On jump conditions at non-material singular curves in the resultant shell thermomechanics

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ABSTRACT: The global, refined, resultant, two-dimensional (2D) balance laws of mass, linear and angular momenta, and energy as well as the entropy inequality were formulated by Pietraszkiewicz (2011) as exact implications of corresponding laws of 3D rational thermomechanics. In case of a shell with the regular base surface and all resultant surface fields differentiable everywhere on it and at any time instant, the local laws of the resultant shell thermomechanics in the referential (Lagrangian) description were also given. In the present contribution, on the undeformed base surface a moving, non-material, singular surface curve representing a discontinuous thermomechanical process is allowed at which some resultant surface fields may not be differentiable. In such a case, to derive the local field equations we have extended the surface transport relation and the surface divergence theorems. With these extensions, the referential local laws of the resultant shell thermomechanics are supplemented here by the corresponding referential jump conditions at the non-material singular curve moving relative to the reference base surface.

1 INTRODUCTION

The jump (also called the continuity) conditions at singular 2D surfaces are used in 3D continuum thermomechanics for proper modeling of such phenomena as wave propagation, phase transition, strain localization, fracture, etc. We refer for example to Truesdell & Toupin (1960), Truesdell & Noll (1965), Kosiński (1986), Abeyaratne & Knowles (2006), Gurtin et al. (2010), and the references given there.

In 2D shell thermomechanics some jump conditions at singular 1D surface curves were formulated in the report by Makowski & Pietraszkiewicz (2002) and modified versions of the conditions were used by Eremeyev & Pietraszkiewicz (2009, 2011) to model phase transition phenomena in shells.

Pietraszkiewicz (2011) worked out the refined resultant thermomechanics of shells by direct through-the-thickness integration of corresponding laws of 3D rational thermomechanics proposed by Truesdell & Toupin (1960). The resultant 2D balance of energy in Pietraszkiewicz (2011) was completed with an additional 2D stress power called an interstitial working after Dunn & Serrin (1985). Such resultant 2D balance laws and the entropy inequality of the resultant shell thermomechanics can be regarded as exact implications of corresponding 3D laws of rational thermomechanics.

2 NOTATION

In the undeformed (reference) placement the shell is represented by the regular smooth base surface M. It is assumed that in the deformed (current) placement the base surface becomes also the regular smooth surface $M(t)=\chi(M,t)$, where χ is the deformation function and t is time. By $x\in M$ and $y=\chi(x,t)\in M(t)$ we denote corresponding placements of a material particle of the base surface in the 3D physical space $\mathcal{E}$ with E as its translation vector space. Then $\boldsymbol{x}=x-\mathrm{o}\in E$ and $\boldsymbol{y}=y-\mathrm{o}\in E$ are the respective position vectors of the surface points x and y in an inertial frame $(\mathrm{o},\mathbf{e}_i)$, where $\mathrm{o}\in\mathcal{E}$ is an origin and $\mathbf{e}_i\in E$, $i=1,2,3$, are orthonormal vectors. The base surface M may be explicitly defined by $\boldsymbol{x}=\boldsymbol{x}(\theta^\alpha)$, where $\theta^\alpha, \alpha=1,2$, are curvilinear surface coordinates. The surface M is oriented by a choice of unit normal vector $\boldsymbol{n}(x)$. The space of all vectors perpendicular to $\boldsymbol{n}(x)$ is then the tangent space T_xM at $x\in M$, and a vector field $\boldsymbol{t}$ on M is tangential if $\boldsymbol{t}(x)\in T_xM$ at every $x\in M$. Given a regular smooth part $\Pi\subset M$ with a piecewise smooth boundary $\partial\Pi$, the outward unit normal $\boldsymbol{\nu}$ at regular $x\in\partial\Pi$ is directed outward of $\partial\Pi$ and tangent to M.

Let $\varphi(x)\in R$, $\boldsymbol{a}(x)\in E$, and $\boldsymbol{T}(x)\in E\otimes E$ be smooth scalar-valued, vector-valued, and 2$^{\mathrm{nd}}$-order tensor-valued fields on M, respectively. Then the surface gradient operator $Grad$ applied to the fields

$\varphi, \boldsymbol{a}, \boldsymbol{T}$ leads to $Grad\,\varphi(x) \in T_x M$, a tangential vector field, $Grad\,\boldsymbol{a}(x) \in E \otimes T_x M$, a mixed 2^{nd}-order tensor field, and $Grad\,\boldsymbol{T}(x) \in E \otimes E \otimes T_x M$, a mixed 3^{rd}-order tensor field. Such surface gradient fields can be defined applying results given in Gurtin & Murdoch (1975) and Gurtin et al. (2010).

The surface divergence Div of a vector $\boldsymbol{a}(x) \in E$ and a mixed 2^{nd}-order tensor $\boldsymbol{S}(x) \in E \otimes T_x M$ fields on M are defined respectively by $Div\,\boldsymbol{a}(x) = \mathrm{tr}\left[\boldsymbol{P}\,Grad\,\boldsymbol{a}(x)\right]$ and $[Div\boldsymbol{S}(x)] \cdot \boldsymbol{c} = Div[\boldsymbol{S}^T(x)\boldsymbol{c}]$ for any $\boldsymbol{c} \in E$, where $\boldsymbol{P}$ is the perpendicular projection onto M.

With the above definitions the surface divergence theorems valid on regular smooth parts Π of M can be given as modifications of those presented in Gurtin and Murdoch (1975),

$$\int_{\partial\Pi} \boldsymbol{a} \cdot \boldsymbol{\nu}\, ds = \iint_{\Pi} \left(Div\,\boldsymbol{a} + 2Ha_n \right) da\,,$$

$$\int_{\partial\Pi} \boldsymbol{S}\boldsymbol{\nu}\, ds = \iint_{\Pi} Div\,\boldsymbol{S}\, da\,,$$

$$\int_{\partial\Pi} \boldsymbol{a} \times \boldsymbol{S}\boldsymbol{\nu}\, ds = \tag{2}$$

$$\iint_{\Pi} \left\{ \boldsymbol{a} \times \left(Div\,\boldsymbol{S} \right) + \mathrm{ax}\left[\boldsymbol{S}(Grad\,\boldsymbol{a})^T - (Grad\,\boldsymbol{a})\boldsymbol{S}^T \right] \right\} da$$

where $a_n = \boldsymbol{n} \cdot \boldsymbol{a}$ and $H = -(1/2)\,\mathrm{tr}\left(\boldsymbol{P}Grad\boldsymbol{n}\right)$ is the mean curvature at $x \in \Pi$.

3 MOVING NON-MATERIAL CURVE

The global, resultant balance laws and entropy inequality of shell thermomechanics formulated in Pietraszkiewicz (2011) each involves the material time derivative of a surface integral. In the absence of singular curves, the standard transport theorem on any fixed part of M allows one to change the order of surface integration and material time differentiation. But for the reference shell base surface containing a moving, non-material, singular curve the transport relation as well as the divergence theorems (2) have to be carefully extended to take into account the effect of the moving discontinuity.

A surface curve moving on M over a time interval $I = [t_0, t_1],\, t_0 < t_1$, is a one–parametric family $C(t)$ of piecewise smooth surface curves oriented consistently with the orientation of M, which are parameterized by the arc length coordinate s introduced by $\theta^\alpha = \theta^\alpha(s)$. With each regular point $x_C \in C(t)$ we can associate the triad of orthonormal vectors: the tangent $\boldsymbol{\tau}_C$, the normal $\boldsymbol{n}_C = \boldsymbol{n}$, and the exterior normal $\boldsymbol{\nu}_C = \boldsymbol{\tau}_C \times \boldsymbol{n}$. Velocity of $C(t)$ relative to M is a tangential vector field $\boldsymbol{\upsilon}$ and its exterior normal component $V = \boldsymbol{\upsilon} \cdot \boldsymbol{\nu}_C$ measures the speed with which the curve $C(t)$ transverses the surface M.

Let $\Pi \subset M$ be an arbitrary fixed, regular, closed region of M containing a portion of $C(t)$ in its interior. The surface curve $C(t)$ separates the region Π into two time-dependent, closed, complementary subregions $\Pi^-(t)$ and $\Pi^+(t)$ such that $\Pi^-(t) \cap \Pi^+(t) = C(t)$. Their boundaries consist of two parts $\partial\Pi^\mp(t) = \left(\partial\Pi^\mp(t) \setminus C(t)\right) \cup C(t)$. At each regular point of $C(t)$ the exterior normal vector $\boldsymbol{\nu}^-$ of $\partial\Pi^-(t)$ coincides with the unit vector $\boldsymbol{\nu}_C$ of $C(t)$. Thus, the exterior normal velocity of $\partial\Pi^-(t)$ is equal to V on $C(t)$ and vanishes on $\partial\Pi^-(t) \setminus C(t)$. Likewise, the vector $\boldsymbol{\nu}^+$ of $\partial\Pi^+(t)$ coincides with $-\boldsymbol{\nu}_C$ of $C(t)$, so that the exterior normal velocity of $\partial\Pi^+(t)$ becomes $-V$ on $C(t)$ and vanishes on $\partial\Pi^+(t) \setminus C(t)$.

Let a smooth time-dependent field $\boldsymbol{\Phi}(x,t)$, with $\boldsymbol{\Phi}$ belonging to any finite-dimensional vector space such as R, $T_x M$, E, $E \otimes T_x M$ etc., is defined only in the interior of $M \setminus C(t)$, but it need not be defined on $C(t)$. But we suppose that at each instant $t \in I$ one-sided finite limits of $\boldsymbol{\Phi}(x,t)$ exist at regular $x_C \in C(t)$. We write $\boldsymbol{\Phi}^-$ for the finite limit of $\boldsymbol{\Phi}$ as C is approached from Π^- and $\boldsymbol{\Phi}^+$ for the one as C is approached from Π^+. Then $[[\boldsymbol{\Phi}]] = \boldsymbol{\Phi}^+ - \boldsymbol{\Phi}^-$ denotes the jump of $\boldsymbol{\Phi}$ at $C(t)$. If $[[\boldsymbol{\Phi}]]$ does not vanish identically, the curve $C(t)$ is said to be singular with respect to $\boldsymbol{\Phi}(x,t)$ at time t.

For the field $\boldsymbol{\Phi}(x,t)$ smooth on the closed subregions $\Pi^-(t)$ and $\Pi^+(t)$, by the Reynolds trans-port theorem for the smoothly evolving subregions $\Pi^\mp(t)$ with moving boundaries $\partial\Pi^\mp(t)$ we have

$$\frac{d}{dt}\iint_{\Pi^-(t)} \boldsymbol{\Phi}\, da = \iint_{\Pi^-(t)} \dot{\boldsymbol{\Phi}}\, da + \int_{\partial\Pi^-(t) \cap C(t)} V\boldsymbol{\Phi}^-\, ds\,, \tag{3}$$

$$\frac{d}{dt}\iint_{\Pi^+(t)} \boldsymbol{\Phi}\, da = \iint_{\Pi^+(t)} \dot{\boldsymbol{\Phi}}\, da - \int_{\partial\Pi^+(t) \cap C(t)} V\boldsymbol{\Phi}^+\, ds\,. \tag{4}$$

From the equations (3) and (4) follows the referential form of the surface transport relation valid for any piecewise smooth field $\boldsymbol{\Phi}(x,t)$ given on Π in the presence of the singular curve $C(t)$:

$$\frac{d}{dt}\iint_{\Pi} \boldsymbol{\Phi}\, da = \iint_{\Pi} \dot{\boldsymbol{\Phi}}\, da - \int_{\Pi \cap C(t)} V[[\boldsymbol{\Phi}]]\, ds. \tag{5}$$

Let us now extend the surface divergence theorems (2) in the presence of the singular surface curve. As there is no time differentiation here, our discussion is confined to a fixed time.

For example, let the surface mixed 2^{nd}-order tensor field $\boldsymbol{S}(x,t) \in E \otimes T_x M$ be piecewise smooth on any fixed, closed $\Pi \subset M$ divided into two regular, closed, complementary parts Π^- and Π^+ as above by the singular curve C. Again, $\boldsymbol{\nu}^-$ of $\partial\Pi^-$ becomes $\boldsymbol{\nu}_C$ on C and $\boldsymbol{\nu}$ elsewhere on $\partial\Pi^-$, and $\boldsymbol{\nu}^+$ of $\partial\Pi^+$ becomes $-\boldsymbol{\nu}_C$ on C and $\boldsymbol{\nu}$ elsewhere on $\partial\Pi^+$. Applying the surface divergence theorem $(2)_2$ separately on the parts Π^- and Π^+ of Π, we obtain

$$\int_{\partial\Pi} \boldsymbol{S}\boldsymbol{\nu} ds = \int_{\partial\Pi^-} \boldsymbol{S}\boldsymbol{\nu}^- ds + \int_{\partial\Pi^+} \boldsymbol{S}\boldsymbol{\nu}^+ ds$$
$$- \int_{\partial\Pi^- \cap C} \boldsymbol{S}^-\boldsymbol{\nu}^- ds - \int_{\partial\Pi^+ \cap C} \boldsymbol{S}^+\boldsymbol{\nu}^+ ds$$

$$= \iint_{\Pi^-} Div\boldsymbol{S}\,da + \iint_{\Pi^+} Div\boldsymbol{S}\,da + \int_{\Pi\cap C}\left(\boldsymbol{S}^+ - \boldsymbol{S}^-\right)\boldsymbol{\nu}_C\,ds\,, \tag{6}$$

so that

$$\int_{\partial\Pi} \boldsymbol{S\nu}\,ds = \iint_{\Pi} Div\boldsymbol{S}\,da + \int_{\Pi\cap C} [\![\boldsymbol{S}]\!]\boldsymbol{\nu}_C\,ds\,. \tag{7}$$

Analogous arguments lead to the following extensions of the surface divergence theorems $(2)_1$ and $(2)_3$ in the presence of the singular surface curve:

$$\int_{\partial\Pi} \boldsymbol{t}\cdot\boldsymbol{\nu}\,ds = \iint_{\Pi} Div\boldsymbol{t}\,da + \int_{\Pi\cap C} [\![\boldsymbol{t}]\!]\cdot\boldsymbol{\nu}_C\,ds\,, \tag{8}$$

$$\int_{\partial\Pi} \boldsymbol{a}\times\boldsymbol{S\nu}\,ds = \iint_{\Pi}\Big\{\boldsymbol{a}\times\left(Div\boldsymbol{S}\right) + \mathrm{ax}\Big[\boldsymbol{S}\left(Grad\,\boldsymbol{a}\right)^T - \left(Grad\,\boldsymbol{a}\right)\boldsymbol{S}^T\Big]\Big\}da + \int_{\Pi\cap C} [\![\boldsymbol{a}\times\boldsymbol{S}]\!]\boldsymbol{\nu}_C\,ds\,. \tag{9}$$

4 JUMP CONDITIONS AT NON-MATERIAL SINGULAR CURVE

In the refined, resultant thermomechanics of shells developed by Pietraszkiewicz (2011) three surface fields on M were used as independent field variables: the position vector $\boldsymbol{y}(x,t)\in E$ of the deformed base surface $M(t)$ (or equivalently the translation vector $\boldsymbol{u}(x,t)=\boldsymbol{y}(x,t)-\boldsymbol{x}$ of M), the gross rotation tensor $\boldsymbol{Q}(x,t)\in \mathrm{Orth}^+$ of the shell cross section, and the mean referential temperature field $\theta(x,t)>0$. In order to ensure that the resultant 2D balance of energy be an exact implication of 3D energy balance of rational thermomechanics, an interstitial working flux vector field $\boldsymbol{w}(x,t)\in T_xM$ was added to the resultant 2D balance of energy. Then for any regular part $\Pi\subset M$ the referential 2D laws of shell thermomechanics - the balances of mass, linear and angular momenta, and energy as well as the entropy inequality - became the following exact resultant implications of corresponding 3D laws of rational thermomechanics:

$$\frac{d}{dt}\int_{\Pi}\rho\,da - \iint_{\Pi} c\,da = 0\,, \tag{10}$$

$$\iint_{\Pi}\rho\boldsymbol{f}\,da - \frac{d}{dt}\iint_{\Pi}\boldsymbol{l}\,da + \int_{\partial\Pi\backslash\partial M_f}\boldsymbol{N\nu}\,ds + \int_{\partial\Pi\cap\partial M_f}\boldsymbol{n}^*\,ds = \boldsymbol{0}\,, \tag{11}$$

$$\iint_{\Pi}\rho\boldsymbol{c}\,da - \frac{d}{dt}\iint_{\Pi}\boldsymbol{k}\,da + \iint_{\Pi}\left(\boldsymbol{y}+\rho\boldsymbol{f}\right)da - \frac{d}{dt}\iint_{\Pi}\left(\boldsymbol{y}\times\boldsymbol{l}\right)da + \int_{\partial\Pi\backslash\partial M_f}\left(\boldsymbol{M\nu}+\boldsymbol{y}\times\boldsymbol{N\nu}\right)ds + \int_{\partial\Pi\cap\partial M_f}\left(\boldsymbol{m}^*+\boldsymbol{y}\times\boldsymbol{n}^*\right)ds = \boldsymbol{0}\,, \tag{12}$$

$$\frac{d}{dt}\iint_{\Pi}\rho\varepsilon\,da - \iint_{\Pi}\left(\boldsymbol{N}\bullet\boldsymbol{E}^{\circ}+\boldsymbol{M}\bullet\boldsymbol{K}^{\circ}\right)da - \int_{\partial\Pi}\boldsymbol{w}\cdot\boldsymbol{\nu}\,ds - \iint_{\Pi}\rho r\,da + \int_{\partial\Pi\backslash\partial M_h}\boldsymbol{q}\cdot\boldsymbol{\nu}\,ds - \int_{\partial\Pi\cap\partial M_h} q^*\,ds = 0, \tag{13}$$

$$\frac{d}{dt}\iint_{\Pi}\rho\eta\,da \geq \iint_{\Pi}\rho\left(\frac{r}{\theta}-s\right)da - \int_{\partial\Pi\backslash\partial M_h}\left(\frac{\boldsymbol{q}}{\theta}+\boldsymbol{s}\right)\cdot\boldsymbol{\nu} - \int_{\partial\Pi\cap\partial M_h}\left(\frac{q^*}{\theta^*}+s^*\right)ds\,. \tag{14}$$

In the resultant laws of shell mechanics (10) – (12) the following mechanical fields have been used: $\rho(x,t)>0$ and $c(x,t)$ are the (referential) resultant surface mass and mass production (densities), $\boldsymbol{f}(x,t)$ and $\boldsymbol{c}(x,t)$ are the resultant surface force and couple vectors per unit mass of M, $\boldsymbol{l}(x,t)$ and $\boldsymbol{k}(x,t)$ are the resultant surface linear momentum and angular momentum vectors per unit area of M, $\boldsymbol{N}(x,t)\in E\otimes T_xM$ and $\boldsymbol{M}(x,t)\in E\otimes T_xM$ are the referential surface stress resultant and stress couple tensors of the Piola type with corresponding work-conjugate referential surface stretch $\boldsymbol{E}(x,t)\in E\otimes T_xM$ and bending $\boldsymbol{K}(x,t)\in E\otimes T_xM$ tensors, while $(.)^{\circ}=\boldsymbol{Q}d/dt\left(\boldsymbol{Q}^T(.)\right)$ is the co-rotational time derivative, respectively.

The resultant energy balance (13) and the entropy inequality (14) are expressed through additional resultant surface fields: $\varepsilon(x,t)$ and $\eta(x,t)$ are the surface internal energy and entropy (densities), $r(x,t)$ and $s(x,t)$ are the surface heat and extra surface entropy supply (densities), all per unit mass of M, while $\boldsymbol{q}(x,t)\in T_xM$ and $\boldsymbol{s}(x,t)\in T_xM$ are the surface heat flux and extra entropy supply vectors per unit area of M, respectively.

In the present contribution, within any fixed regular $\Pi\in M$ we allow a moving, non-material, singular surface curve $C(t)$ on which some fields appearing in (10) – (14) may not be differentiable, see section 3. In this case, applying (5) from (10) we obtain the local referential balance of mass and jump condition. If we assume, as is usual in solid mechanics, that mass is not created during the thermomechanical process, so that $c=0$, then $\dot{\rho}=0$, and $\rho=\rho(x)$. Additionally, $[\![\rho]\!]=0$.

When a singular surface curve $C(t)$ is admitted, in 2D balances of momenta (11) and (12) some terms containing time derivatives can be transformed with the help of the transport relation (5) as follows:

$$\frac{d}{dt}\iint_{\Pi}\boldsymbol{l}\,da = \iint_{\Pi}\dot{\boldsymbol{l}}\,da + \int_{\Pi\cap C(t)} V[\![\boldsymbol{l}]\!]\,ds\,, \tag{16}$$

$$\frac{d}{dt}\iint_{\Pi}\left(\boldsymbol{k}+\boldsymbol{y}\times\boldsymbol{l}\right)da = \iint_{\Pi}\left(\dot{\boldsymbol{k}}+\dot{\boldsymbol{y}}\times\boldsymbol{l}+\boldsymbol{y}\times\dot{\boldsymbol{l}}\right)da + \int_{\Pi\cap C(t)} V[\![\boldsymbol{k}+\boldsymbol{y}\times\boldsymbol{l}]\!]\,ds\,. \tag{17}$$

To some other terms we apply the extended surface divergence theorems (7) and (8) which yields

$$\int_{\partial\Pi} \boldsymbol{N}\boldsymbol{\nu}\, ds = \iint_{\Pi} Div\, \boldsymbol{N}\, da - \int_{\Pi\cap C(t)} [\![\boldsymbol{N}]\!]\cdot \boldsymbol{\nu}_C\, ds \ ,$$
$$\int_{\partial\Pi} \boldsymbol{M}\boldsymbol{\nu}\, ds = \iint_{\Pi} Div\, \boldsymbol{M}\, da - \int_{\Pi\cap C(t)} [\![\boldsymbol{M}]\!]\cdot \boldsymbol{\nu}_C ds \ ,$$
$$\int_{\partial\Pi} \boldsymbol{y}\times \boldsymbol{N}\boldsymbol{\nu}\, ds = \iint_{\Pi} \left\{ \boldsymbol{y}\times\left(Div\, \boldsymbol{N}\right) \right. \tag{18}$$
$$\left. + \operatorname{ax}\left(\boldsymbol{N}\boldsymbol{F}^T + \boldsymbol{F}\boldsymbol{N}^T\right)\right\} da - \int_{\Pi\cap C(t)} [\![\boldsymbol{y}\times\boldsymbol{N}]\!]\cdot \boldsymbol{\nu}_C ds \ ,$$

where $\boldsymbol{F} = Grad\, \boldsymbol{y} \in E\otimes T_x M$ is the surface deformation gradient. Introducing (16), (17) and (18) into (11) and (12), and taking into account that for a coherent singular curve $[\![\boldsymbol{y}]\!] = \boldsymbol{0}$, we obtain the referential, local, resultant balances of momenta and dynamic boundary conditions derived in Pietraszkiewicz (2011), eqs. (30) and $(33)_{1,2}$, and additionally the following dynamic jump conditions along $C(t)$:

$$[\![\boldsymbol{N}]\!]\cdot\boldsymbol{\nu}_C + V[\![\boldsymbol{l}]\!] = \boldsymbol{0}\,, \quad [\![\boldsymbol{M}]\!]\cdot\boldsymbol{\nu}_C + V[\![\boldsymbol{k}]\!] = \boldsymbol{0}\ . \tag{19}$$

Similarly, in the presence of the singular surface curve $C(t)$ some terms in the 2D energy balance (13) are transformed as follows:

$$\frac{d}{dt}\iint_{\Pi} \rho\varepsilon\, da = \iint_{\Pi} \rho\dot{\varepsilon}\, da + \int_{\Pi\cap C(t)} V[\![\rho\varepsilon]\!]\, ds, \tag{20}$$

$$\int_{\partial\Pi} \boldsymbol{w}\cdot\boldsymbol{\nu}\, ds = \iint_{\Pi} Div\, \boldsymbol{w}\, da - \int_{\Pi\cap C(t)} [\![\boldsymbol{w}]\!]\cdot\boldsymbol{\nu}_C\, ds\,,$$
$$\int_{\partial\Pi} \boldsymbol{q}\cdot\boldsymbol{\nu}\, ds = \iint_{\Pi} Div\, \boldsymbol{q}\, da - \int_{\Pi\cap C(t)} [\![\boldsymbol{q}]\!]\cdot\boldsymbol{\nu}_C\, ds\,. \tag{21}$$

Introducing (20) and (21) into (14) we obtain again the referential, local, resultant balance of energy and the thermal boundary conditions (48) and $(34)_1$ of Pietraszkiewicz (2011), and additionally the following energetic jump condition along $C(t)$, with account of $[\![\rho]\!] = 0$:

$$\rho V[\![\varepsilon]\!] + [\![\boldsymbol{w} - \boldsymbol{q}]\!]\cdot\boldsymbol{\nu}_C = 0\ . \tag{22}$$

Finally, in the presence of the singular surface cur-e $C(t)$ some terms in the resultant entropy inequality (14) are transformed similarly as in (21) and (22). Then (14) leads to the referential, local, resultant entropy inequality and entropic boundary inequality given in Pietraszkiewicz (2011), eqs. (57) and (58), and additionally to the following entropic jump inequality along $C(t)$, with account of $[\![\rho]\!] = 0$:

$$\rho V[\![\eta]\!] - [\![\frac{\boldsymbol{q}}{\theta} + \boldsymbol{s}]\!]\cdot\boldsymbol{\nu}_C \geq 0. \tag{23}$$

5 CONCLUSIONS

Within the resultant shell thermomechanics of Pietraszkiewicz (2011), we have formulated the referential jump conditions at the non-material moving singular surface curve. They will allow one to model some discontinuous thermomechanical processes in regular shells such as wave propagation or phase transition. In the derivation process it has been assumed that the singular surface curve is coherent during the process, and that the base surface itself does not contain branchings, self-intersections, abrupt thickness changes, technological junctions and other curvilinear irregularities of this type. Removing any of these assumptions would lead to more complex jump conditions to be discussed elsewhere.

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