformation gradients $\mathbf{F}_0$, $\mathbf{F}$ and $\mathbf{F}^*$, respectively, we have the relations [11]

$$\mathbf{F}^* = \mathbf{F}\mathbf{F}_0, \qquad \mathbf{E}^* = \mathbf{F}_0^T \mathbf{E} \mathbf{F}_0 + \mathbf{E}_0, \tag{2}$$

where for $\mathbf{F}$ we have

$$\begin{aligned} &\mathbf{F} = \mathbf{1} + \mathbf{H}, \qquad \mathbf{B} = \mathbf{F}\mathbf{F}^T, \\ &\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{1}), \qquad \widetilde{\mathbf{E}} = \frac{1}{2}(\mathbf{H} + \mathbf{H}^T), \end{aligned} \tag{3}$$

and similar relations for $\mathbf{F}_0$ and $\mathbf{F}^*$.

The material constitutive equation for the elastic solid has the form

$$\mathbf{S} = \rho_{\varkappa} \tau_{,\mathbf{E}}(\mathbf{E}), \tag{4}$$

where [2, 5]: $\mathbf{S} = J\mathbf{F}^{-1}\mathbf{T}_{\gamma}\mathbf{F}$ — the second Piola—Kirchhoff stress tensor, $\tau \equiv \tau_{\varkappa}$ — the strain energy function, both defined in respect to $\boldsymbol{\varkappa}$ as the reference configuration.

For solids with a specified symmetry group the representation theorems are known mainly for the strain energy function $\tau_0 \equiv \tau_{\varkappa_0}$ defined in respect to the natural state $\boldsymbol{\varkappa}_0$ as the reference configuration, where the following relation holds

$$\tau_{,\mathbf{E}}(\mathbf{E}) = \mathbf{F}_0 \, \tau_{0,\mathbf{E}^*}(\mathbf{E}^*) \, \mathbf{F}_0^T. \tag{5}$$

For an isotropic elastic solid τ_0 is an orthogonal invariant

$$\tau_0(\mathbf{E}^*) = \tau_0(I_{\mathbf{E}^*}, II_{\mathbf{E}^*}, III_{\mathbf{E}^*}), \tag{6}$$

where $I_{\mathbf{E}^*}$, $II_{\mathbf{E}^*}$, $III_{\mathbf{E}^*}$ are the principal invariants of $\mathbf{E}^* \in {}^s\mathcal{T}_2$ [2, 5].

3. Elasticity tensors

Expanding (4) into Taylor series in the neighbourhood of $\boldsymbol{\varkappa}$, we have

$$\mathbf{S}=\mathbf{T}+\mathbf{L}\cdot\mathbf{E}+\frac{1}{2}\mathbf{M}\cdot(\mathbf{E}\otimes\mathbf{E})+\ldots, \tag{7}$$

where

$$\begin{aligned}\mathbf{T}&\equiv\mathbf{T}_{\varkappa}=\rho_{\varkappa}\,\tau_{,\mathbf{E}}(\mathbf{0}),\\ \mathbf{L}&\equiv\mathbf{L}_{\varkappa}=\rho_{\varkappa}\,\tau_{,\mathbf{EE}}(\mathbf{0}),\\ \mathbf{M}&\equiv\mathbf{M}_{\varkappa}=\rho_{\varkappa}\,\tau_{,\mathbf{EEE}}(\mathbf{0})\end{aligned} \tag{8}$$

are the elasticity tensors (elasticities according to [2]) of the zero, the first and the second order in the reference configuration $\boldsymbol{\varkappa}$.

Let the deformation χ_0 be assumed a fixed one. Using (1) and the formulae for derivatives of tensor functions [5, 8, 13], for isotropic elastic solids, we can define the functions:

$$\begin{aligned}&g_1(\mathbf{E})=I_{\mathbf{E}^*,\mathbf{E}}=\mathbf{B}_0,\\ &g_2(\mathbf{E})=II_{\mathbf{E}^*,\mathbf{E}}=\mathbf{B}_0\,I_{\mathbf{E}^*}-\left[\mathbf{B}_0\,\mathbf{E}\mathbf{B}_0+\frac{1}{2}(\mathbf{B}_0^2-\mathbf{B}_0)\right],\\ &g_3(\mathbf{E})=III_{\mathbf{E}^*,\mathbf{E}}=\mathbf{B}_0\,II_{\mathbf{E}^*}-\left[\mathbf{B}_0\,\mathbf{E}\mathbf{B}_0+\frac{1}{2}(\mathbf{B}_0^2-\mathbf{B}_0)\right]I_{\mathbf{E}^*}+\\ &\qquad+\mathbf{B}_0\,\mathbf{E}\mathbf{B}_0\,\mathbf{E}\mathbf{B}_0+\frac{1}{2}(\mathbf{B}_0^2\,\mathbf{E}\mathbf{B}_0+\mathbf{B}_0\,\mathbf{E}\mathbf{B}_0^2)-\mathbf{B}_0\,\mathbf{E}\mathbf{B}_0+\frac{1}{4}(\mathbf{B}_0^3-2\mathbf{B}_0^2+\mathbf{B}_0).\end{aligned} \tag{9}$$

From (2), (5), (8) and (9), using (1) and the formulae for derivatives of tensor functions [5, 8, 13], after transformations we arrive at the relations in the following form [13]

$$\begin{aligned}\mathbf{T}&=\sum_{r=1}^{3}\tau_r\,\mathbf{G}_r,\\ \mathbf{L}&=\sum_{r=1}^{3}\left(\tau_r\,\mathbf{K}_r+\sum_{s=1}^{3}\tau_{rs}\,\mathbf{G}_r\otimes\mathbf{G}_s\right),\\ \mathbf{M}&=\sum_{r=1}^{3}\left\{\tau_r\,\mathbf{N}_r+\sum_{s=1}^{3}\left[\tau_{rs}(\mathbf{K}_r\otimes\mathbf{G}_s+\{\mathbf{K}_r\otimes\mathbf{G}_s\}^{\overset{3,5}{T}\,\overset{4,6}{T}}+\mathbf{G}_r\otimes\mathbf{K}_s)+\right.\right.\\ &\qquad\left.\left.+\sum_{t=1}^{3}\tau_{rst}\,\mathbf{G}_r\otimes\mathbf{G}_s\otimes\mathbf{G}_t\right]\right\},\end{aligned} \tag{10}$$

where

$$\begin{aligned}\mathbf{G}_1&=g_1(\mathbf{0})=\mathbf{B}_0,\\ \mathbf{G}_2&=g_2(\mathbf{0})=\mathbf{B}_0\,I_{\mathbf{E}_0}-\frac{1}{2}(\mathbf{B}_0^2-\mathbf{B}_0),\\ G_3&=g_3(\mathbf{0})=\mathbf{B}_0\,II_{\mathbf{E}_0}-\frac{1}{2}(\mathbf{B}_0^2-\mathbf{B}_0)\,I_{\mathbf{E}_0}+\frac{1}{4}(\mathbf{B}_0^3-2\mathbf{B}_0^2+\mathbf{B}_0);\end{aligned} \tag{11}$$

$$\mathbf{K}_1 = g_{1,\mathbf{E}}(\mathbf{0}) = \mathbf{0},$$

(12)
$$\mathbf{K}_2 = g_{2,\mathbf{E}}(\mathbf{0}) = \mathbf{B}_0\otimes\mathbf{B}_0 - \frac{1}{2}\,[(\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,3}{T}}],$$

$$\mathbf{K}_3 = g_{3,\mathbf{E}}(\mathbf{0}) = \mathbf{B}_0\otimes\mathbf{B}_0\, I_{\mathbf{E}_0} - \frac{1}{2}\,[(\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,3}{T}}]\,(I_{\mathbf{E}_0}+1) -$$
$$-\frac{1}{2}\,[\mathbf{B}_0\otimes(\mathbf{B}_0^2-\mathbf{B}_0)+(\mathbf{B}_0^2-\mathbf{B}_0)\otimes\mathbf{B}_0] + \frac{1}{4}\,[(\mathbf{B}_0^2\otimes\mathbf{B}_0+\mathbf{B}_0\otimes\mathbf{B}_0^2)^{\overset{1,4}{T}} +$$
$$+(\mathbf{B}_0^2\otimes\mathbf{B}_0+\mathbf{B}_0\otimes\mathbf{B}_0^2)^{\overset{1,3}{T}}].$$

$$\mathbf{N}_1 = g_{1,\mathbf{EE}}(\mathbf{0}) = \mathbf{0}, \qquad \mathbf{N}_2 = g_{2,\mathbf{EE}}(\mathbf{0}) = \mathbf{0},$$

(13)
$$\mathbf{N}_3 = g_{3,\mathbf{EE}}(\mathbf{0}) = \mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0 - \frac{1}{2}\,\{\mathbf{B}_0\otimes[(\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,3}{T}}] +$$
$$+[(\mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,6}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,5}{T}}] + [(\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,3}{T}}]\otimes\mathbf{B}_0\} +$$
$$+\frac{1}{8}\,\{(\mathbf{B}_0\otimes[(\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,3}{T}}] + [(\mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,6}{T}} +$$
$$+(\mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,5}{T}}])^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes[(\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,4}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,3}{T}}] +$$
$$+[(\mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,6}{T}} + (\mathbf{B}_0\otimes\mathbf{B}_0\otimes\mathbf{B}_0)^{\overset{1,5}{T}}])^{\overset{1,3}{T}}\};$$

(14)
$$\tau_1 = \rho_{\varkappa}\,\frac{\partial\tau_0}{\partial I_{\mathbf{E}*}}\bigg|_{\mathbf{E}=\mathbf{0}},\ \ldots,\ \tau_{212} = \rho_{\varkappa}\,\frac{\partial^3\tau_0}{\partial III_{\mathbf{E}*}\,\partial I_{\mathbf{E}*}\,\partial II_{\mathbf{E}*}}\bigg|_{\mathbf{E}=\mathbf{0}},\ \ldots\,.$$

The obtained relations (10) represent the exact form of the elasticity tensors of the isotropic elastic solid in arbitrarily deformed reference configuration $\varkappa$. The relation analogous to $(10)_1$ is known in the literature (see [2] for references). Explicit absolute forms for $\mathbf{L}$ and $\mathbf{M}$ given in $(10)_{2,3}$ seem to be novel ones.

If $\varkappa$ is the natural state, $\varkappa=\varkappa_0$, then $\mathbf{B}_0\equiv\mathbf{1}$, $\mathbf{E}\equiv\mathbf{0}$, $\mathbf{T}=\mathbf{T}_0\equiv\mathbf{0}$ and

(15)
$$\mathbf{L}_0 = \lambda(\mathbf{1}\otimes\mathbf{1}) + \mu[(\mathbf{1}\otimes\mathbf{1})^{\overset{1,4}{T}} + (\mathbf{1}\otimes\mathbf{1})^{\overset{1,3}{T}}],$$
$$\mathbf{M}_0 = \nu_1(\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1}) + \nu_2\,\{\mathbf{1}\otimes[(\mathbf{1}\otimes\mathbf{1})^{\overset{1,4}{T}} + (\mathbf{1}\otimes\mathbf{1})^{\overset{1,3}{T}}] +$$
$$+[(\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})^{\overset{1,6}{T}} + (\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})^{\overset{1,5}{T}}] + [(\mathbf{1}\otimes\mathbf{1})^{\overset{1,4}{T}} + (\mathbf{1}\otimes\mathbf{1})^{\overset{1,3}{T}}]\otimes\mathbf{1}\} +$$
$$+\nu_3\,\{(\mathbf{1}\otimes[(\mathbf{1}\otimes\mathbf{1})^{\overset{1,4}{T}} + (\mathbf{1}\otimes\mathbf{1})^{\overset{1,3}{T}}] + [(\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})^{\overset{1,6}{T}} + (\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})^{\overset{1,5}{T}}])^{\overset{1,4}{T}} +$$
$$+(\mathbf{1}\otimes[(\mathbf{1}\otimes\mathbf{1})^{\overset{1,4}{T}} + (\mathbf{1}\otimes\mathbf{1})^{\overset{1,3}{T}}] + [(\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})^{\overset{1,6}{T}} + (\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})^{\overset{1,5}{T}}])^{\overset{1,3}{T}}\},$$

where

(16)
$$\lambda = \theta_{11} + \theta_2\,, \qquad \mu = -\frac{1}{2}\,\theta_2,$$
$$\nu_1 = \theta_{111} + 3\theta_{12} + \theta_3, \qquad \nu_2 = -\frac{1}{2}\,(\theta_{12}+\theta_3), \qquad \nu_3 = \frac{1}{4}\,\theta_3,$$

(17)
$$\theta_r = \tau_r\big|_{\varkappa=\varkappa_0},\ \ldots,\ \theta_{rst} = \tau_{rst}\big|_{\varkappa=\varkappa_0}.$$

The second order elastic constants ν_1, ν_2, ν_3 and the relation $(15)_2$ written in components in respect to the natural basis of a coordinate system given in $\varkappa_0$, has been introduced in [12].

4. Infinitesimally strained reference configuration

Although the relations (10) and (15) are exact, they can be derived analytically only in the case when the explicit form of the strain energy function τ_0 is known.

For a wide class of elastic solids the explicit form of the strain energy function has not been established as yet. The elastic constants of the first, and sometimes also of the second order in the natural state $\varkappa_0$ have been found by experimental tests.

In such a case, for a deformation χ_0 with small displacement gradient $\mathbf{H}_0$, $\|\mathbf{H}_0\| \sim \varepsilon \ll 1$, it is possible to derive analytically the approximate values of the elasticity tensors in $\varkappa$, expanding them into Taylor series in respect to $\mathbf{H}_0$ in the neighbourhood of $\varkappa_0$. Omitting the details of the expansion procedure [5, 11, 13], we present here the results for isotropic elastic solid [13].

If only the λ and μ are given, we obtain the results known from the classical linear theory of elasticity

$$
\begin{aligned}
&\mathbf{T} = \lambda I_{\tilde{\mathbf{E}}_0} \mathbf{1} + 2\mu \widetilde{\mathbf{E}}, \\
&\mathbf{L} = \mathbf{L}_0 .
\end{aligned} \tag{18}
$$

When also the constants ν_1, ν_2 and ν_3 are given, we obtain

$$
\begin{aligned}
\mathbf{T} = &\left\{ \lambda + \left(\frac{\nu_1}{2} + \nu_2 - \lambda \right) I_{\tilde{\mathbf{E}}_0} \right\} I_{\tilde{\mathbf{E}}_0} \mathbf{1} + 2\left\{ \mu + (\nu_2 + \lambda - \mu) I_{\mathbf{E}_0} \right\} \widetilde{\mathbf{E}}_0 + \\
&+ \left(\frac{\lambda}{2} I_{\mathbf{H}_0 \mathbf{H}_0^T} - 2\nu_2 \, II_{\tilde{\mathbf{E}}_0} \right) \mathbf{1} + \mu \mathbf{H}_0 \mathbf{H}_0^T + 4(\mu + \nu_3) \widetilde{\mathbf{E}}_0^2, \\
\mathbf{L} = &\{ \lambda + (\nu_1 - \lambda) I_{\tilde{\mathbf{E}}_0} \} (\mathbf{1} \otimes \mathbf{1}) + \{ \mu + (\nu_2 - \mu) I_{\tilde{\mathbf{E}}_0} \} [\overset{1,4}{(\mathbf{1} \otimes \mathbf{1})^T} + \overset{1,3}{(\mathbf{1} \otimes \mathbf{1})^T}] + \\
&+ 2(\nu_2 + \lambda)(\mathbf{1} \otimes \widetilde{\mathbf{E}}_0 + \widetilde{\mathbf{E}}_0 \otimes \mathbf{1}) + 2(\nu_3 + \mu) [\overset{1,4}{(\mathbf{1} \otimes \widetilde{\mathbf{E}}_0 + \widetilde{\mathbf{E}}_0 \otimes \mathbf{1})^T} + \\
&+ \overset{1,3}{(\mathbf{1} \otimes \widetilde{\mathbf{E}}_0 + \widetilde{\mathbf{E}} \otimes \mathbf{1})^T}], \\
\mathbf{M} = &\mathbf{M}_0 .
\end{aligned} \tag{19}
$$

The result analogous to $(19)_1$ with other second order constants is known from [14, 2]. The explicit absolute form for $\mathbf{L}$ given in $(19)_2$ seems to be a novel one.

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В. Петрашкевич, **О тензорах упругости деформированных изотропных твердых тел**

Содержание. Опираясь на материальную форму определяющего уравнения для изотропного упругого тела, при помощи абсолютного тензорного анализа. В настоящей работе выведены точные формулы для тензоров упругости до второго порядка включительно по отношению к произвольно и инфинитезимально деформированной конфигурации упругого твердого тела.