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Shell Structures: Theory and Applications

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Preface

This volume contains full texts of 114 papers that have been selected for presentation at the 8th Conference "Shell Structures: Theory and Applications", 12-14 October, 2005, Jurata (Poland), called briefly SSTA2005.

Shells are basic structural elements of modern technology and everyday life. Examples of shell structures include automobile bodies, domes, water and oil tanks, pipelines, silos, ship hulls, aircraft fuselages, turbine blades, nanotubes, but also loudspeaker cones, balloons, parachutes, biological membranes, a human skin, a bottle of wine, or a beer can.

SSTA conferences are traditionally organized by the Section of Structural Mechanics of the Committee for Civil Engineering of the Polish Academy of Sciences in co-operation with other scientific and technological organizations. Previous SSTA conferences were held in Cracow (1974), Gołuń (1978), Opole (1982), Szklarska Poręba (1986), Janowice (1992), and Jurata (1998, 2002). The aim of the meetings is always the same: to bring together scientists, engineers, and other specialists in shell structures in order to discuss important results and new ideas in the field. The goal is to pursue more accurate theoretical models, to develop more powerful and versatile methods of analysis, and to disseminate expertise in design and maintenance of shell structures.

All abstracts sent to organizers of SSTA2005 and the full-text manuscripts submitted to this volume were reviewed by members of the International Advisory Board. The editors are deeply indebted to all members of the Board for their efforts and important contribution to publication of this volume.

The final corrected versions of the manuscripts had to be prepared in a camera-ready form suitable for publication by Taylor & Francis (incorporating A.A. Balkema Publishers). However, many final texts submitted did not adjust to the standard suggested by the publisher. Therefore, most of the final texts have additionally been adjusted to the publishers requirements using DTP techniques, obvious printing errors were corrected, and the English of some texts was refined. We believe that all those technical and linguistic improvements have made some papers better readable an more understandable. We would like to thank very much indeed our associates Dr. J. Górski, Dr. W. Witkowski, Ms. V. Konopińska, and Mr. Sz. Opoka for their assistance and help in bringing the volume to its final form.

The papers published in this volume reflect a wide spectrum of scientific and engineering problems of shell structures. The papers are divided into six broad groups: general lectures, theoretical modelling, stability, dynamics, finite element analyses, and engineering design. In each group there is a number of papers containing original and/or interesting results in the field. We feel that such a grouping of papers allows the reader to find more easily the results he is interested in.

We would like to express our gratitude to all authors for their contributions, and for their willingness and efforts to share their research and development activities with the community of shell structures. We are deeply grateful to the authors of the general lectures Prof. I. Andrianov (Ukraine), Prof. VA. Eremeyev (Russia), Prof. A. Ibrahimbegović (France), Prof. P. Kłosowski (Poland), Prof. B.H. Kröplin (Germany), Prof. H.A. Mang (Austria), and Prof. J.M. Rotter (UK) for their particularly valuable and extensive contributions to this volume.

For organization of SSTA2005, active involvement of the local Organizing Committee has been of primary importance. We thank very much indeed all the members of OC for their long-lasting engagement and exceptional efforts. Last but not least, we would like to gratefully acknowledge the financial support of our sponsors, in particular the Centre for Urban Construction and Rehabilitation, SOFiSTiK AG, and the Foundation for Civil Engineering Development.

Gdańsk, July 2005
Wojciech Pietraszkiewicz
Czesław Szymczak

Shell Structures: Theory and Applications - W. Pietraszkiewicz & C.Szymczak (eds)

On exact dynamic continuity conditions in the theory of branched shells

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ABSTRACT: We formulate the exact, local, resultant, dynamic continuity conditions along the singular surface curve modeling the branched shell structure. The conditions are derived by performing direct through-the-thickness integration in the global equilibrium conditions of continuum mechanics. Possible inaccuracies following from additional fictitious surface loads and double integration over a part of the branching region are compensated by some statically equivalent force and couple fields defined along the singular curve.

1 INTRODUCTION

The general theory of irregular shell structures was initiated by Makowski & Stumpf (1994), and Chróścielewski et al. (1997), developed by Pietraszkiewicz (2001), and summarized and extended by Chróścielewski et al. (2004). In such a shell theory one has to provide also some resultant continuity conditions to be satisfied along the singular surface curves modeling the irregularities.

Chróścielewski et al. (2004) noted that for shells with folds and stepwise thickness, curvature, and/or material changes it is possible to formulate *exactly* the resultant dynamic continuity conditions by a direct through-the-thickness integration of corresponding local balance laws of continuum mechanics. However, in the case of shell branching, self-intersection, complex stiffening and/or rigid or deformable junctions there is some ambiguity in defining both the shell base surface and thickness in the regions of irregularity. Different approximate mechanical models are used in the literature to treat these regions leading to inaccuracies in the resulting dynamic continuity conditions.

In this paper we refine the through-the-thickness reduction procedure and apply it to the formulation of the resultant dynamic continuity conditions at the singular surface curve modeling the shell branching. It is noted that in reducing 3D fields to their resultants prescribed at the base surface there may appear additional fictitious surface loads and double integration over a part of the branching region. We compensate the possible surplus of the resultant fields by subtracting some statically equivalent resultant force and couple fields applied along the singular curve. As a result, our local, resultant, dynamic continuity conditions for the branched shell become *exact implications* of the global equilibrium conditions of continuum mechanics.

2 NOTATION AND PRELIMINARY RELATIONS

A shell is a 3D solid body identified in a reference (undeformed) placement with a region B of the physical space $\mathcal{E}$. The shell boundary ∂B consists of three separable parts: the upper M^+ and lower M^- shell faces, and the lateral boundary surface $\partial\mathrm{B}^*$ such that $\partial\mathrm{B} = M^+ \cup M^- \cup \partial\mathrm{B}^*$, $\quad M^+ \cap M^- = \emptyset$.

The position vector $\mathbf{x}$ of any shell point $\mathrm{x} \in \mathrm{B}$ can be described by $\mathbf{x}(x,\xi) = \boldsymbol{x}(x) + \xi\boldsymbol{t}(x)$. Here $\boldsymbol{x}(x) = \mathbf{x}(x,0)$ is the position vector of a point x on some reference base surface M arbitrarily located in B, $-h^- \leq \xi \leq +h^+$ is the distance along ξ from M with $h = h^- + h^+ > 0$ the initial shell thickness, and $\boldsymbol{t}(x)$ is the unit vector of the rectilinear coordinate line ξ not necessarily normal to M.

The position vector $\mathbf{y} = \chi(\mathbf{x})$ of the shell in the deformed placement $\bar{\mathrm{B}} = \chi(\mathrm{B})$ can formally be represented by

$$\mathbf{y}(x,\xi) = \boldsymbol{y}(x) + \boldsymbol{\zeta}(x,\xi), \quad \boldsymbol{\zeta}(x,0) = \mathbf{0}, \tag{1}$$

where $\boldsymbol{y}$ is the position vector of the deformed base surface $\bar{M} = \chi(M)$, which is a material surface during deformation process.

Let $\mathrm{P} \subset \mathrm{B}$ be an arbitrary part of the 3D shell B with the boundary consisting of three separable parts: $\partial\mathrm{P} = \Pi^+ \cup \Pi^- \cup \partial\mathrm{P}^*$. Then in the referential description the 3D global equilibrium conditions can be expressed by vanishing of the total force vector $\mathbf{F}(\mathrm{P})$ and the total torque vector $\mathbf{T}_\mathrm{o}(\mathrm{P})$ taken relative to an arbitrary point

$o \in \mathcal{E}$ of all forces acting on P :

$$\mathbf{F}(\mathrm{P}) = \iiint_{\mathrm{P}} \mathbf{f}\, \mathrm{d}v + \iint_{\partial \mathrm{P} \setminus \partial \mathrm{B}_f} \mathbf{t}_n\, \mathrm{d}a + \iint_{\partial \mathrm{P} \cap \partial \mathrm{B}_f} \mathbf{t}^*\, \mathrm{d}a = \mathbf{0}, \tag{2}$$

$$\mathbf{T}_o(\mathrm{P}) = \iiint_{\mathrm{P}} \mathbf{y} \times \mathbf{f}\, \mathrm{d}v + \iint_{\partial \mathrm{P} \setminus \partial \mathrm{B}_f} \mathbf{y} \times \mathbf{t}_n\, \mathrm{d}a + \iint_{\partial \mathrm{P} \cap \partial \mathrm{B}_f} \mathbf{y} \times \mathbf{t}^*\, \mathrm{d}a = \mathbf{0}.$$

In (2), $\partial \mathrm{B}_f$ is that part of $\partial \mathrm{B}$ on which the external surface force field $\mathbf{t}^*(\mathrm{x})$ is prescribed, $\mathbf{f}(\mathrm{x})$ is the volume force field, and $\mathbf{t}_n(\mathrm{x})$ is the contact force field.

If (1) and $\mathbf{x} = \boldsymbol{x} + \xi \boldsymbol{t}$ are introduced into (2) one can perform an exact through-the-thickness integration with regard to the coordinate ξ. The global equilibrium conditions (2) can then be expressed through the resultant fields defined entirely on the reference base surface M, and this is an appropriate formulation for the 2D theory of shells. In case of a regular shell, such an exact reduction procedure with regard to a non-material weighted surface of mass was suggested by Simmonds (1984), see also Libai & Simmonds (1983, 1998). In what follows we perform such an exact reduction of the equilibrium conditions (2) with regard to the material base surface in the special case of the branched shell.

3 BRANCHED SHELL

Let the shell B consist of three regular parts B_1, B_2, B_3 connected together along a common junction. The base surface M of B consists now of three regular parts M_1, M_2, M_3 connected along the surface curve $\Gamma = \partial M_1 \cap \partial M_2 \cap \partial M_3$. Cutting off an arbitrary part P of B containing the junction, let us discuss the reduction of its global equilibrium conditions (2).

By the through-the-thickness reduction procedure the shell parts P_1, P_2, P_3 are reduced to their images Π_1, Π_2, Π_3 and Γ at $\Pi \subset M$. In the reduction procedure there are two parts P_{1d} and P_{2d} of the branching region where the through-the-thickness integration is performed twice: once when reducing spatial forces given in P_1, P_2 and on $\partial \mathrm{P}_1$, $\partial \mathrm{P}_2$ to their resultants defined on Π_1, Π_2 and along $\partial \Pi_1$, $\partial \Pi_2$, respectively, and the second time when reducing spatial forces given in P_3 and on $\partial \mathrm{P}_3$ to their resultants defined on Π_3 and along $\partial \Pi_3$, see Figure 1. Additionally, when extending the parts P_1, P_2, P_3 into the junction region some fictitious surface forces are implicitly applied on the extended surfaces Π^+_{1d}, Π^+_{2d}, Π^+_{3d}, Π^-_{3d} marked in Figure 1. In order to compensate the possible surplus of the resultant surface forces and couples on Π one has to subtract some resultant forces and couples along Γ following from the loads taken twice in P_{1d}, P_{2d} and on $\partial \mathrm{P}_{1d}$, $\partial \mathrm{P}_{2d}$ as well as the ones applied on the extended surfaces.

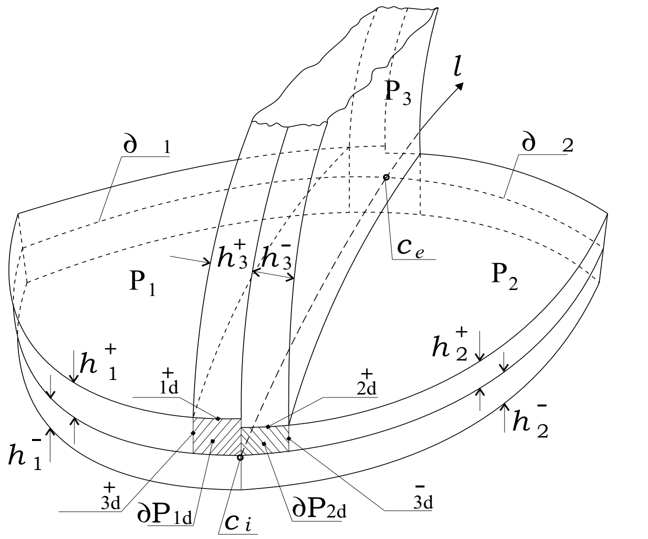

Figure 1. The part of branched shell: regions of double integration.

After performing integration with regard to ξ, the total force vector $\mathbf{F}_1(\mathrm{P}_1)$ defined in (2) of all spatial forces acting in P_1 and on $\partial \mathrm{P}_1$ is given by

$$\mathbf{F}_1(\Pi_1) = \iint_{\Pi_1} \boldsymbol{f}_1\, \mathrm{d}a + \int_{\partial \Pi_1 \setminus \partial M_f} \boldsymbol{n}_{1\nu}\, \mathrm{d}l + \int_{\partial \Pi_1 \cap \partial M_f} \boldsymbol{n}_1^*\, \mathrm{d}l - \int_{\Gamma} \boldsymbol{f}_{1\Gamma}\, \mathrm{d}l - \boldsymbol{n}_{1e} + \boldsymbol{n}_{1i}, \tag{3}$$

where

$$\boldsymbol{f}_1 = \int_{-h_1^-}^{+h_1^+} \mathbf{f}_1 \mu_1\, \mathrm{d}\xi + \alpha_1^+ \mathbf{t}_1^{*+} - \alpha_1^- \mathbf{t}_1^{*-},$$

$$\boldsymbol{n}_{1\nu} = \int_{-h_1^-}^{+h_1^+} \alpha_1^* \mathbf{t}_{1n}\, \mathrm{d}\xi, \quad \boldsymbol{n}_1^* = \int_{-h_1^-}^{+h_1^+} \alpha_1^* \mathbf{t}_1^*\, \mathrm{d}\xi,$$

$$\boldsymbol{f}_{1\Gamma} = \int_0^{+h_3^+} \alpha_1^+ \mathbf{t}_1^{*+} \alpha_3^*\, \mathrm{d}\xi + \int_0^{+h_1^+} \alpha_3^+ \mathbf{t}_3^{*+} \alpha_1^*\, \mathrm{d}\xi + \int_0^{+h_1^+} \left(\int_0^{+h_3^+} \mathbf{f}_1 \mu_3\, \mathrm{d}\xi \right) \alpha_1^*\, \mathrm{d}\xi, \tag{4}$$

$$\boldsymbol{n}_{1i} = \iint_{\partial \mathrm{P}_{1d}} \mathbf{t}_{1n} \mathrm{d}a^* \quad \text{at} \quad c_i\,,$$

$$\boldsymbol{n}_{1e} = \iint_{\partial \mathrm{P}_{1d}} \mathbf{t}_{1n} \mathrm{d}a^* \quad \text{at} \quad c_e\,.$$

In (4) we have used the geometric expansion coefficients μ, $\alpha^{\pm}$, α^{*} relating differentials outside Π to those on Π and along $\partial\Pi$:

$$\mathrm{d}v = \mu \mathrm{d}\xi \mathrm{d}a, \quad \mathrm{d}a^{\pm} = \alpha^{\pm}\mathrm{d}a, \quad \mathrm{d}a^{*} = \alpha^{*}\mathrm{d}\xi \mathrm{d}l. \tag{5}$$

When $c_i \in \Gamma \cap \partial M_f$ and/or $c_e \in \Gamma \cap \partial M_f$, the vectors $\boldsymbol{n}_{1i}$ and $\boldsymbol{n}_{1e}$ in $(4)_{4,5}$ should be calculated from the known traction $\mathbf{t}_1^{*}$ applied on $\partial \mathrm{B}_f$, instead of $\mathbf{t}_{1n}$.

In exactly the same way we can calculate the total force vector $\mathbf{F}_2(\Pi_2)$ of all spatial forces acting on P_2. The result is expressed through the fields $\boldsymbol{f}_2$, $\boldsymbol{n}_{2\nu}$, $\boldsymbol{n}_2^{*}$, $\boldsymbol{f}_{2\Gamma}$, $\boldsymbol{n}_{2i}$, $\boldsymbol{n}_{2e}$ defined on Π_2 and $\partial\Pi_2$ in complete analogy to those given in (4).

The total force vector $\mathbf{F}_3(\Pi_3)$ is calculated as for the regular shell region by direct integration in (2) with regard to ξ leading to

$$\mathbf{F}_3(\Pi_3) = \iint_{\Pi_3} \boldsymbol{f}_3 \,\mathrm{d}a + \int_{\partial\Pi_3 \setminus \partial M_f} \boldsymbol{n}_{3\nu} \,\mathrm{d}l + \int_{\partial\Pi_3 \cap \partial M_f} \boldsymbol{n}_3^{*} \,\mathrm{d}l. \tag{6}$$

whare the fields $\boldsymbol{f}_3$, $\boldsymbol{n}_{3\nu}$, $\boldsymbol{n}_3^{*}$ are defined analogously to (4).

Summing up the results for $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ we can write

$$\mathbf{F}(\Pi) = \iint_{\Pi\setminus\Gamma} \boldsymbol{f} \,\mathrm{d}a + \int_{\partial\Pi\setminus\partial M_f} \boldsymbol{n}_{\nu} \,\mathrm{d}l + \int_{\partial M_f} \boldsymbol{n}^{*} \,\mathrm{d}l - \int_{\Gamma} \boldsymbol{f}_{\Gamma} \,\mathrm{d}l - \boldsymbol{n}_e + \boldsymbol{n}_i. \tag{7}$$

In (7) the resultant surface forces $\boldsymbol{f}$, the surface stress resultants $\boldsymbol{n}_{\nu}$, and the resultant boundary forces $\boldsymbol{n}^{*}$ are defined in all three parts of P, while the compensating curvilinear force resultants $\boldsymbol{f}_{\Gamma}$ and concentrated forces $\boldsymbol{n}_i$, $\boldsymbol{n}_e$ follow only from P_{1d}, P_{2d}, Π_{1d}^{+}, Π_{2d}^{+}, Π_{3d}^{+}, Π_{3d}^{-} and $\partial\mathrm{P}_{1d}$, $\partial\mathrm{P}_{2d}$ taken into account in $\mathbf{F}_1(\Pi_1)$ and $\mathbf{F}_2(\Pi_2)$, respectively.

The total torque vector $\mathbf{T}_{\mathrm{o}}(\Pi)$ relative to $\mathrm{o} \in \mathcal{E}$ of all spatial forces acting on P can again be calculated by direct integration in (2) with regard to ξ. The procedure is exactly the same as in (3)-(7), only when calculating the surface couples one has to introduce the following exact relations for the 3D position vector in the deformed placement:

$$\mathbf{y} = \boldsymbol{y} + \boldsymbol{\zeta}, \quad \mathbf{y}^{+} = \boldsymbol{y} + \boldsymbol{\zeta}^{+}, \quad \mathbf{y}^{-} = \boldsymbol{y} + \boldsymbol{\zeta}^{-}. \tag{8}$$

In the regions P_{1d} and P_{2d} the compensating couples should be reduced relative to the singular curve Γ, and the deformed position vectors should be taken in the forms

$$\mathbf{y} = \boldsymbol{y}_{\Gamma} + \boldsymbol{\zeta}_{\Gamma}, \;\; \mathbf{y}^{+} = \boldsymbol{y}_{\Gamma}^{+} + \boldsymbol{\zeta}_{\Gamma}^{+}, \;\; \mathbf{y}^{-} = \boldsymbol{y}_{\Gamma}^{-} + \boldsymbol{\zeta}_{\Gamma}^{-}. \tag{9}$$

Performing appropriate transformations similar to those leading to (7) we obtain the total torque vector $\mathbf{T}_{\mathrm{o}}(\Pi)$ of the branched shell expressed only by fields defined on the base surface

$$\begin{aligned}\mathbf{T}_{\mathrm{o}}(\Pi) = &\iint_{\Pi\setminus\Gamma} (\boldsymbol{c} + \boldsymbol{y} \times \boldsymbol{f}) \,\mathrm{d}a \\ &+ \int_{\partial\Pi\setminus\partial M_f} (\boldsymbol{m}_{\nu} + \boldsymbol{y} \times \boldsymbol{n}_{\nu}) \,\mathrm{d}l \\ &+ \int_{\partial M_f} (\boldsymbol{m}^{*} + \boldsymbol{y} \times \boldsymbol{n}^{*}) \,\mathrm{d}l \\ &- \int_{\Gamma} (\boldsymbol{c}_{\Gamma} + \boldsymbol{y}_{\Gamma} \times \boldsymbol{f}_{\Gamma}) \,\mathrm{d}l \\ &- (\boldsymbol{m}_e + \boldsymbol{y}_{\Gamma e} \times \boldsymbol{n}_e) + (\boldsymbol{m}_i + \boldsymbol{y}_{\Gamma i} \times \boldsymbol{n}_i).\end{aligned} \tag{10}$$

Again, in (10) the resultant surface couples $\boldsymbol{c}$, stress couples $\boldsymbol{m}_{\nu}$, and boundary couples $\boldsymbol{m}^{*}$ are defined in all three parts of P, while the compensating curvilinear couple resultants $\boldsymbol{c}_{\Gamma}$ and the concentrated couples $\boldsymbol{m}_i$, $\boldsymbol{m}_e$ follow only from P_{1d}, P_{2d}, Π_{1d}^{+}, Π_{2d}^{+}, Π_{3d}^{+}, Π_{3d}^{-} and $\partial\mathrm{P}_{1d}$, $\partial\mathrm{P}_{2d}$ taken into account in $\mathbf{T}_{\mathrm{o}1}(\Pi_1)$ and $\mathbf{T}_{\mathrm{o}2}(\Pi_2)$, respectively.

The relations (7) and (10) are exact 2D static equivalents of $\mathbf{F}(\mathrm{P})$ and $\mathbf{T}_{\mathrm{o}}(\mathrm{P})$ appearing in the 3D global equilibrium conditions (2) for an arbitrary part P of the shell B treated as a 3D solid.

4 DYNAMIC CONTINUITY CONDITIONS

The global equilibrium conditions (2) with the total force and torque given through the surface fields by (7) and (10), should now be appropriately transformed. In particular, we have to:

a) use the surface Cauchy postulates $\boldsymbol{n}_{\nu} = \boldsymbol{N}\boldsymbol{\nu}$, $\boldsymbol{m}_{\nu} = \boldsymbol{M}\boldsymbol{\nu}$, with $\boldsymbol{\nu}$ the unit external normal vector at $\partial\Pi$, which allow us to introduce the surface stress resultant $\boldsymbol{N}$ and resultant couple $\boldsymbol{M}$ tensors;

b) apply appropriate generalized surface divergence theorems (see Chróścielewski et al. 2004) which allow us to represent the curvilinear integrals over $\partial\Pi$ of the fields $\boldsymbol{N}\boldsymbol{\nu}$ and $\boldsymbol{M}\boldsymbol{\nu}$ by the surface integrals over Π of the surface divergences $Div\,\boldsymbol{N}$ and $Div\,\boldsymbol{M}$ together with additional jump terms along Γ;

c) integrate by parts along Γ, which allow us to represent the concentrated vectors $\boldsymbol{n}_i$, $\boldsymbol{n}_e$ and $\boldsymbol{m}_i$, $\boldsymbol{m}_e$ by curvilinear integrals over Γ of some distributed force and couple vectors $\boldsymbol{n}$ and $\boldsymbol{m}$.

As a result of all the transformations the global equilibrium conditions for the branched shell take the forms (for details see Konopińska and

Pietraszkiewicz 2005)

$$\mathbf{F}(\Pi) = \iint_{\Pi\setminus\Gamma} (Div\,\boldsymbol{N} + \boldsymbol{f})\,\mathrm{d}a + \int_{\partial M_f} (\boldsymbol{n}^* - \boldsymbol{N\nu})\,\mathrm{d}l$$

$$- \int_{\Pi\cap\Gamma} \left(\boldsymbol{n}' + [\boldsymbol{N\nu}] + \boldsymbol{f}_\Gamma\right)\,\mathrm{d}l, \tag{11}$$

$$\mathbf{T}_o(\Pi) = \iint_{\Pi\setminus\Gamma} \{Div\,\boldsymbol{M} + ax(\boldsymbol{NF}^T - \boldsymbol{FN}^T) + \boldsymbol{c}$$

$$+ \boldsymbol{y} \times (Div\,\boldsymbol{N} + \boldsymbol{f})\}\,\mathrm{d}a$$

$$+ \int_{\partial M_f} \{(\boldsymbol{m}^* - \boldsymbol{M\nu}) + \boldsymbol{y} \times (\boldsymbol{n}^* - \boldsymbol{N\nu})\}\,\mathrm{d}l$$

$$- \int_{\Gamma} \{\boldsymbol{m}' + \boldsymbol{y}'_\Gamma \times \boldsymbol{n} + [\boldsymbol{M\nu}] + \boldsymbol{c}_\Gamma \tag{12}$$

$$+ \boldsymbol{y}_\Gamma \times \left(\boldsymbol{n}' + [\boldsymbol{N\nu}] + \boldsymbol{f}_\Gamma\right)\}\,\mathrm{d}l,$$

where by $ax(.)$ we understand the axial vector of the skew tensor $(.)$, and by **[.]** we denote jumps along the singular curve Γ defined by

$$[\boldsymbol{N\nu}] = \sum_{k=1}^{3} \boldsymbol{N}^{(k)}\boldsymbol{\nu}^{(k)},$$

$$[\boldsymbol{M\nu}] = \sum_{k=1}^{3} \boldsymbol{M}^{(k)}\boldsymbol{\nu}^{(k)}. \tag{13}$$

In (13) the upper index (k) means the finite limit of the corresponding field when approaching the common singular curve Γ from inside of $M^{(k)}$, $k = 1, 2, 3$.

Vanishing of the first two integrals of (11) and (12) leads to the known local equilibrium equations to be satisfied at each regular point of M and dynamic boundary conditions to be satisfied at each regular point of ∂M_f, see Libai & Simmonds (1983, 1998), Simmonds (1984), Pietraszkiewicz (2001), and Chróścielewski et al. (2004).

Vanishing of the third integrals of (11) and (12) is assured by the local, resultant, dynamic continuity conditions

$$\boldsymbol{n}' + [\boldsymbol{N\nu}] + \boldsymbol{f}_\Gamma = \boldsymbol{0},$$

$$\boldsymbol{m}' + \boldsymbol{y}'_\Gamma \times \boldsymbol{n} + [\boldsymbol{M\nu}] + \boldsymbol{c}_\Gamma = \boldsymbol{0} \tag{14}$$

to be satisfied at each regular point of Γ.

The conditions (14) are the ordinary differential equations along Γ which differ from the equilibrium equations of rods by the jump terms describing interaction between regular shell parts along the junction.

5 CONCLUSIONS

We have presented new and exact expressions (14) for the local, resultant, dynamic continuity conditions to be satisfied along the singular curve Γ being the common boundary of three regular branches of the shell base surface. The conditions have been derived by performing a direct through-the-thickness integration in the global equilibrium conditions of continuum mechanics. Therefore, our dynamic continuity conditions are valid for an arbitrary shell thickness, internal through-the-thickness structure and/or material properties, as well as for unrestricted values of translations, rotations, strains and/or bendings of the shell material elements.

ACKNOWLEDGEMENTS

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