BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
11. Série des sciences techniques
Volume XX, No. 4 — 1972

V A R I A

(QUANTUM ELECTRONICS)

System of Integral Equations and Equivalence Parameters of an Optical Resonator with Inhomogeneous Medium — Approximation Resulting from Quasi-Geometric Optics

by

J. MIZERACZYK

Presented by R. SZEWALSKI on December 18, 1971

Summary. Using the quasi-geometric optics formulation for media with inhomogeneous index of refraction, the problem of determining the electromagnetic field distribution at the mirrors of an optical resonator filled with a medium displaying a parabolic variation of the index of refraction has been reduced to a simpler problem of an equivalent empty optical resonator. From the derived system of integral equations the equivalent generalized parameters of the considered resonator are determined. The presented approach allows to compare the diffraction losses, the resonant conditions and the mode patterns of a resonator in question with respect to an equivalent one.

Introduction. The majority of papers on the subject of laser resonators deal with the theory of empty optical resonators or resonators filled with optically homogeneous media. However, the active media of lasers may, for various reasons (e.g. [1—7]), be optically inhomogeneous. An inhomogeneous medium inserted between the resonator mirrors affects the properties of the optical resonator, i.e. it changes the mode patterns, diffraction losses and conditions of resonance.

Following the increasing interest in resonators, filled with inhomogeneous media, the demand arises to solve the problem of the field distribution, diffraction losses and resonant frequency for such resonators. The exact solution of Maxwell's equation with appropriate boundary conditions, even when available, is too complicated to be used for determining the information about the resonator in question. In the literature, there is the Kogelnik's well-known approach [8] based on the ray matrix formulation of geometric optics and the imaging rules obtained with the use of formalism of Fresnel diffraction theory.

The purpose of this paper is to present the theory of an optical resonator with a medium displaying a parabolic variation of the index of refraction developed from the formulation of quasi-geometric optics for inhomogeneous media proposed by Eichmann [9]. The choice of such a form of the index of refraction is justified since the parabolic variation of the index comprises a broad variety of functions describing the actual distribution of the index of refraction. Eichmann's formulation of quasi-geometric optics for inhomogeneous media is analogous to Feynman's approach [10] to quantum mechanics.

Quasi-geometric-optics approach for media of inhomogeneous index of refraction. Assume that at any surface σ_1 of an inhomogeneous medium, the distribution of the electromagnetic field $\Psi(x_1, y_1, z_1)$ is known. The distribution $\psi(x, y, z_1+\epsilon)$, where ϵ is a small distance in the direction of coordinate z can be found from the relation [9, 10]:

$$\psi(x, y, z_1+\epsilon) = \iint_{\sigma_1} \exp[-ikS(x, y, x_1, y_1, \epsilon)]\, \psi(x_1, y_1, z_1) \frac{dx_1}{A} \frac{dy_1}{A}, \tag{1}$$

where A is an ϵ-dependent normalization constant and k is the wave number in the medium. The optical path length

$$S(x, y, x_1, y_1, \epsilon) = \int_{z_1}^{z_1+\varepsilon} L(x, y, \dot{x}, \dot{y}, z)\, dz \tag{2}$$

is the integral of the optical lagrangian

$$L(x, y, \dot{x}, \dot{y}, z) = n(x, y, z)(1+\dot{x}^2+\dot{y}^2)^{\frac{1}{2}} \tag{3}$$

taken along the path which makes the optical path length (2) an extremum; $n(x, y, z)$ is the inhomogeneous index of refraction of the medium. The dots represent differentation with respect to z. Eq. (1) is true in the limit $\epsilon \to 0$.

The integral (1) has been presented for the first time by Feynman [10] as the expression of the Huygens' principle for matter waves in his approach to quantum mechanics. Basing on Eq. (1) Eichmann [9] proposed the formulation of quasi-geometric optics for media with inhomogeneous index of refraction.

At any arbitrary distance z, $\psi(x, y, z)$ can be obtained by iterating Eq. (1) by distances ϵ along the z direction until z is reached, resulting in

$$\psi(x, y, z) = \iint_{\sigma_1} \mathcal{K}(x, y, z, x_1, y_1, z_1)\, \psi(x_1, y_1, z_1)\, dx_1\, dy_1, \tag{4}$$

where the kernel of integral (4)

$$\mathcal{K}(x, y, z, x_1, y_1, z_1) = \lim_{\varepsilon\to 0} \frac{1}{A}\frac{1}{A} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \exp\left[-ik \sum_{j=1}^{l} S(x_{j+1}, y_{j+1}, x_j, y_j, \epsilon\right] \times$$
$$\times \frac{dx_2}{A} \cdot \frac{dx_3}{A} \dots \frac{dx_l}{A} \cdot \frac{dy_2}{A} \cdot \frac{dy_3}{A} \dots \frac{dy_l}{A} =$$
$$= \int \exp[-ikS(x, y, z, x_1, y_1, z_1)\, Dx(z) \cdot Dy(z) \tag{5}$$

is the continuous-path integral [11]. In Eq. (5) it is assumed that x_{l+1}, y_{l+1} are the coordinates of the surface at which the disturbance $\psi(x, y, z)$ is searched for. The integral equation (4) gives the field distribution $\psi(x, y, z)$ at any surface $\sigma(x, y, z)$ if distribution $\psi(x_1, y_1, z_1)$ at a surface σ_1 is known.

The problem now reduces to the evaluation of an appropriate kernel function for each particular inhomogeneous medium. At present, it can be done only in very special cases [11].

The kernel of integral equation of a medium having a parabolically varying index of refraction. Assume that the inhomogeneous index of refraction of a medium can be expressed as:

$$n(x, y) = 1 - \frac{1}{2}\omega^2 x^2 - \frac{1}{2}\omega^2 y^2, \tag{6}$$

where ω is the characteristic parameter of the medium. It is also assumed that the medium is the weakly focusing one, i.e. the parameter ω is small against the unity

$$|\omega| \ll 1. \tag{7}$$

Expanding the optical lagrangian (3) in a series and taking also into account that for paraxial rays, the ray slopes are

$$|\dot{x}| \ll 1, \; |\dot{y}| \ll 1, \tag{8}$$

yields

$$L(x, y, \dot{x}, \dot{y}) = 1 + L_{0h}(x, y, \dot{x}, \dot{y}), \tag{9a}$$

where

$$L_{0h}(x, y, \dot{x}, \dot{y}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 + \frac{1}{2}\dot{y}^2 - \frac{1}{2}\omega^2 y^2 \tag{9b}$$

is the optical analog of the classical lagrangian for two-dimensional harmonic oscillator.

Inserting Eqs. (9) into Eq. (2) and integrating from z_1 to z, one obtains

$$S(x, y, z, x_1, y_1, z_1) = z - z_1 + S_{0h}(x, y, x_1, y_1), \tag{10a}$$

where

$$S_{0h}(x, y, x_1, y_1) = \int_{z_1}^{z} L_{0h}(x, y, \dot{x}, \dot{y})\, dz \tag{10b}$$

is the optical analog to the classical action for two-dimensional harmonic oscillator.

For the optical lagrangian L and optical path S given by Eqs. (9) and (10), respectively, the continuous path integral (5) can be solved [11] to yield the kernel function of integral equation of a medium displaying a parabolic variation of the index of refraction in a closed form:

$$\mathcal{K}(x, y, z, x_1, y_1, z_1) = \left[\frac{i\omega k}{2\pi \sin\omega(z-z_1)}\right] \cdot \exp[-ik(z-z_1)] \times$$
$$\times \exp\left\{\frac{-i\omega k}{2\sin\omega(z-z_1)}[(x^2+y^2+x_1^2+y_1^2)\cos\omega(z-z_1) - 2(xx_1+yy_1)]\right\}. \tag{11}$$

The kernel (11) is the same as in the case of a two-dimensional harmonic oscillator [11] with the exception of the factor $\exp[-ik(z-z_1)]$, which represents the effect of the disturbance propagation in z direction.

System of integral equations and equivalence parameters of an optical resonator filled with a medium with index of refraction varying parabolically. Consider an optical resonator formed by two spherical (or flat) mirrors M_1 and M_2 spaced at a distance d, as shown in Fig. 1. The radii of curvature of the mirrors are R_1 and

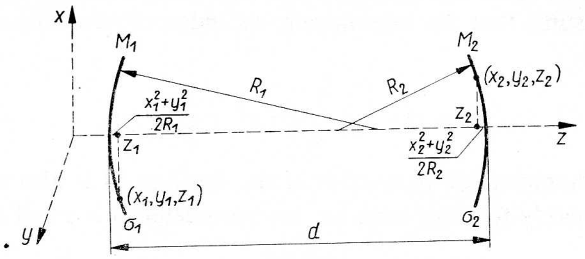

Fig. 1. Geometry of an optical resonator. The mirrors are assumed to be rectangular of widths $2a_1$, $2b_1$ and $2a_2$, $2b_2$, respectively

R_2. The mirrors are assumed to be rectangular*) of the dimensions $2a_1$, $2b_1$ and $2a_2$, $2b_2$, respectively. All the resonator dimensions are assumed large against the wavelength. It is assumed, additionally, that

$$a_j \ll d, \quad b_j \ll d, \quad (j=1, 2) \tag{12}$$

which is typical for the optical resonators. All the space between the mirrors is filled with the medium defined by the parabolically varying index of refraction (6). Identifying the surfaces of the mirrors M_1 and M_2 with the surfaces σ_1 and σ from the previous section, it is seen that the integral equation (4) with kernel function (11) gives the distribution of electromagnetic field at mirror M_2 of the resonator with a medium described by Eq. (6) if the distribution at mirror M_1 is known. Analogously, the distribution of electromagnetic field at the mirror M_1 can be expressed by the one at mirror M_2.

Introducing the definition of the resonator mode in a steady state [12]

$$\psi_q^{(j)} = [\gamma^{(j)}]^q \psi^{(j)}, \quad (j=1 \text{ or } 2) \tag{13}$$

the resulting system of integral equations describing the distribution of electromagnetic field at mirrors M_1 and M_2 yields

$$\begin{aligned} \gamma^{(2)*}\psi^{(2)}(x_2, y_2) &= \frac{ik}{2\pi d^*}\exp(-ikd^*)\int\limits_{\sigma_1}\int \exp\left\{-\frac{ik}{2d^*}\left[(x_1^2+y_1^2)g_1^* +\right.\right. \\ &\left.\left. +(x_2^2+y_2^2)g_2^* - 2(x_1x_2+y_1y_2)\right]\right\}\cdot\psi^{(1)}(x_1, y_1)\,dx_1\,dy_1, \\ \gamma^{(1)*}\psi^{(1)}(x_1, y_1) &= \frac{ik}{2\pi d^*}\exp(-ikd^*)\int\limits_{\sigma_2}\int \exp\left\{-\frac{ik}{2d^*}\left[(x_1^2+y_1^2)g_1^* +\right.\right. \\ &\left.\left. +(x_2^2+y_2^2)g_2^* - 2(x_1x_2+y_1y_2)\right]\right\}\cdot\psi^{(2)}(x_2, y_2)\cdot dx_2\,dy_2, \end{aligned} \tag{14}$$

*) The presented consideration is valid evidently for the circular mirrors of diameters $2a_1$ and $2a_2$, respectively.

where:

(15) $$\gamma^{(1)*} = \exp\,[-ik\,(d^* - d)]\,\gamma^{(1)},$$

(16) $$\gamma^{(2)*} = \exp\,[-ik\,(d^* - d)]\,\gamma^{(2)},$$

(17) $$d^* = \frac{\sin \omega d}{\omega},$$

(18) $$g_1^* = \cos \omega d - \frac{d^*}{R_1},$$

(19) $$g_2^* = \cos \omega d - \frac{d^*}{R_2}.$$

The indices 1 and 2 are referring to mirrors M_1 and M_2, respectively. The system of integral equations (14) has been obtained from Eqs. (4) and (11) with the aid of the following approximations:

(20) $$z_1 \cong \frac{x_1^2 + y_1^2}{2R_1},$$

(21) $$z_2 \cong d - \frac{x_2^2 + y_2^2}{2R_2},$$

and

(22) $$z_2 - z_1 \cong d$$

in the trigonometric functions, reasonable in view of the assumptions (12).

By definition (13) it is meant that in a steady state the field distribution at the j-th mirror reproduces itself within the multiplicative constant $\gamma^{(j)}$ after each wave transit in the resonator, the number q specifying the successive transition of the wave.

The factors $\gamma^{(1)}$ and $\gamma^{(2)}$, named the resonator eigenvalues specify the diffraction losses and phase shift, to which the waves are exposed during each transit in the resonator. Apart from the difference of resonator eigenvalues, the system of integral equations (14) describing the distribution of the electromagnetic field at mirrors of the optical resonator with the medium having the parabolically varying index of refraction (6) is the same as that for an empty optical resonator [13] described by the generalized parameters [14]*):

*) The resonator with circular mirrors is described by three parameters only, namely by:

$$N = \frac{k a_1^* a_2^*}{2\pi d^*}, \qquad G_1 \frac{a_1^*}{a_2^*} g_1^*, \qquad G_2 = \frac{a_2^*}{a_1^*} g_2^*,$$

where $2a_2^*$ and $2a_2^*$ are the diameters of the mirrors.

(23)
$$N=\frac{k\sqrt{a_1^* a_2^* b_1^* b_2^*}}{2\pi d^*}=\frac{\omega k\sqrt{a_1 a_2 b_1 b_2}}{2\pi \sin \omega d},$$

(24)
$$G_1=\frac{a_2^*}{a_2^*}g_1^*=\frac{a_1}{a_2}\left(\cos \omega d-\frac{1}{R_1}\frac{\sin \omega d}{\omega}\right),$$

(25)
$$G_2=\frac{a_2^*}{a_1^*}g_2^*=\frac{a_2}{a_1}\left(\cos \omega d-\frac{1}{R_2}\frac{\sin \omega d}{\omega}\right),$$

(26)
$$G_3=\frac{b_1^*}{b_2^*}g_1^*=\frac{b_1}{b_2}\left(\cos \omega d-\frac{1}{R_1}\frac{\sin \omega d}{\omega}\right),$$

(27)
$$G_4=\frac{b_2^*}{b_1^*}g_2^*=\frac{b_2}{b_1}\left(\cos \omega d-\frac{1}{R_2}\frac{\sin \omega d}{\omega}\right),$$

where $2a_1^*$, $2b_1^*$ and $2a_2^*$, $2b_2^*$ are the dimensions of mirrors of such resonator, d^* is the distance between these mirrors, and the radii of curvature R_1^* and R_2^* of mirrors are contained in conventional parameters [13]

(28)
$$g_1^*=1-\frac{d^*}{R_1^*},$$

(29)
$$g_2^*=1-\frac{d^*}{R_2^*}.$$

From the integral equations (14) it follows that the mode patterns of considered resonator and the empty one are scaled versions of each other. The differences between the empty resonator eigenvalues $\gamma^{(1)*}$, $\gamma^{(2)*}$ and the eigenvalues $\gamma^{(1)}$, $\gamma^{(2)}$ of the resonator in question are expressed by the factor $\exp[-ik(d^*-d)]$. Physically it means that the diffraction losses of both resonators are the same, and there is only a small difference in the corresponding resonant frequency.

Therefore, any empty optical resonator having the generalized parameters (23)—(27) is equivalent, in a sense given above (with exception of the resonant conditions), to the considered resonator with the medium having the parabolically varying index of refraction (6).

Discussion. It has been shown that the problem of the field distribution at the mirrors of a resonator filled with a medium of the index of refraction (6) may be reduced to a simpler one of the field distribution at the mirrors of an equivalent empty optical resonator described by the parameters (23)—(27).

The values of the equivalent resonator parameters N, G_1, G_2, G_3, G_4 (23)—(27) are equal to the appropriate parameters of the resonator with an internal lens-like medium derived by Kogelnik [8] on the basis of the ray matrix formulation of geometric optics and the imaging rules following from the formalism of Fresnel diffraction theory. The other conclusions presented here are in good agreement with those resulting from Kogelnik's theory. These facts suggest the equivalence of Kogelnik's approach and the one developed here.

The direct advantage of the presented approach is that the information about the resonator of interest are enclosed in the system of integral equations for an empty optical resonator, the properties of which have been studied intensively for over ten years.

It is obvious that the quasi-geometric approach is also valid for media with homogeneous index of refraction ($\omega \equiv 0$). In the case of homogeneous medium one can obtain the well known system of integral equations for an empty resonator, earlier developed from the Huygens—Fresnel principle or other methods more complicated mathematically.

Appendix

A partly filled resonator. The purpose of this appendix is to present the integral equations and equivalence parameters of an optical resonator partly filled with an inhomogeneous medium.

Consider an optical resonator with an inhomogeneous medium occupying only a part of the space between the resonator mirrors, as shown in Fig. 2. All the resonator dimensions are assumed

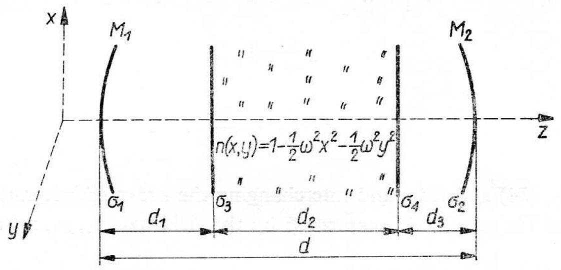

Fig. 2. Geometry of an optical resonator partly filled with an inhomogeneous medium. All dimensions of mirrors are the same as in Fig. 1

σ_3 and σ_4 denote the transverse surfaces of a medium; d_2 is a length of a medium. d_1 and d_3 specify the location of the medium in a resonator

to be just the same as in previous sections (see Fig. 1). The transverse dimensions of an inhomogeneous medium is regarded as infinitely large. It is assumed that the medium displays a parabolic variation of the index of refraction.

According to Eqs. (4) and (11) (taking $\omega=0$) of the main text, the field distribution $\psi^{(3)}(x_3, y_3, z_3)$ at the surface σ_3 can be expressed as

$$\psi^{(3)}(x_3, y_3, z_3) = \int_{\sigma_1} \mathcal{K}_{31}(x_3, y_3, z_3, x_1, y_1, z_1)\, \psi^{(1)}(x_1, y_1, z_1)\, dx_1\, dy_1 , \tag{30}$$

where the kernel function is

$$\mathcal{K}_{31} = \frac{ik}{2\pi d_1} \exp(-ikd_1) \exp\left\{-\frac{ik}{2d_1}\left[g_{31}^{(1)}(x_1^2+y_1^2)+g_{31}^{(2)}(x_3^2+y_3^2)-2(x_1 x_3+y_1 y_3)\right]\right\} \tag{31}$$

and

$$g_{31}^{(1)} = 1 - \frac{d_1}{R_1} , \tag{32}$$

$$g_{31}^{(2)} = 1 . \tag{33}$$

Taking $\omega \neq 0$ or $\omega = 0$ where necessary, one can analogously obtain the field distributions $\psi^{(4)}(x_4, y_4, z_4)$ at the surface σ_4 and $\psi^{(2)}(x_2, y_2, z_2)$ at σ_2 in the forms:

(34) $$\psi^{(4)}(x_4, y_4, z_4) = \int_{\sigma_3} \mathcal{K}_{43}(x_4, y_4, z_4, x_3, y_3, z_3)\, \psi^{(3)}(x_3, y_3, z_3)\, dx_3\, dy_3\,,$$

(35) $$\psi^{(2)}(x_2, y_2, z_2) = \int_{\sigma_4} \mathcal{K}_{24}(x_2, y_2, z_2, x_4, y_4, z_4)\, \psi^{(4)}(x_4, y_4, z_4)\, dx_4\, dy_4\,,$$

where

(36) $$\mathcal{K}_{43} = \frac{ik}{2\pi d_2^*} \exp(-ikd_2) \exp\left\{-\frac{ik}{2d_2^*}\left[g_{43}^{(1)}(x_3^2+y_3^2)+g_{43}^{(2)}(x_4^2+y_4^2)-2(x_3 x_4+y_3 y_4)\right]\right\},$$

(37) $$d_2^* = \frac{\sin \omega\, d_2}{\omega}\,,$$

(38) $$g_{43}^{(1)} = g_{43}^{(2)} = \cos \omega\, d_2\,,$$

(39) $$\mathcal{K}_{24} = \frac{ik}{2\pi d_3} \exp(-ikd_3) \exp\left\{-\frac{ik}{2d_3}\left[g_{24}^{(1)}(x_4^2+y_4^2)+g_{24}^{(2)}(x_2^2+y_2^2)-2(x_4 x_2+y_4 y_2)\right]\right\},$$

(40) $$g_{24}^{(1)} = 1\,,$$

(41) $$g_{24}^{(2)} = 1 - \frac{d_3}{R_2}\,.$$

Combining Eqs. (30), (34) and (35), and interchanging the order of integration, one can find the field $\psi_2(x_2, y_2, z_2)$ at the mirror M_2 expressed by the field $\psi_1(x_1, y_1, z_1)$ at the mirror M_1 as

(42) $$\psi^{(2)}(x_2, y_2, z_2) = \int_{\sigma_1} \mathcal{K}(x_2, y_2, z_2, x_1, y_1, z_1)\, \psi^{(1)}(x_1, y_1, z_1)\, dx_1\, dy_1\,,$$

where

(43) $$\mathcal{K} = \int_{\sigma_4}\int_{\sigma_3} \mathcal{K}_{24} \cdot \mathcal{K}_{43} \cdot \mathcal{K}_{31}\, dx_3\, dy_3\, dx_4\, dy_4\,.$$

The integration of Eq. (43) can be performed by noting that [15]

(44) $$\int_{-\infty}^{+\infty} \exp(-iax^2+iyx)\, dx = \sqrt{\frac{\pi}{ia}} \exp\left(i\frac{y^2}{4a}\right).$$

With this Eq. (43) becomes

(45) $$\mathcal{K} = \frac{ik}{2\pi d^*} \exp(-ikd) \exp\left\{-\frac{ik}{2d^*}\left[g_1^*(x_1^2+y_1^2)+g_2^*(x_2^2+y_2^2)-2(x_1 x_2+y_1 y_2)\right]\right\},$$

where:

(46) $$d^* = (d_1+d_3)\cos \omega d_2 + \frac{\sin \omega d_2}{\omega}(1-\omega^2 d_1 d_3)\,,$$

(47) $$g_1^* = \cos \omega\, d_2 - \omega\, d_3 \sin \omega d_2 - \frac{d^*}{R_1}\,,$$

(48) $$g_2^* = \cos \omega d_2 - \omega\, d_1 \sin \omega d_2 - \frac{d^*}{R_2}\,.$$

Introducing the definition of the resonator mode (13) as before, one can obtain the resulting system of integral equations describing the distribution of electromagnetic field at mirrors M_1 and M_2 of an optical resonator filled with a medium of parabolically varying index of refraction in the form of Eqs. (14) with (15) and (16) where the parameters d^*, g_1^* and g_2^* are expressed by Eqs. (46), (47) and (48). Therefore, according to the considerations of previous sections any empty optical resonator having the generalized parameters N, G_1, G_2, G_3, G_4 with the values of d^*, g_1^* and g_2^* given by Eqs. (46)—(48) is equivalent to the resonator filled partly with the medium of parabolically varying index of refraction (6). The problem has been reduced again to the case of an empty optical resonator.

Following Kogelnik's approach [8] to a resonator partly filled with lens-like medium one can show that the parameters N, G_1, G_2, G_3, G_4 predicted by the present analysis are identical with those resulting from Kogelnik's theory.

In the discussion of parameters (46)—(48) it appears that the properties of a resonator with medium occupying a part of the space between the mirrors are not only determined by the length of medium but also by the location of medium with respect to the mirrors.

The author is grateful to Dr. Z. Rozkwitalski for his interest in this work.

INSTITUTE OF FLUID FLOW MACHINES, POLISH ACADEMY OF SCIENCES, GDAŃSK, FISZERA 14
(INSTYTUT MASZYN PRZEPŁYWOWYCH, PAN, GDAŃSK)

REFERENCES

[1] L. A. Schlie, J. T. Verdeyen, *Radial profile of neon* $1s_5$ *atoms in a He—Ne active discharge and their lens effect on lasing at* 6401 Å, IEEE J. Quantum Electronics, **QE-5**, (1969) 21—29.

[2] D. G. Smith, *Thermal defocusing of* CO_2 *laser radiation in gases*, ibid., 600—607.

[3] H. K. V. Lotsch, W. C. Davis, *The lensing effect of* CO_2 *laser plasma*, Appl. Optics, **12** (1970), 2725—2728.

[4] L. Casperson, A. Yariv, *The gaussian mode in optical resonators with a radial gain profile*, Appl. Phys. Lett., **10** (1968), 355—357.

[5] A. J. Laderman, S. R. Byron, *Temperature rise and radial profiles in* CO_2 *lasers*, J. Appl. Phys., **42** (1971), 3138—3144.

[6] B. N. Malishev, V. A. Salyuk, *Deistvie effektivnoi linzy aktivnogo elementa na raskhodimost vykhodnogo izlucheniya zhidkostnogo OKG*, Zh.T.F., **41** (1971), 1690—1698.

[7] H. H. Witte, *Influence of thermal lens effects on the resonator geometry gas lasers*, Opt. Laser Technol., **3** (1971), 31—35.

[8] H. Kogelnik, *Imaging of optical modes-resonators with internal lenses*, B.S.T.J., **44** (1965), 455—494.

[9] G. Eichmann, *Quasi-geometric optics of media with inhomogeneous index of refraction*, J.O.S.A., **61** (1971), 161—168.

[10] R. P. Feynman, *Space-time approach to non-relativistic quantum mechanics*, Rev. Mod. Physics, **20** (1948), 367—387.

[11] R. P. Feynman, A. R. Hibbs, *Quantum mechanics and path integrals*, Mc Graw-Hill, New York, 1965.

[12] A. G. Fox, T. Li, *Resonant modes in a maser interferometer*, B.S.T.J., **40** (1961), 453—488.

[13] — , — , *Modes in a maser with curved and tilted mirrors*, Proc. IEEE, **55** (1963), 80—89.

[14] J. P. Gordon, H. Kogelnik, *Equivalence relations among spherical mirror optical resonators*, B.S.T.J., **43** (1964), 2873—2886.

[15] A. Erderly, W. Magnus, F. Oberhettinger, F. G. Tricomi, *Tables of integral transforms*, vol. 1, McGraw Hill, New York — Toronto — London, 1954, pp. 7—122.

Ю. Мизэрачик, **Система интегральных уравнений и параметры эквивалентности оптического резонатора с неоднородной средой — приближение квази-геометрической оптики**

Содержание. На основе формализма квази-геометрической оптики для среды с неоднородным коэффициентом преломления, проблема распределения электромагнитного поля на зеркалах оптического резонатора, который выполнен средой с коэффициентом преломления изменяющимся согласно параболе, сводится к упрощенной задаче так называемого эквивалентного пассивного резонатора. Из полученных интегральных уравнений определены эквивалентные обобщенные параметры резонатора. Представленный в настоящей работе подход дает возможность сравнить диффракционные потери, условия резонанса, а также распределение поля модов рассматриваемого здесь и эквивалентного резонатора.