Lecture Notes in Applied and Computational Mechanics

Volume 16

Theories of Plates and Shells

Critical Review and New Applications

Reinhold Kienzler
Holm Altenbach
Ingrid Ott (Editors)

Springer

Professor Dr. Reinhold Kienzler
Dipl.-Ing. Ingrid Ott
Universität Bremen
Fachbereich 4, Produktionstechnik
Technische Mechanik, Strukturmechanik
28334 Bremen
Germany

Professor Holm Altenbach
Martin-Luther-Universität
Halle-Wittenberg
Fachbereich Ingenieurwissenschaften
06099 Halle (Saale)
Germany

With 75 Figures

Cataloging-in-Publication Data applied for
Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;
detailed bibliographic data is available in the Internet at <http://dnb.de>

ISBN 3-540-20997-2 Springer Verlag Berlin Heidelberg New York

Cover design: design & production GmbH, Heidelberg
Typesetting: Digital data supplied by editors

Printed on acid-free paper 62/3020Rw -5 4 3 2 1 0

EUROMECH COLLOQUIUM 444

Critical Review of the Theories of Plates and Shells and New Application

EUROMECH 444

September 22-25, 2002

Bremen / Germany

organized by

R. Kienzler, Universität Bremen

H. Altenbach, Martin-Luther-Universität, Halle-Wittenberg

Universität Bremen

Martin-Luther-Universität
Halle-Wittenberg

FEM and Time Stepping Procedures in Non-Linear Dynamics of Flexible Branched Shell Structures

J. Chróścielewski[1], I. Lubowiecka[1] and W. Pietraszkiewicz[2]

[1] Gdańsk University of Technology, Poland
[2] Institute of Fluid-Flow Machinery, PASci., Gdańsk, Poland

1 Introduction

Non-linear dynamic behaviour of flexible irregular shell structures was discussed recently by Chróścielewski et al. [3,4], Lubowiecka [7], and Lubowiecka and Chróścielewski [8], where references to other related papers are given. The shell evolution in time was described by two fields: the vector $\boldsymbol{u}$ of the translatory motion of the shell base surface, and the proper orthogonal tensor $\boldsymbol{Q}$ of the mean rotation of the shell cross sections. As a result, the rotation group $SO(3)$ entered the definition of the configuration space. In such problems of structural mechanics finite-dimensional approximations, like the finite element method or the time-stepping algorithm, require non-standard approaches.

We worked out in [3,4,7,8] the extended time-stepping algorithm for the non-linear dynamic analysis on the configuration space containing the rotation group $SO(3)$. Applying the algorithm, several FE numerical simulations of 3D large overall motion of flexible elastic shell structures have been performed. In particular, we have successfully simulated the dynamic behaviour of branched shell structures forced to free flight in space by the pair of forces initially applied during a short time period. The structures perform many turns in space, exhibiting large relative deformations as well.

In this report we want to draw attention to two critical aspects of the numerical analyses performed in [3,4,7,8] which can considerably influence the conservation of total potential energy of the system. These are: a) the influence of the time step size, and b) the influence of the FE mesh condensation.

For any time step size below a critical one, the numerical tests show that values of potential and kinetic energies may oscillate in time, but the total energy remains conserved in a considered time period. The rapid increase of the total energy indicates the appearance of numerical instability of the solution. It has been noted that the numerical instability in the analysis of shell structures undergoing free motion in space is preceded by a rising trend of the potential energy. This fact makes it

possible to predict the moment of numerical instability and to appropriately reduce in advance the time step size. This allows one to prolong considerably the time duration of stable numerical simulations without any special procedures.

By refining the FE mesh, or by using more complex elements, we increase the number of degrees of freedom (dof), which dramatically raises the highest eigenfrequency of the discrete system. This, in turn, makes the system more sensitive to the time step size as well.

2 Weak formulation of non-linear shell dynamics

Motion of the irregular shell structure can in general be described by the displacement vector field $\boldsymbol{u}(\boldsymbol{x},t)=\boldsymbol{y}(\boldsymbol{x},t)-\boldsymbol{x}$, where $\boldsymbol{x}\in M$ and $\boldsymbol{y}(\boldsymbol{x},t)$ are position vectors of the undeformed shell base surface and the base surface at time t, respectively, together with the independent proper orthogonal tensor field $\boldsymbol{Q}(\boldsymbol{x},t)$ representing the mean rotary motion of the shell cross sections. The fields $\boldsymbol{y}(\boldsymbol{x},t)$ and $\boldsymbol{Q}(\boldsymbol{x},t)$ are assumed to be continuous during the motion, and on any stationary singular curve $\Gamma\subset M$ determining an irregularity (fold, intersection, branching, etc., see [5,6]), i.e. $\boldsymbol{y}_{\Gamma}(\boldsymbol{x}_{\Gamma},t)=\boldsymbol{y}(\boldsymbol{x},t)|_{\Gamma}$, $\boldsymbol{Q}_{\Gamma}(\boldsymbol{x}_{\Gamma},t)=\boldsymbol{Q}(\boldsymbol{x},t)|_{\Gamma}$.

For the translational $\boldsymbol{p}(\boldsymbol{x},t)$ and rotational $\boldsymbol{m}(\boldsymbol{x},t)$ momentum surface vectors we take simple kinetic constitutive relations $\boldsymbol{p}(\boldsymbol{x},t)=m_0\boldsymbol{v}=\rho_0 h_0\boldsymbol{v}$ and $\boldsymbol{m}(\boldsymbol{x},t)=I_0\boldsymbol{\omega}=(\rho_0 h_0^3/12)\boldsymbol{\omega}$ (see Libai and Simmonds [7]), where $\rho_0(\boldsymbol{x})$ is the initial shell mass density, $h_0(\boldsymbol{x})$ the initial shell thickness, $\boldsymbol{v}(\mathbf{x},t)=\dot{\boldsymbol{y}}(\boldsymbol{x},t)=\dot{\boldsymbol{u}}(\boldsymbol{x},t)$ the linear velocity vector, and $\boldsymbol{\omega}(\boldsymbol{x},t)$ the angular velocity vector in the spatial representation, with $\mathrm{ad}\,\boldsymbol{\omega}=\dot{\boldsymbol{Q}}\boldsymbol{Q}^{T}$, $\mathrm{ad}: E^3\to so(3)$.

We confine our considerations to hyper-elastic shells for which there exists a 2D strain energy density $W(\boldsymbol{\varepsilon}_{\beta},\boldsymbol{\kappa}_{\beta};\boldsymbol{x})$ of the shell strain vectors $\boldsymbol{\varepsilon}_{\beta}=\boldsymbol{y}_{,\beta}-\boldsymbol{Q}\boldsymbol{x}_{,\beta}$ and $\boldsymbol{\kappa}_{\beta}=\mathrm{ad}^{-1}(\boldsymbol{Q}_{,\beta}\boldsymbol{Q}^{\mathrm{T}})$. Then the constitutive relations of the shell material are given by $\boldsymbol{n}^{\beta}=\partial W/\partial\boldsymbol{\varepsilon}_{\beta}$, $\boldsymbol{m}^{\beta}=\partial W/\partial\boldsymbol{\kappa}_{\beta}$, where $\boldsymbol{n}^{\beta}(\boldsymbol{x},t)$ and $\boldsymbol{m}^{\beta}(\boldsymbol{x},t)$ are the internal stress and couple resultant vectors, respectively.

When expressed in the weak form, the initial-boundary value problem for the branched shell-like structure can be formulated as follows. Given the external resultant force and couple vector fields $\boldsymbol{f}(\boldsymbol{x},t)$ and $\boldsymbol{c}(\boldsymbol{x},t)$ on $\boldsymbol{x}\in M\setminus\Gamma$, $\boldsymbol{n}^{*}(\boldsymbol{x},t)$ and $\boldsymbol{m}^{*}(\boldsymbol{x},t)$ along ∂M_f, $\boldsymbol{f}_{\Gamma}(\boldsymbol{x}_{\Gamma},t)$ and $\boldsymbol{c}_{\Gamma}(\boldsymbol{x}_{\Gamma},t)$ along the singular curve $\Gamma\subset M$, and the initial values $\boldsymbol{u}_0(\boldsymbol{x})$, $\boldsymbol{Q}_0(\boldsymbol{x})$, $\dot{\boldsymbol{u}}_0(\boldsymbol{x})$, $\dot{\boldsymbol{Q}}_0(\boldsymbol{x})$ at $t=0$, find a curve $\mathbb{u}(\boldsymbol{x},t)=(\boldsymbol{u}(\boldsymbol{x},t),\boldsymbol{Q}(\boldsymbol{x},t))$ on the configuration space $C(M,E^3\times SO(3))$ such that for any continuous kinematically admissible virtual vector field $\mathbb{w}(\boldsymbol{x})\equiv(\boldsymbol{v}(\boldsymbol{x}),\boldsymbol{w}(\boldsymbol{x}))$ the following principle of virtual work is satisfied:

$$G[\mathbb{u};\mathbb{w}] = \iint_{M\setminus\Gamma} [m_0 \dot{\boldsymbol{v}}\bullet\boldsymbol{v} + I_0\dot{\boldsymbol{\omega}}\bullet\boldsymbol{w}]\,da$$
$$+ \iint_{M\setminus\Gamma} [\boldsymbol{n}^{\beta}\bullet(\boldsymbol{v}_{,\beta} + \boldsymbol{y}_{,\beta}\times\boldsymbol{w}) + \boldsymbol{m}^{\beta}\bullet\boldsymbol{w}_{,\beta}]\,da - \iint_{M\setminus\Gamma} (\boldsymbol{f}\bullet\boldsymbol{v} + \boldsymbol{c}\bullet\boldsymbol{w})\,da \qquad (1)$$
$$- \int_{\partial M_f} (\boldsymbol{n}^{*}\bullet\boldsymbol{v} + \boldsymbol{m}^{*}\bullet\boldsymbol{w})ds - \int_{\Gamma} (\boldsymbol{f}_{\Gamma}\bullet\boldsymbol{v}_{\Gamma} + \boldsymbol{c}_{\Gamma}\bullet\boldsymbol{w}_{\Gamma})ds \quad = 0\,.$$

In (1) it is implicitly assumed that the kinematic boundary conditions $\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{u}^{*}(\boldsymbol{x},t)$ and $\boldsymbol{Q}(\boldsymbol{x},t) = \boldsymbol{Q}^{*}(\boldsymbol{x},t)$ are satisfied along the complementary part $\partial M_d = \partial M\setminus\partial M_f$ of the shell boundary, and the virtual vector fields are kinematically admissible if $\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{0}$ and $\boldsymbol{w}(\boldsymbol{x}) = \boldsymbol{0}$ along ∂M_d.

3 Temporal and spatial discretisation in SO(3)

The solution of (1) is achieved by an iterative procedure which reduces the problem to a sequence of solutions of linearised problems. Each linearised problem is formulated at discrete values of both temporal and spatial variables. The main difficulty of the solution procedure is associated with the structure of the configuration space involving the rotation group $SO(3)$.

The Newmark type algorithm extended to the rotation group was developed in [3,4,7,8]. It is based on ideas suggested among others by Simo and Vu-Quoc [10], and Cardona and Geradin [1]. In the algorithm the linearised equations of shell dynamics are written in the spatial representation relative to the last instantaneous configuration, while the angular velocity and acceleration vectors are approximated in time in the material representation.

With the time-stepping algorithm, the solution of the non-linear problem (1) is constructed by the incremental-iterative procedure based on the Newton-Kantorovich method applied in the configuration space $C(M, E^3\times SO(3))$. Let an *i*-th approximation $\mathbb{u}_{n+1}^{(i)}$ to the solution $\mathbb{u}_{n+1}$ has been found. In order to calculate the correction $\delta\mathbb{u}_{n+1}^{(i)}$, which would allow us to find the successive approximation $\mathbb{u}_{n+1}^{(i+1)}$ to an unknown solution $\mathbb{u}_{n+1}$, we linearise G_{n+1} at the approximation $\mathbb{u}_{n+1}^{(i)}$:

$$G[\mathbb{u}_{n+1}^{(i)}, t_{n+1}; \mathbb{w}] + \delta G[\mathbb{u}_{n+1}^{(i)}, t_{n+1}; \mathbb{w}, \delta\mathbb{u}_{n+1}^{(i)}] = 0 \qquad (2)$$

The second term in (2) denotes directional derivative of the functional G_{n+1}, taken at the point $\mathbb{u}_{n+1}^{(i)} \in C$ in the direction $\delta\mathbb{u}_{n+1}^{(i)} \in T_{\mathbb{u}_{n+1}^{(i)}} C$. This term yields the so-called tangent operator of the non-linear problem, calculated at the approximation $\mathbb{u}_{n+1}^{(i)}$. The first term in (2) represents unbalanced forces at the approximation point $\mathbb{u}_{n+1}^{(i)}$. For spatial discretisation the finite element method is applied, [2].

Stability of the solution is verified by the energy criterion given by inequality

$$U_{n+1} - U_n + K_{n+1} - K_n \leq \Delta G_{ext} \tag{3}$$

where $U_{n+1} \simeq U(t_{n+1})$ and $U_n \simeq U(t_n)$ are values of the strain energy $U(t)$ numerically calculated at the beginning t_n and at the end t_{n+1} of the time step Δt, respectively, K_{n+1} and K_n are the corresponding values of the kinetic energy $K(t)$, and ΔG_{ext} denotes an increment of the work done by external loads within the time step.

4 Influence of time step size

The twisted T-shaped shell (Fig. 1) exhibits the complex 3D motion comprising large elastic relative deformation and multiple global turns. Two concentrated forces of the ramp function type (from 0 to 10^4 in $1s$ and back down to 0 in another $1s$) are applied at the points (a) and (b). After $2s$ the shell is free from external loading and moves freely in the space undergoing also large local deformations.

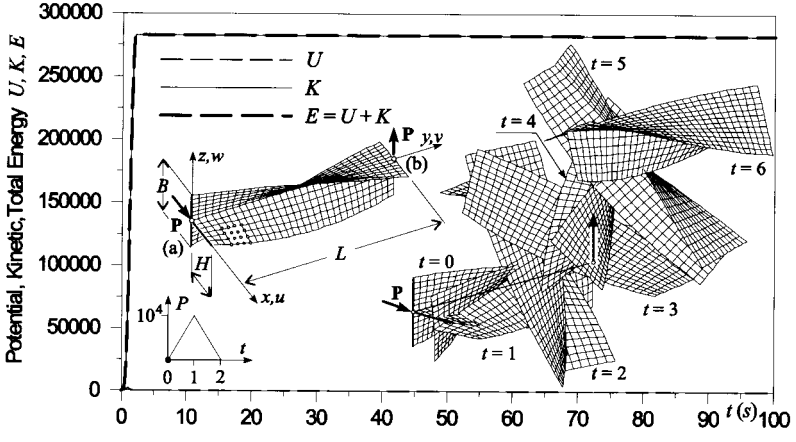

Fig. 1. T-shaped shell structure: $L=50$, $H=10$, $B=14.0112$, $\alpha=90°$, $h_0=0.25$, $E=2\times10^7$, $\nu=0.3$, $\rho_0 h_0=1$; discretisation $(2+2+2)\times10=60e16\,FI$ (16–nodes elements with full integration); external load function; some configurations in time; energy history

The dynamic analysis is performed for three time steps: $\Delta t=0.1s$, $\Delta t=0.025s$, and $\Delta t=0.005s$. During simulations with the largest time step an instability is noticed after about first $10s$ of the motion (see Fig. 2). The reduction of the step size allows one to extend duration of the stable simulation provided that the reduction is executed at an appropriate moment. Although the simulation

becomes stable again during the analysis with the smaller time steps, visible differences appear in the potential energy values (see Fig. 3).

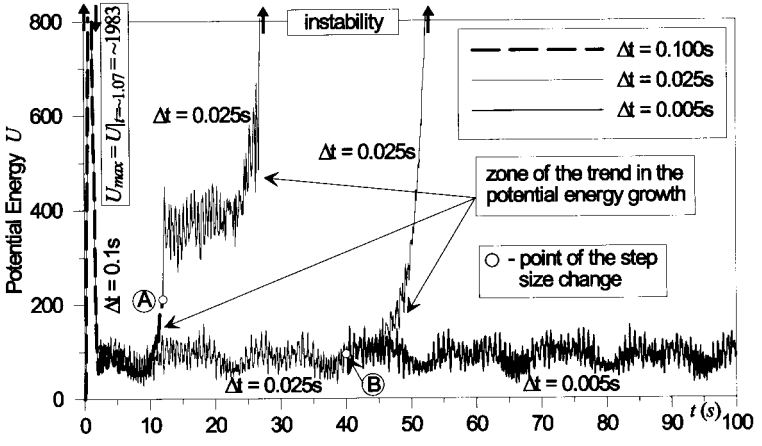

Fig. 2. Potential energy history of twisted T-shaped shell structure

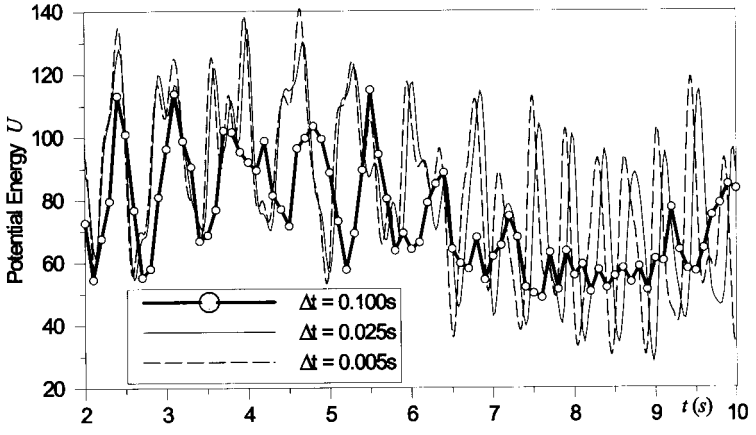

Fig. 3. Magnified potential energy history of twisted T-shaped shell structure for three different time steps Δt

5 Influence of FE mesh refinement

Large overall motion of a flexible cylindrical panel with change of the curvature sign, reinforced by a plate rib (Fig. 4), is analysed. The structure is loaded as in the previous example, but the load function rises here from 0 to 10. The structure, as

in the first example, moves freely in the space undergoing multiple turns and large relative deformations.

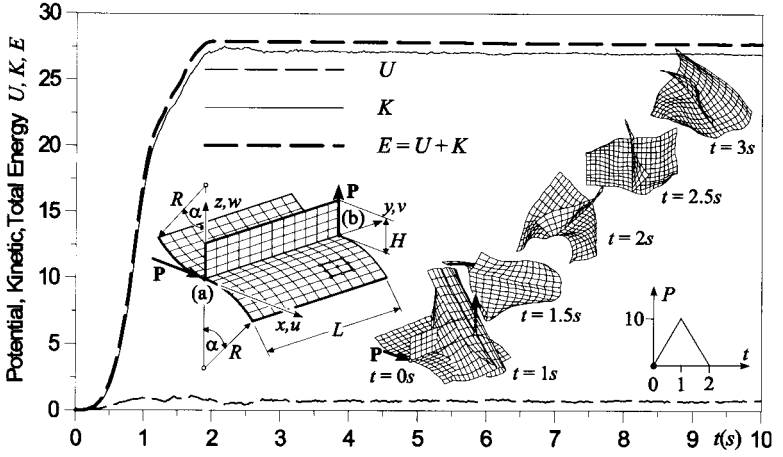

Fig. 4. Wavy cylindrical branched panel: $L=2$, $\alpha=0.4$, $R=1$, $H=0.4$, $h_0=0.01$, $E=10^5$, $\nu=0.25$, $\rho_0 h_0=1$; discretisation; external load function; some configurations in time; energy history

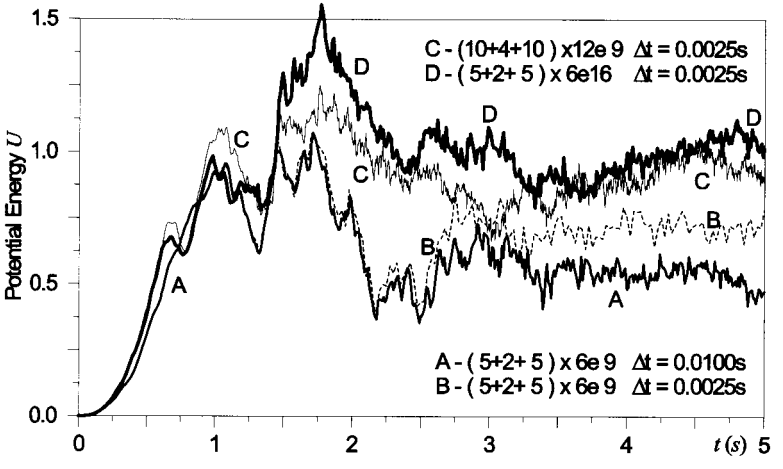

Fig. 5. Variations of potential energy of the cylindrical panel for different meshes, elements, and time steps

The panel is analysed with different discretisations: by $(5+2+5)\times 6=72e9$ *FI* (9–nodes elements) with totally 1950 dof, by $(5+2+5)\times 6=72e16$ *FI* with totally 4218 dof, and by $(10+4+10)\times 12=288e16$ *FI* with totally 7350 dof. The analysis is performed for two different time steps $\Delta t=0.01s$ and $\Delta t=0.0025s$.

The energy histories given in Fig. 5 indicate that there are visible differences in the total energy values, both for different time steps with the same elements and for one time step with different elements. We observed that with the growth of number of dof the numerical problem becomes very fast unstable and requires shortening of the time step size already at the early simulation stage. Moreover, unfortunately, Fig. 5 indicates that by using the same shorter time step $\Delta t = 0.0025s$ different discretisations lead to different plots of the potential energy, and thus to different time changes of stresses in the panel. More information about these examples is included in [7,8].

6 Conclusions

From our experience it follows that the 6-parameter shell theory (with 6 dof in each node of FE mesh) is appropriate to analyse branched shell structures of any shape. The large overall motion of the structures can be analysed without any limits in translation and rotation. When external forces are removed, the convergence of simulated dynamic behaviour is confirmed by two facts: a) the global centre of mass moves along the straight line, and b) the total energy of the system is conserved.

Under considered external loads, as long as they are applied, the total energy initially increases. After removing the loads small oscillations of kinetic and potential energies can still be observed, but the sum of both energy components remains constant. The numerical instability of the solution demonstrates itself after some period of simulation as an explosive increase of the total energy. In these simulations we have noted that the instability is preceded by the rising trend in the potential energy growth (Fig. 2). This observation allows one to choose an appropriate time instant to shorten the size of the time step (point B in Fig. 2) and to proceed with further stable dynamic analysis. However, too late reduction of size of the integration step, already in the rising trend phase of the potential energy growth (point A in Fig. 2), does not assure correct results. By condensing the FE mesh, or using more complex elements, we increase the number of dof, which makes the system more sensitive to the step size (the highest eigenfrequency of the discrete system raises dramatically). As a result, the dynamic analysis with longer time steps is practically impossible without applying special methods. The instability appears here already at a very early stage of numerical simulation.

Significant differences in time changes of potential energy (and thus also in the stresses) obtained for different FE mesh and time step sizes in Fig. 3 and Fig. 5 allow one to question the correctness of the typical results obtained for longer integration steps and coarse mesh, even if the usual stability criteria are satisfied.

Acknowledgement

The author I. Lubowiecka is awarded by Foundation for Polish Science.

References

1. Cardona C, Geradin C (1988) A beam finite element non-linear theory with finite rotations. Int J Num Meth Engng 26: 2403-2438
2. Chróścielewski J (1996) The family of C^0 finite elements in the non-linear sixparameter shell theory (in Polish). Zesz. Nauk. Politechniki Gdańskiej Nr 540, Budownictwo Lądowe LIII: 1-291
3. Chróścielewski J, Makowski J, Pietraszkiewicz W (2002) Non-linear dynamics of flexible shell structures. Computer Assisted Mechanics and Engineering Sciences 9: 341-357
4. Chróścielewski J, Makowski J, Pietraszkiewicz W (2000) Large overall motion of flexible branched shell structures. In: Ambrosio J A C, Kleiber M (eds.) Computational aspects of nonlinear structural systems with large rigid body motion. NATO Advanced Research Workshop, Pułtusk (Poland), July 2-7 2000. IDMEC, Lisboa, pp 201-218
5. Chróścielewski J, Makowski J, Stumpf H (1997) Finite element analysis of smooth, folded and multi-shell structures. Comp Meth Appl Mech Engng 141: 1-46
6. Libai A, Simmonds J G (1998) The nonlinear theory of elastic shells, 2nd ed. Cambridge University Press, Cambridge
7. Lubowiecka I (2001) Integration of nonlinear dynamic equations of motion in structural mechanics. Dynamics of elastic shell structures (in Polish). PhD Dissertation, Gdańsk University of Technology
8. Lubowiecka I, Chróścielewski J (2002) On dynamics of flexible branched shell structures undergoing large overall motion using finite elements. Comp Struct 80: 891-898
9. Makowski J, Pietraszkiewicz W, Stumpf H (1999) Jump conditions in the non-linear theory of thin irregular shells. J Elasticity 54: 1-26
10. Simo JC, Vu-Quoc L (1988) On the dynamics in space of rods undergoing large motions - a geometrically exact approach. Comp Meth Appl Mech Engng 66: 125-161