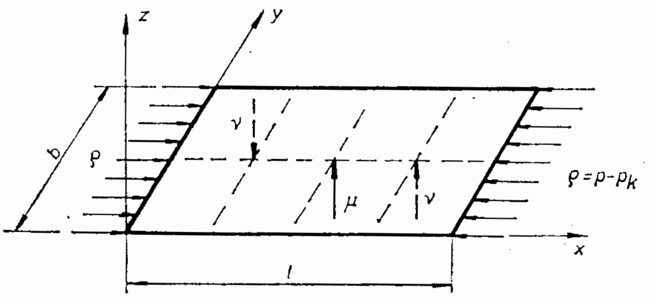

Abb. 1. Rechteckplatte mit den vier Parametern p, l, μ, ν

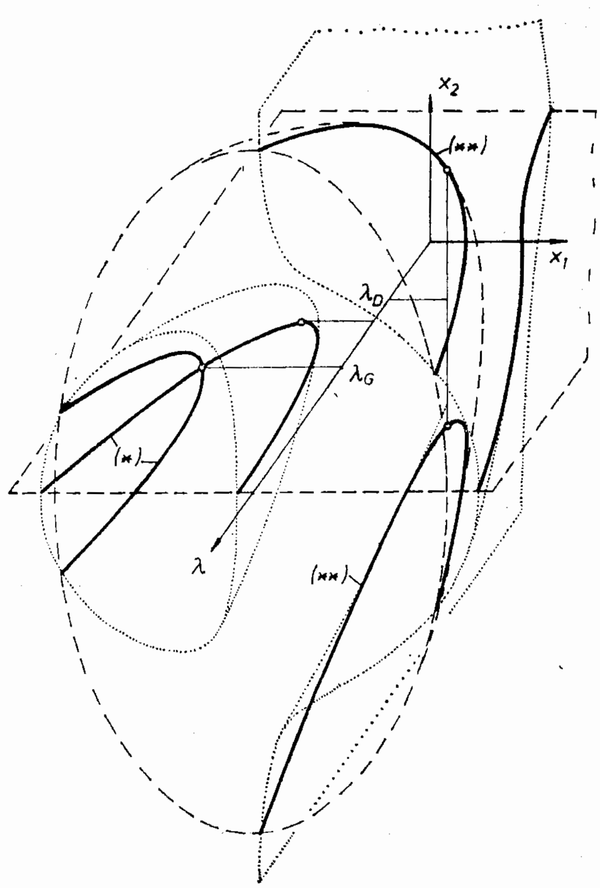

Abb. 2. Nicht beschränkt-generischer Verzweigungsfall ($\nu = 0$), da für $\lambda = \lambda_G$ eine Gabelverzweigung vorliegt (Fall A) und für $\lambda = \lambda_D$ vier Lösungen gleichzeitig auftreten (Fall B) ($f_1 = 0$: punktiert, $f_2 = 0$: strichliert)

gungspunkte gleichzeitig auf, d. h. es kommen vier Lösungen (**) hinzu (Fall B). Beschränkt-generische Verzweigungen erhält man nur für $\mu \neq 0$, $\nu \neq 0$ und unter Ausschluß spezieller Werte von μ/ν ([6]). Dann ist nämlich der Fall ausgeschlossen, daß sich die beiden Lösungskurven $f_1 = 0$, $f_2 = 0$ von (4) für ein $\lambda = \lambda_D$ in zwei verschiedenen Punkten berühren.

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J. Makowski / W. Pietraszkiewicz

Incremental Formulation of the Non-Linear Theory of Thin Shells in the Total Lagrangian Description

Consider the middle surface of a thin shell in three different configurations: the undeformed $\mathcal{M}$, the deformed $\bar{\mathcal{M}}$ and the superposed $\tilde{\mathcal{M}}$. The geometric parameters associated with $\mathcal{M}$ and $\bar{\mathcal{M}}$ are described in detail in [1], while those associated with $\tilde{\mathcal{M}}$ will be marked by a tilde over the symbol. With the superposed deformation $\chi\colon \bar{\mathcal{M}} \to \tilde{\mathcal{M}}$ we associate the following deformation parameters which are referred to the geometry of $\mathcal{M}$: the displacement vector $\boldsymbol{u} = \tilde{\boldsymbol{r}} - \bar{\boldsymbol{r}}$, the difference vector $\boldsymbol{\beta} = \tilde{\boldsymbol{n}} - \bar{\boldsymbol{n}}$, the strain tensor $\gamma = \frac{1}{2}\,(\boldsymbol{G}_1^T\boldsymbol{G}_1 - \boldsymbol{G}_0^T\boldsymbol{G}_0)$ and the tensor of change of curvature $\varkappa = -(\boldsymbol{G}_1^T\tilde{\boldsymbol{b}}\boldsymbol{G}_1 - \boldsymbol{G}_0^T\bar{\boldsymbol{b}}\boldsymbol{G}_0)$. Analogous parameters associated with the initial deformation $\chi_0\colon \mathcal{M} \to \bar{\mathcal{M}}$

and with the total deformation $\chi_1 : \mathcal{M} \to \tilde{\mathcal{M}}$ will be distinguished by naught or unity as subscripts, respectively. In particular, $\boldsymbol{G}_1 = \tilde{\boldsymbol{a}}_\alpha \otimes \boldsymbol{a}^\alpha + \tilde{\boldsymbol{n}} \otimes \boldsymbol{n}$ and $\boldsymbol{G}_0 = \bar{\boldsymbol{a}}_\alpha \otimes \boldsymbol{a}^\alpha + \bar{\boldsymbol{n}} \otimes \boldsymbol{n}$ are here the deformation gradient tensors of χ_1 and χ_0, respectively, and $\boldsymbol{G}_1 = \boldsymbol{G}\boldsymbol{G}_0$.

It is easy to show [2] the following additive decomposition

$$\boldsymbol{u}_1 = \boldsymbol{u}_0 + \boldsymbol{u}\,, \qquad \boldsymbol{\beta}_1 = \boldsymbol{\beta}_0 + \boldsymbol{\beta}\,, \qquad \gamma_1 = \gamma_0 + \gamma\,, \qquad \varkappa_1 = \varkappa_0 + \varkappa\,. \tag{1}$$

Let $\tilde{\mathcal{M}}$ be the middle surface of a shell in an equilibrium state, under the surface force $\boldsymbol{p}_1$, per unit area of $\mathcal{M}$, and under the boundary force $\boldsymbol{T}_1$ and the boundary static moment $\boldsymbol{H}_1$, both per unit length of $\mathcal{C}$. The associated with $\tilde{\mathcal{M}}$ symmetric Lagrangian internal stress resultant and stress couple tensors $\boldsymbol{N}_1$ and $\boldsymbol{M}_1$, referred to $\mathcal{M}$, are related to the Eulerian stress measures of $\tilde{\mathcal{M}}$ by the formulae $\tilde{\boldsymbol{N}}_1 = \sqrt{a/\tilde{a}}\, \boldsymbol{G}_1\boldsymbol{N}_1\boldsymbol{G}_1^T$ and $\tilde{\boldsymbol{M}}_1 = \sqrt{a/\tilde{a}}\, \boldsymbol{G}_1\boldsymbol{M}_1\boldsymbol{G}_1^T$. If $\boldsymbol{p}_0$, $\boldsymbol{T}_0$, $\boldsymbol{H}_0$, $\boldsymbol{N}_0$, $\boldsymbol{M}_0$ are analogous Lagrangian force parameters, referred to $\mathcal{M}$, associated with the equilibrium state of $\bar{\mathcal{M}}$, the appropriate force parameters associated with $\tilde{\mathcal{M}}$ can be decomposed additively as follows

$$\boldsymbol{p}_1 = \boldsymbol{p}_0 + \boldsymbol{p}\,, \qquad \boldsymbol{T}_1 = \boldsymbol{T}_0 + \boldsymbol{T}\,, \qquad \boldsymbol{H}_1 = \boldsymbol{H}_0 + \boldsymbol{H}\,, \qquad \boldsymbol{N}_1 = \boldsymbol{N}_0 + \boldsymbol{N}\,, \qquad \boldsymbol{M}_1 = \boldsymbol{M}_0 + \boldsymbol{M} \tag{2}$$

where the unmarked parameters appear as a result of the superposed deformation χ.

Since the initial deformation χ_0 does not change during the variation, the principle of virtual displacements for the superposed configuration $\tilde{\mathcal{M}}$, but referred to the undeformed configuration $\mathcal{M}$, takes the form [2, 3]

$$\iint_{\mathcal{M}} [(\boldsymbol{N}_0 + \boldsymbol{N}) \cdot \delta\gamma + (\boldsymbol{M}_0 + \boldsymbol{M}) \cdot \delta\varkappa]\, \mathrm{d}A =$$
$$= \iint_{\mathcal{M}} (\boldsymbol{p}_0 + \boldsymbol{p}) \cdot \delta\boldsymbol{u}\, \mathrm{d}A + \int_{\mathcal{C}_f} [(\boldsymbol{T}_0 + \boldsymbol{T}) \cdot \delta\boldsymbol{u} + (\boldsymbol{H}_0 + \boldsymbol{H}) \cdot \delta\boldsymbol{\beta}]\, \mathrm{d}s\,. \tag{3}$$

In general, γ, $\varkappa$ and $\boldsymbol{\beta}$ are complex non-linear functions of $\boldsymbol{u}$ in $\mathcal{M}$ and of $\boldsymbol{u}$, $\beta_\nu = \boldsymbol{\nu} \cdot \boldsymbol{\beta}$ at $\mathcal{C}$, [1, 3]. The exact transformation of (3) for an arbitrary superposed deformation χ leads to extremely complex local shell equations which will not be presented here. In the case of a small superposed deformation χ the principle (3) may be linearized to the form

$$\iint_{\mathcal{M}} (\boldsymbol{N}_0 \cdot \delta\gamma^Q + \boldsymbol{M}_0 \cdot \delta\varkappa^Q + \boldsymbol{N} \cdot \delta\gamma^L + \boldsymbol{M} \cdot \delta\varkappa^L)\, \mathrm{d}A - \iint_{\mathcal{M}} \boldsymbol{p} \cdot \delta\boldsymbol{u}\, \mathrm{d}A - \int_{\mathcal{C}_f} (\boldsymbol{T} \cdot \delta\boldsymbol{u} + \boldsymbol{H} \cdot \delta\boldsymbol{\beta}^L)\, \mathrm{d}s\, +$$
$$+ \iint_{\mathcal{M}} (\boldsymbol{N}_0 \cdot \delta\gamma^L + \boldsymbol{M}_0 \cdot \delta\varkappa^L)\, \mathrm{d}A - \iint_{\mathcal{M}} \boldsymbol{p}_0 \cdot \delta\boldsymbol{u}\, \mathrm{d}A - \int_{\mathcal{C}_f} (\boldsymbol{T}_0 \cdot \delta\boldsymbol{u} + \boldsymbol{H}_0 \cdot \delta\boldsymbol{\beta}^L)\, \mathrm{d}s = 0 \tag{4}$$

where

$$\left.\begin{aligned} &\delta\gamma^L_{\alpha\beta} = \tfrac{1}{2}(\bar{\boldsymbol{a}}_\alpha \cdot \delta\boldsymbol{u}_{,\beta} + \bar{\boldsymbol{a}}_\beta \cdot \delta\boldsymbol{u}_{,\alpha})\,, \qquad \delta\varkappa^L_{\alpha\beta} = \bar{\boldsymbol{n}} \cdot (\bar{a}^{\lambda\varkappa}\overset{0}{\gamma}_{\varkappa\alpha\beta}\delta\boldsymbol{u}_{,\lambda} - \delta\boldsymbol{u}_{|\alpha\beta})\,, \\ &\delta\boldsymbol{\beta}^L = \frac{1}{a^0_\nu}\left[(\boldsymbol{\nu} \times \bar{\boldsymbol{n}})\left(\bar{\boldsymbol{n}} \cdot \frac{\mathrm{d}}{\mathrm{d}s}\delta\boldsymbol{u}\right) + \bar{\boldsymbol{a}}_\nu(\boldsymbol{\nu} \cdot \delta\boldsymbol{\beta}^L)\right], \qquad a^0_\nu = \boldsymbol{\nu} \cdot \bar{\boldsymbol{a}}_\nu\,, \end{aligned}\right\} \tag{5}$$

$$\delta\gamma^Q_{\alpha\beta} = \tfrac{1}{2}\,\delta\boldsymbol{u}_{,\alpha} \cdot \delta\boldsymbol{u}_{,\beta}\,, \qquad \delta\varkappa^Q_{\alpha\beta} = \tfrac{1}{2}\,\bar{b}_{\alpha\beta}\bar{a}^{\lambda\mu}(\bar{\boldsymbol{n}} \cdot \delta\boldsymbol{u}_{,\lambda})\, \bar{\boldsymbol{n}} \cdot \delta\boldsymbol{u}_{,\mu} + (\bar{\boldsymbol{n}} \cdot \delta\boldsymbol{u}_{,\lambda})\, \bar{\boldsymbol{a}}^\lambda \cdot \delta\boldsymbol{u}_{||\alpha\beta}\,, \tag{6}$$

and $\delta\boldsymbol{\beta}$ is approximated at $\mathcal{C}_f$ only by its linear part.

The last line of (4) represents the condition for $\bar{\mathcal{M}}$ to be in equilibrium. Therefore, it should vanish for exact values of the initial stress measures $\boldsymbol{N}_0$ and $\boldsymbol{M}_0$. However, due to the numerical errors in computing procedures and due to the approximation involved in the linearized principle (4), some residual surface, boundary and corner forces may appear, which are defined by [1, 3]

$$\left.\begin{aligned} &\boldsymbol{p}_R = (\boldsymbol{G}_0\boldsymbol{N}_0^\beta)_{|\beta} + \boldsymbol{p}_0 \quad \text{in } \mathcal{M}\,, \\ &\boldsymbol{P}_R = \left(\boldsymbol{G}_0\boldsymbol{N}_0^\beta\nu_\beta + \frac{\mathrm{d}\boldsymbol{F}_0}{\mathrm{d}s}\right) - \left(\boldsymbol{T}_0 + \frac{\mathrm{d}\boldsymbol{F}_0^*}{\mathrm{d}s}\right), \qquad M_R = M_0 - M_0^* \quad \text{on } \mathcal{C}_f\,, \\ &\boldsymbol{F}_{Rj} = \boldsymbol{F}_{0j} - \boldsymbol{F}_{0j}^* \quad \text{at each corner } M_j \in \mathcal{C}_f \end{aligned}\right\} \tag{7}$$

where

$$\left.\begin{aligned} &\boldsymbol{N}_0^\beta = (N_0^{\alpha\beta} - \bar{b}^\alpha_\lambda M_0^{\lambda\beta})\, \boldsymbol{a}_\alpha + [M_0^{\alpha\beta}|_\alpha + \bar{a}^{\alpha\beta}(2\overset{0}{\gamma}_{\alpha\lambda|\mu} - \overset{0}{\gamma}_{\lambda\mu|\alpha})\, M_0^{\lambda\mu}]\, \boldsymbol{n}\,, \\ &\boldsymbol{F}_0 = -\frac{1}{a^0_\nu}[(\bar{\boldsymbol{n}} \times \bar{\boldsymbol{a}}_\alpha) \cdot \boldsymbol{\nu}]\, M_0^{\alpha\beta}\nu_\beta\bar{\boldsymbol{n}}\,, \qquad M_0 = \frac{1}{a^0_\nu}(\bar{\boldsymbol{n}} \times \bar{\boldsymbol{a}}_\alpha) \cdot \bar{\boldsymbol{a}}_t M_0^{\alpha\beta}\nu_\beta\,. \end{aligned}\right\} \tag{8}$$

Applying Stokes' theorem to other terms of (4) and integrating by parts we obtain the following local equilibrium equations and associated static and geometric boundary conditions

$$\left.\begin{aligned} &(\boldsymbol{G}_0\boldsymbol{N}^\beta + \boldsymbol{S}_0^\beta)_{|\beta} + \boldsymbol{p} + \boldsymbol{p}_R = \boldsymbol{0} \quad \text{in } \mathcal{M}\,, \\ &(\boldsymbol{G}_0\boldsymbol{N}^\beta + \boldsymbol{S}_0^\beta)\,\nu_\beta + \frac{\mathrm{d}}{\mathrm{d}s}(\boldsymbol{F} + \boldsymbol{C}_0) = \boldsymbol{T} + \frac{\mathrm{d}\boldsymbol{F}^*}{\mathrm{d}s} - \boldsymbol{P}_R\,, \qquad M + K_0 = M^* - M_R \quad \text{on } \mathcal{C}_f\,, \\ &\boldsymbol{F}_j + \boldsymbol{C}_{0j} = \boldsymbol{F}_j^* - \boldsymbol{F}_{Rj} \quad \text{at each corner } M_j \in \mathcal{C}_f\,, \\ &\boldsymbol{u} = \boldsymbol{u}^*\,, \qquad \beta_\nu = \beta_\nu^* \quad \text{on } \mathcal{C}_u; \qquad u_i = u_i^* \quad \text{at each corner } M_i \in \mathcal{C}_u\,. \end{aligned}\right\} \tag{9}$$

Here $\boldsymbol{N}^\beta$, $\boldsymbol{F}$ and M have the structure of (8) but expressed in $N^{\alpha\beta}$, $M^{\alpha\beta}$ and

$$\left.\begin{aligned} \boldsymbol{S}_0^\beta &= N_0^{\alpha\beta}\boldsymbol{u}_{,\alpha} + \bar{a}^{\alpha\beta}\bar{b}_{\lambda\mu}M_0^{\lambda\mu}(\bar{\boldsymbol{n}}\cdot\boldsymbol{u}_{,\alpha})\,\bar{\boldsymbol{n}} - [M_0^{\alpha\beta}(\bar{\boldsymbol{n}}\cdot\boldsymbol{u}_{,\lambda})\,\bar{\boldsymbol{a}}^\lambda]_{|\alpha} + M_0^{\lambda\mu}(\bar{\boldsymbol{a}}^\beta\cdot\boldsymbol{u}_{||\lambda\mu})\,\bar{\boldsymbol{n}} - \\ &\quad - \bar{a}^{\alpha\beta}(2\gamma^0_{\alpha\varkappa|\varrho} - \gamma^0_{\varkappa\varrho|\alpha})\,M_0^{\varkappa\varrho}(\bar{\boldsymbol{n}}\cdot\boldsymbol{u}_{,\lambda})\,\bar{\boldsymbol{a}}^\lambda\,, \\ \boldsymbol{C}_0 &= -\frac{1}{a_\nu^0}[(\bar{\boldsymbol{n}}\times\bar{\boldsymbol{a}}_\lambda)\cdot\boldsymbol{\nu}](\bar{\boldsymbol{a}}^\lambda\cdot\boldsymbol{u}_{,\alpha})\,M_0^{\alpha\beta}\nu_\beta\bar{\boldsymbol{n}}\,, \qquad K_0 = \frac{1}{a_\nu^0}[(\bar{\boldsymbol{n}}\times\bar{\boldsymbol{a}}_\lambda)\cdot\bar{\boldsymbol{a}}_t]\,(\bar{\boldsymbol{a}}^\lambda\cdot\boldsymbol{u}_{,\alpha})\,M_0^{\alpha\beta}\nu_\beta\,. \end{aligned}\right\} \tag{10}$$

The geometric parameters associated with $\bar{\mathscr{M}}$ and $\bar{\mathscr{C}}$ are understood in (8—10) to be expressed in terms of $\boldsymbol{u}_0$ in $\mathscr{M}$ and $\boldsymbol{u}_0$, n_ν^0 at $\mathscr{C}$, [1, 3]. Some of the parameters may also be expressed partly or entirely in terms of the strain measures γ_0 and $\varkappa_0$ as intermediate variables, which allows for a considerable reduction of numerical calculations in the computerized analysis of shell structures.

The incremental shell equations (9) are valid for an arbitrary initial deformation χ_0. They allow for an equivalent formulation in the updated Lagrangian description [5], where the deformed surface $\bar{\mathscr{M}}$ is taken as the reference configuration.

Applying consistent approximations to the initial deformation χ_0, which are discussed in [1, 4], many approximate versions of the incremental shell equations may be constructed. In particular, when the theory of shells undergoing moderate rotations is applied, the equations (9) reduce to the shell stability equations discussed in [2, 6].

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H. G. Matthies / C. Nath

Dynamische Stabilität nichtlinearer Systeme mit periodischer Erregung, dargestellt am Beispiel der großen Windenergieanlage (Growian)

Für Systeme mit periodischen Lösungen, wie zum Beispiel mechanische Systeme mit rotierenden Teilen, ist es oft wichtig festzustellen, unter welchen Bedingungen diese periodischen Lösungen stabil bleiben.

Stabilitätstheorie

Wir gehen aus von den Bewegungsgleichungen eines diskreten mechanischen Systems (bei kontinuierlichen Systemen nehmen wir an, daß sie durch geeignete Diskretisierungsverfahren auf eine endliche Zahl von Freiheitsgraden reduziert worden sind, z. B. mit Hilfe der Methode der finiten Elemente)

$$M\ddot{\bar{u}}(t) + C\dot{\bar{u}}(t) + S(\bar{u}(t)) = F(t) \tag{1}$$

mit den Anfangs- bzw. Periodizitätsbedingungen

$$\bar{u}(0) = \bar{u}(T) = u^0\,, \qquad \dot{\bar{u}}(0) = \dot{\bar{u}}(T) = v^0 \tag{2}$$

und der periodischen Erregung $F(t)$.

Hier bedeuten M und C die Massen- und Dämpfungsmatrix, S den Vektor der inneren Kräfte (hier steckt die Nichtlinearität des Systems) und T die Periodendauer. Wir setzen im folgenden die Existenz der periodischen Lösung $\bar{u}$ voraus, ebenso wie die Differenzierbarkeit von $S(u)$ mit der Ableitung $K(u)$, der tangentialen Steifigkeits-