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Junctions in shell structures: A review

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ABSTRACT Many shell structures used in modern technology consist of regular shell parts joined together along their common boundaries. We review different theoretical, numerical, and experimental approaches to modelling, analyses and design of the compound shell structures with junctions. Several alternative forms of boundary, continuity and jump conditions at the singular midsurface curves modelling the shell junction are reviewed. We also analyse the results obtained for special shell structures containing the cylinder-cylinder intersections, cone- , sphere- , and plate-cylinder junctions, tubular joints as well as other special types of junctions appearing in complex multi-shell structures.

Keywords: shell junction, continuity condition, jump conditions, cylinder-cylinder intersection, cone-cylinder junction, sphere-cylinder junction, shell of revolution, tubular joint

1 Introduction

Many complex shell structures used in modern technology consist of regular shell parts joined together along their common boundaries. Automobile bodies, aircraft fuselages, water and oil tanks, silos, branching and intersecting pipelines, pressure vessels with nozzles etc. are just a few typical examples of such thin-walled constructions. Even supposedly regular shell parts may contain stiffeners, stepwise thickness changes, parts made of different materials or reinforced in different directions etc., which may cause some kinematic and/or dynamic fields of the boundary value problem to be discontinuous at specific curves or points on the shell base surface. Some shell junctions may have their own mechanical properties allowing the adjacent shell elements to deform (usually rotate) one with respect to another, or such deformability develops at some level of deformation, as in the case of phase transition or plastic hinge of the shell material.

It was recognized long ago that in order to analyse the compound shell structures it is necessary to complete the BVP of the regular shell structure with additional relations – the continuity conditions – by adjusting the boundary conditions along the joined parts of regular shell elements. The form of these conditions depends on the shell model used in the analysis. In most discussions the compound irregular shell structure is divided into regular parts each having a smooth and regular surface as its base surface. The regular shell parts are modelled separately, and all the parts are then assembled into the whole structure by adjusting boundary conditions of the adjacent shell elements along the junctions, with possible account of additional different mechanical properties of the junction itself.

In the approach proposed by Makowski et al. [1,2] within the non-linear theory of thin elastic shells, the whole compound shell structure was modelled by a material surface-like continuum capable of resisting to stretching and bending, Fig. 1. From the Principle of Virtual Work written for the whole shell structure several forms of static and kinematic jump conditions at the junctions were formulated, besides the equilibrium equations and static boundary conditions known from the theory of thin regular shells.

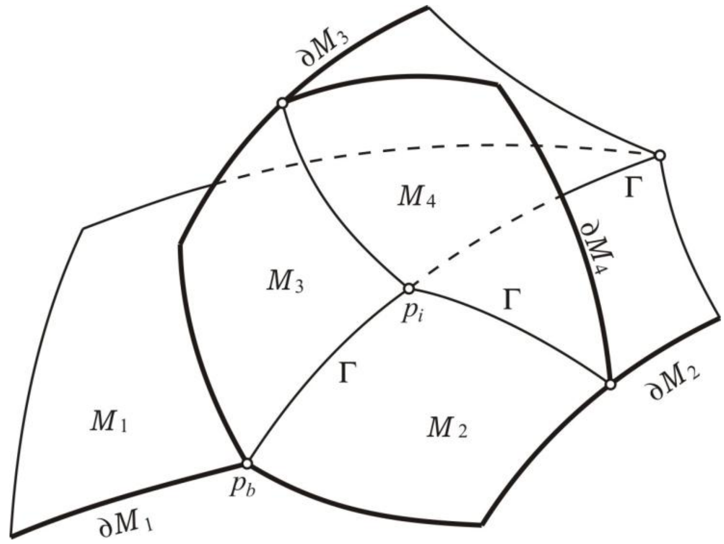

Figure 1. Irregular piecewise smooth shell base surface composed of regular surfaces M_n joined along singular curves Γ

Within the resultant six-field non-linear shell theory, a refined approach was used by Chróścielewski et al. [3,4]. Geometry of the undeformed irregular shell structure was described by a union of piecewise smooth surfaces and curves. The curves with their intersections formed a complex reference network. Each curve in the network may represent a stationary singular surface curve modelling the fold, junction, intersection etc., but also a one-dimensional continuum endowed with its own dynamic, kinematic, and/or material properties. Then an integral identity with meaning of the Principle of Virtual Work allowed one to formulate the equilibrium equations, dynamic boundary conditions and unique definitions of the surface strain measures known from the six-parameter model of the regular shell parts. Additionally, one obtained the appropriate jump conditions at the shell junctions. Konopińska and Pietraszkiewicz [5,6] formulated the exact, resultant dynamic jump conditions for the branching (see Fig. 2) and intersecting shells, with the unique work-conjugate kinematics at the singular curves constructed in Konopińska and Pietraszkiewicz [7] and Pietraszkiewicz and Konopińska [8] (see Figs. 3 and 4). The most general jump conditions for the surface mass, linear momentum, angular momentum, energy and entropy at the moving singular surface curves have recently been formulated by Pietraszkiewicz and Konopińska [9] within the refined resultant thermodynamics of shells worked out by Pietraszkiewicz [10].

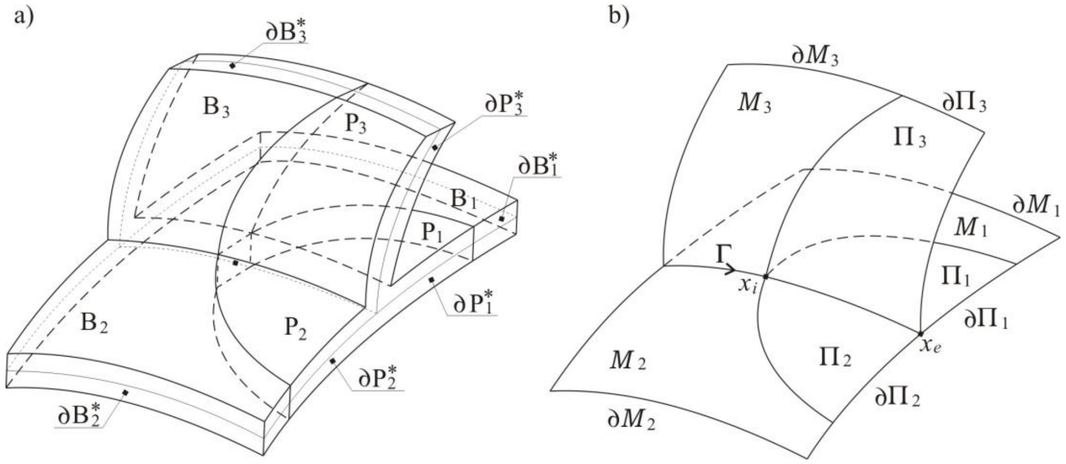

Figure 2. The branching structure: (a) the 3D shell, and (b) the corresponding 2D base surface

Development of the numerical finite element method (FEM) considerably changed modelling and analysis of complex shell structures with junctions. In the direct numerical approach the continuity (or jump) conditions are supposed to be automatically satisfied in the process of assembling the global stiffness matrix from local matrices of individual finite elements. One should note, however, that accuracy of such numerical satisfaction of continuity at the junction depends on the type of finite elements used in the numerical simulations. Any all-purpose computer FEM code offers many types of membrane, plate, shell and 3D finite elements of different complexity and accuracy. It is not apparent in advance which of the FEs available is suited best for assuring the most complete continuity of the analysed compound shell structure. Thus, in many recent design cases of responsible shell structures with junctions, different methods of analyses – analytical, numerical, and/or experimental – are still used independently and parallely, and the results are compared in order to achieve the best security of structural design.

In this paper we review some achievements on modelling and analysis of shell structures composed of regular shell parts joined together along the common boundaries. In section 2 we recall some books and review papers in which also some compound shell structures with junctions were discussed. Then in section 3 we analyse the results on various forms of boundary, continuity, and jump conditions between regular parts of the shell structure for general geometries of their base surfaces. Finally, in sections 4 – 9 we summarize the published literature on analysis of special shell structures containing the cylinder-cylinder intersections, cone-, sphere-, and plate-cylinder junctions, compound shells of revolution, tubular joints as well as other special types of multi-shells commonly used in modern technology. In most cases the results obtained in each of the referred paper are characterised in descriptive concise way, which should help the reader to gain some information prior to reading the full paper.

2 Books and reviews

Vast majority of monographs and books written on shell structures contain in fact the theory and analyses of only regular shells with reference surface consisting of a single, smooth, and regular surface element. Only some monographs contain special chapters or sections in which shell structures composed of regular shell elements joined along the common edges were discussed. Among them let us mention here the books by Flügge [11], Chernykh [12], Baker et al. [13], Calladine [14], Bushnell [15], Mikhailovskii [16], Novozhilov et al. [17], Bernadou [18], and Zingoni [19].

Harvey [20] presented an early exposition of analysis and design of pressure vessels for nuclear and chemical applications. In particular, an introduction to the flange analysis and design of various pressure vessels was given. Stress concentration at openings, nozzles and other structural junction elements was discussed.

Early Russian results on analysis and design of intersecting shells of vessels were summarized by Voloshin and Samsonov [21]. Mikhailov [22] developed an original analytical method of analysis of the shell and plate structures with folds, stiffeners, holes, subjected to concentrated loads and with other irregularities. The linear classical theory of thin elastic shells was used, and the method was based on application of generalized (non-continuous) functions. The explicit solutions of several engineering problems of complex compound shells were given.

In the literature there are also some review papers on special shell structures containing junctions. In particular, Ellyin [23] delivered a comprehensive review of early papers on plastic analysis of cylinder-sphere intersecting problem. Biron [24] reviewed early results based on the lower-bound limit analysis for pressure vessel junctions. Haswell and Hopkins [25] presented a critical review and comparison of various early models of fracture mechanics of tubular joints. Coffer and Will [26] reviewed an analytical procedure based on FE techniques, which can be used for the ultimate strength analysis of tubular joints. Various finite elements were reviewed as appropriate for different parts of the tubular junctions. Non-linearity was included via an elastoplastic material model. Several experimental tubular joint analyses were reproduced to validate the analytical procedure. Mackerle [27] provided a bibliography (1976-1996) of finite element analysis and simulation of welding processes. Among the issues there were discussed the welded tubular joints in pipes and pressure vessel components, and welds in plates and other structural members. Grigorenko et al. [28] reviewed linear and nonlinear problems on the elastic deformation of complex shells and methods of their numerical solutions. Mackerle [29] compiled a bibliography from the period of 1990-2002 of the FEM applied to the analysis of fastening and joining from theoretical and practical point of view. Some references deal with plate / shell junctions as well. Bedair [30] reviewed the analytical and numerical procedures developed for analysis of stiffened plates and shells. Design philosophies for predicting the ultimate strength of these structures and optimization procedure to minimize the weight of them were also reviewed. Zingoni [31] reviewed some recent research on the strength, stability and vibration behaviour of liquid-containment shells of revolution. Within the review, the junction and discontinuity problems in the cylindrical shells with bottom-plate closure and ring wall, the double-cone pressure vessels, the egg-shaped and multi-segmented spherical shells of revolution, and transition junctions with T-ring beams were discussed.

Teng and Rotter [32] summarized some research on the behaviour and design of steel silos hoppers and transition junctions in the both uniformly supported and column-supported circular steel silos. Teng [33] presented a survey of research, most conducted in the Hong-Kong University, on the stress, stability and strength of silos transition junctions as axisymmetric steel shell intersections. Particular attention was paid to intersections formed from cylindrical and conical segments as these were more common and had been more extensively researched. A simple approximate method for extrapolating the knowledge gained on these intersections to those containing curved shell segments was suggested. Extending the research, Teng [34] provided a summary of knowledge of the buckling and collapse of shell junctions subject to internal pressure. Junctions covered here included cone large end-to-cylinder junctions, sleeved or ring-stiffened cone-cylinder junctions, flat plate-cylinder junctions and junctions of curved shell segments, on which there had been only limited work. A general method for determining the plastic limit loads of complex shell junctions was presented as well. Finally, Teng and Zhao [35] provided a survey of buckling problems of junction transition rings usually constructed in elevated steel silos uniformly supported around the circumference. Some possible application of the results to the design of other shell junction was examined.

3 General results at shell junctions

Within the linear Kirchhoff-Love type theory of thin elastic shells, Chernykh [12,36] introduced three sets of boundary quantities which allowed one to formulate the four boundary conditions (and the junction conditions) in terms of either three displacements and one linear rotation, or three linear rotations and stretch along the edge, or three deformational quantities and the stretch, with corresponding work-conjugate resultant force and moment quantities. Mai [37] discussed boundary conditions along the line of intersection of two statically loaded shells. Mal'kov [38] transformed the four junction conditions of the theory of thin elastic shells so that two of them did not contain the boundary effect. When the basic stress-strain state in the shell is found, the boundary effect could be calculated by the explicitly derived formulae. Mikhailovskii and Chernykh [39] indicated that to assure uniqueness of the solution with the deformational boundary conditions it was necessary to constrain also the principal boundary force and moment vectors, because in some cases the deformational boundary conditions might be linearly dependent. The three conjugate variants of boundary conditions allowed Chernykh [12] and Mikhailovskii [16] to solve a number of non-classical junction shell problems and problems with reinforced shell edge. The solutions were summarised and considerably extended in the monograph by Novozhilov et al. [17].

For the geometrically non-linear theory of thin elastic shells, the thorough description of deformation of the shell boundary element was performed by Novozhilov and Shamina [40] and Pietraszkiewicz [41-43]. In these works three sets of four kinematic boundary quantities, generalizing those of Chernykh [36] – i.e. displacemental, rotational, and deformational – were constructed. Makowski and Pietraszkiewicz [44] applied theory of differential forms and constructed the general form of four displacement boundary conditions which allowed one to formulate the variational principles of the non-linear theory of thin elastic shell. To each set of the four kinematic boundary quantities there corresponded an appropriate work-conjugate set of four dynamic quantities expressed entirely in terms of boundary stress and couple resultants, see Pietraszkiewicz [42,43,45]. Each of these boundary sets allowed one to formulate appropriate junction or jump conditions at the interface or intersection of thin shells. In particular, the rotational boundary quantities are appropriate for formulating the BVP of thin shells cast in terms of finite rotations as primary independent variables, as in Simmonds and Danielson [46] and Pietraszkiewicz [47,48]. For irregular shell problems, the rotational quantities enter the jump conditions at the shell junction, as in Pietraszkiewicz [49].

The deformational boundary quantities should be used in the boundary and jump conditions of non-linear shell problems formulated in intrinsic variables, as proposed by Chien [50], developed by Koiter and Simmonds [51], Pietraszkiewicz [41,42], Opoka and Pietraszkiewicz [52] and summarised by Pietraszkiewicz [53].

Chróścielewski et al. [54] modelled the undeformed base surface of the irregular thin shell by the union of a finite number of regular smooth surface parts joined together along spatial curvilinear midsurface edges. The PVW was then discretised by C^1 finite elements and the incremental-iterative procedure was applied to solve this highly non-linear BVP. As an example, the axisymmetric deformation state was calculated in the casing of a measuring devise having two pairs of circular welded junctions. The elasto-plastic range of material behaviour with linear combination of isotropic and kinematic hardening, and deformability of the junctions was taken into account.

Three corresponding sets of five boundary quantities, appropriate for the linear shell model of Timoshenko–Reissner type (accounting for transverse shears), were constructed by Shamina [55]. These quantities were extended into the non-linear range of shell deformation by Pietraszkiewicz [42,43,56]. For the geometrically non-linear case Mikhailovskii [57] proposed some alternative set of five deformational quantities at the shell boundary, which were shown by Pietraszkiewicz [58] to be equivalent to those proposed in Pietraszkiewicz [42]. Paimushin [59] constructed the non-linear theory of Timoshenko-type shells, the end sections of which were connected by a rod. It enabled all classical and non-classical forms of loss of stability in structures of the class considered to be investigated.

The most advanced, non-linear, resultant mechanical shell model was originally proposed by Reissner [60] and its development was summarized in monographs by Libai and Simmonds [61], Chróścielewski et al. [4], and Eremeyev and Zubov [62]. For such a six-field displacemental shell model (3 translation and 3 finite rotation components), the corresponding jump conditions were initiated by Makowski and Stumpf [63], Chróścielewski et al. [3,64] and developed in the books by Chróścielewski [65], Pietraszkiewicz [66] and Chróścielewski et al. [4]. A number of finite elements with six dof per node were constructed by Chróścielewski [65], Chróścielewski et al. [4,64] and in a number of other works. Many complex non-linear problems of irregular shells in statics and dynamics were solved by FEM in, for example, Chróścielewski et al. [3,4,64,67,68], where further references to other papers were given.

The exact, resultant, dynamic jump conditions for the branching and self-intersecting shells were derived by Konopińska and Pietraszkiewicz [6] by performing direct through-the-thickness integration of the global equilibrium conditions of continuum mechanics. The corresponding unique work-conjugate kinematic jump conditions for such shell structures were constructed by Pietraszkiewicz and Konopińska [8]. Classification of shell junctions was then proposed, among which were the stiff, entirely simply connected, partly simply supported as well as elastically and dissipatively deformable junctions, see Fig. 3 and Fig. 4. The jump conditions were also generalized in several papers by Eremeyev and Pietraszkiewicz [69-71] and Pietraszkiewicz et al. [72] to account for the phase transition phenomena in shells. Hanna [73] presented conditions for compatibility of velocities, conservation of mass, and balance of momentum and energy across moving discontinuities in inextensible strings and sheets of uniform mass density. The balances are derived from an action with a time-dependent, non-material boundary. Two examples of general solutions for conservative sheet motions near a line discontinuity were provided. Finally, the ultimate forms of jump conditions at moving singular surface curves were derived by Pietraszkiewicz and Konopińska [9] within the refined resultant thermomechanics of shells proposed by Pietraszkiewicz [10].

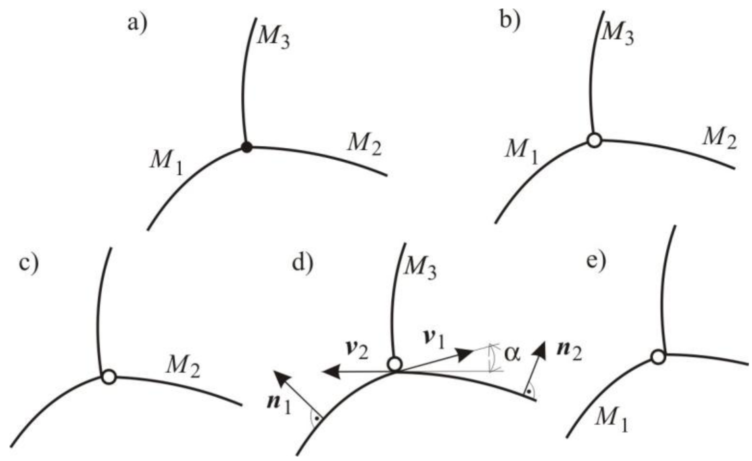

Figure 3. Junctions in the branching shell as in Fig. 2 : a) stiff, b) entirely simply supported, c) – e) partly simply supported

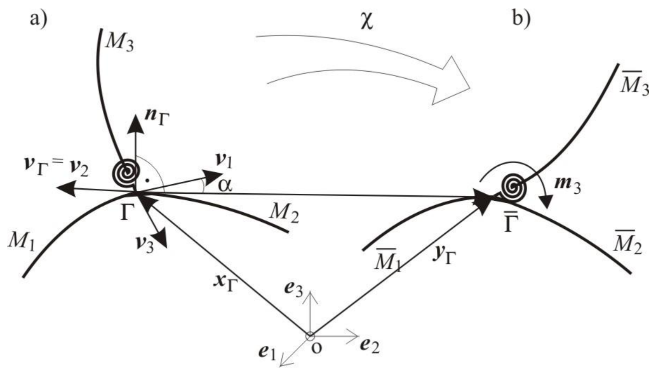

Figure 4. The deformable junction of M_3 with M_1 and M_2 in Fig. 2 : The undeformed (a) and the deformed (b) placements

Other theoretical results on jump conditions in shell structures were discussed by Paimushin [74], who applied variational methods to discuss non-linear problems of solving composite spatial bodies connected with thin shells. Simo [75] showed that the director field in the full nonlinear continuum shell equations must necessarily be described by three rotational degrees of freedom. A computational procedure involving a trivial modification of the global singularity-free update procedure described in that paper was proposed, which allowed to solve the shell intersection problem without introducing 'drill springs' or related ad-hoc devices.

Geymonat and Sanchez-Palencia [76] investigated rigidity of certain surfaces with folds within the framework of Douglis-Nirenberg elliptic systems. It was proved that the folded surfaces were never perfectly stiff. Additional rigidity conditions at the folds were required, and implications for the shell theory were discussed. Akian [77,78] performed an asymptotic analysis of bending-dominated shell junction within the linear isotropic elasticity. Two shells were linked together at the part of boundary, with mid-surfaces having distinct tangent planes. It was shown that when the shell thickness goes to zero the limit displacement field became inextensional and the angle between the shells remained the same during deformation. The limit model coincided with the Koiter type linear theory of thin elastic shells.

Tesar and Kuglerova [79] applied a modified technical torsion-flexural theory for analysis of slender thin-walled structures with elastic joints. The influence of elasticity of joints on the behaviour of the structures was discussed. Ibrahimbegović et al. [80] presented a model for thermomechanical coupling in folded plates or non-smooth shells, which could be used for analysis of fire-resistance of cellular structures. A modified version of shell element including drilling degrees of freedom was proposed. The model performance was illustrated by several numerical examples including the fire-resistance of walls built with clay bricks.

Important progress in analysis of compound complex shell structures was associated with a rapid development of numerical techniques, in particular the finite element method (FEM). Yamazaki and Tsubosaka [81] used isoparametric FE formulation with the penalty function to analyse the junction of plate and shell built-up structures. The connectivity condition was added to the potential energy functional. Special junction elements allowed for direct evaluation of the surface traction at the junction. Yamazaki and Tsubosaka [82] worked out a FE formulation for the exact analysis of junctions of plate and shell built-up structures. The formulation yielded an integral-type stiffness matrix of the special junction elements. A design sensitive formulation of analysis with adhesive special elements was developed. Koves and Nair [83] developed a specialized self-intersection finite element which had the capability of representing the complex 3D geometry and stress state at the shell intersection. The element was applied to the cylinder-sphere and cylinder-cylinder intersection problems as examples. Riccius et al. [84] proposed to combine adaptivity and mesh smoothing to detect shell junctions automatically as common curves of two adjacent shell parts. Shell intersections were treated in a way similar to shell boundaries. Flores and Onate [85] developed a rotation-free shell triangular finite element for the analysis of kinked and branching shells. The element was based on the non-linear theory of thin shells of Kirchhoff-Love type with only displacements and their derivatives as the DOFs satisfying the C^1 interelement continuity. Eberlein and Wriggers [86] outlined problems with FE calculation of finite plastic strains in the shell intersections. The 5/6-parameter, 6-parameter and 3D shell elements were compared on some carefully solved examples. Guo and Ruess [87] proposed a variationally consistent weak coupling method for thin-walled shell patches. The method overcomes the need for C^1-continuity along the patch interface to ensure a corresponding geometric continuity in the deformed configuration and a correct transfer of bending moments across the interface. The good performance of the method for pure Kirchhoff–Love shell models and blended shell models was illustrated with various examples.

Bernadou and Cubier [88] discussed numerical analysis of junction between thin shells. The linear Koiter-type elastic shell model was used. The rigid junction and the elastic hinge were analyzed. Variational formulation of different junction types was proposed. Existence and uniqueness of the solution was shown. Bernadou and Cubier [89] developed the pseudo-conforming FE method associated with the Argyris triangle. Numerical modeling of the junction region at the rigid and elastic junction was analyzed. As example, the axisymmetric stress state in the cylinder with the spherical cap was analyzed. Skopinsky [90] presented a systematic approach to the stress analysis of shell intersections made of composite materials. The FEM based on modified mixed formulation and 2D laminated shell theory allowed one to construct the special code Stress Analysis in Intersecting Shells (SAIS). A parametric study of intersecting composite laminated cylindrical shells was performed. Wang et al. [91] developed a four-node shell element based on 5/6 DOF with a singularity-free description of the rotation tensor in the original state. The numerical examples discussed suitability of the element in shell intersection problems. Porter et al. [92] suggested a procedure based on FEM for evaluating the stresses in the shell/plate nozzle model. The procedure related the stress levels from FE software to the provisions of ASME codes. Di Pisa and Aliabadi [93] analysed by the boundary element method (BEM) the fractured stiffened panels repaired with riveted or adhesively bonded patches. The patches were joined to the panel either with rivets or adhesive. Examples presented included a wing box with a three spar section with fully stiffened skin, which was considered to have a crack, and a repair patch was used on top of the crack to stop its growth.

4 Cylinder – cylinder intersections

The intersection problem of two cylindrical shells was extensively discussed in the literature in various configurations (see Fig. 5). A variety of external and internal loadings could be applied to the cylinder and to the nozzle.

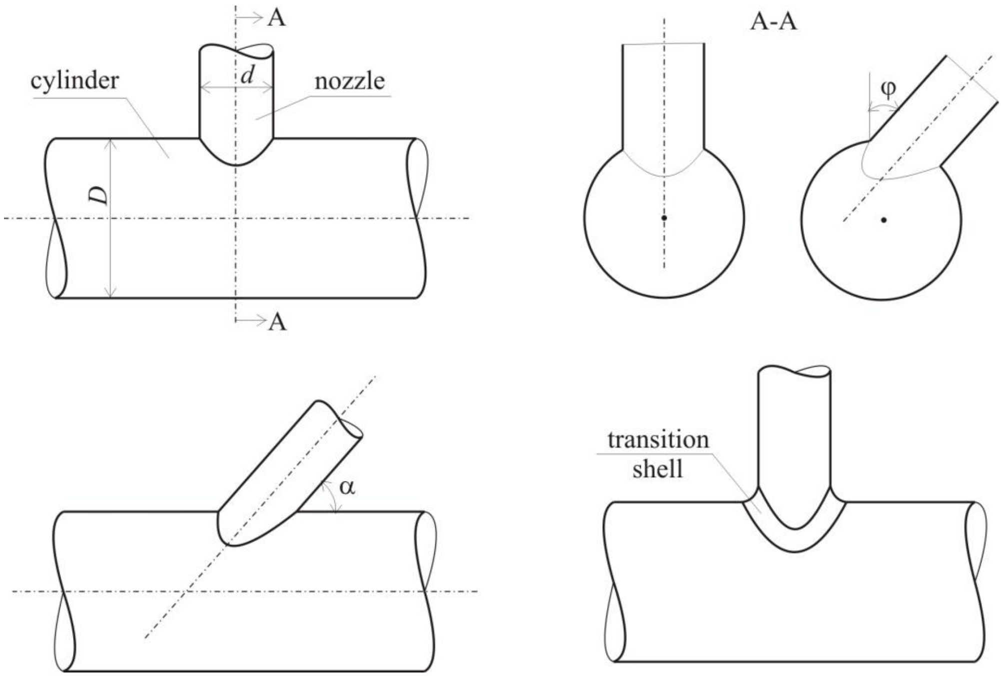

Figure 5. Typical configurations of cylinder – cylinder intersection

In the early period of junction modelling, Samans [94] analysed stresses in a cylindrical shell due to nozzle or pipe connection. Myint et al. [95,96] obtained approximate solutions of an intersection problem for circular cylindrical shells. The Ritz method was applied to the potential energy stored in the boundary region of the deformed shell. Numerical results were presented in the form of tables of integrals which permitted the application of the method to a wide range of problems. Stone and Hochschi [97,98] discussed the effect of nozzles pairing on pressure stresses at intersections of cylindrical nozzles and shells. Bijlaard et al. [99] analysed stresses in junction of nozzle-to-cylindrical pressure vessel for equal diameter of vessel and nozzle. Taniguchi et al. [100] analysed local stresses around nozzles of pressure vessels under external loading. Ando et al. [101] analysed the stress distributions in thin-walled intersecting cylinders subjected to internal pressure and in-plane force. Lekerkerker [102] determined the elastic stresses near cylinder-to-cylinder intersection. Hansberry and Jones [103] calculated elastic stresses due to axial loads on a nozzle intersecting a cylindrical shell. Kulkarni et al. [104] used the principle of virtual work for formulating the problem of two intersecting cylindrical shells. In particular, continuity conditions at the junction were furnished for the theory with transverse shear deformation, from which the Kirchhoff-type continuity relations were then considered. Chan and Sanders [105] performed matched asymptotic expansions based on thin shell theory at a mitered joint of two cylinders under in-plane bending. Steele and Steele [106] carried out the stress analysis for a cylindrical vessel with a nozzle subjected to external loading consisting of longitudinal and circumferential moments and radial force. Example curves and tables of stress factors were included. The comparison with experimental results for eight models was about as good as can be expected. Polevoi and Strelchenko [107] derived static junction conditions for two cylindrical shells intersected under an angle of their axes.

Several papers analysed solutions for stresses in two normally intersecting cylinders. The early papers included Reidelbach [108] who analysed the state of stress in the vicinity of the intersection of two perpendicularly joined circular cylindrical tubes. Eringen and Suhubi [109] formulated the BVP for the state of stress in two normally intersecting circular cylindrical shells subjected to internal pressure. The differential equations of the two cylindrical shells were solved for a small ratio of radii subject to edge conditions along the intersection curve and at the ends of the cylinders. Eringen et al. [110] performed the theoretical and numerical analyses of two normally intersecting cylindrical shells having diameter ratios not greater than 1/3. Both cylinders were considered to be infinitely long, capped at the ends and subjected to internal pressure. The exact solution of the partial differential equation of Donnell's theory of thin cylindrical shells was obtained in the form of infinite series. Yamamoto et al. [111] developed the theory of stress concentration of two normally intersecting cylindrical shells. Pan and Beckett [112] considered the problem of two normally intersecting cylindrical shells subjected to internal pressure. The differential equations used for the shells were solved subject to the boundary conditions imposed along the intersection between the two cylinders. Numerical results for the radius ratio of 1:2 were presented, and the results were compared with experiment. Maye and Eringen [113] proposed the theoretical analysis of two normally intersecting cylindrical shells subjected to internal pressure. Donnell's shell theory was used with the exact solution being obtained in the form of infinite series. Hansberry and Jones [114] made a theoretical study of the elastic behaviour of a joint formed by the normal intersection of a right circular cylindrical shell with another one of larger diameter. The analysis was valid for nozzle-to-cylinder diameter ratios of less than 1:3. Numerical results were given for a number of cases having the radius ratios between 1:10 and 1:4. Naghdi and Gersting [115] provided solutions for stresses and displacements at the perpendicular intersection of two simply supported cylindrical shells subjected to a non-equilibrated loading.

More advanced solutions for stress distribution in the normally intersecting cylinders were given by Moffat et al. [116], who produced design curves and an associated equation allowing the calculation of the effective stress factors for a wide range of piping branch junctions for both internal pressure and all six external moment load categories. The data suggested that modifications were required in design procedures for some geometries with internal pressure loading. Mokhtarian and Endicott [117] developed a method for finding stresses in intersecting cylinders subjected to pressure. Yim et al. [118] obtained the thin shell theoretical solution by solving a complex boundary value problem for a pair of fourth-order complex-valued partial differential Morley equations for the shell and the nozzle. The theoretical results were in a sufficient agreement with those obtained from 3D FE analyses for the case of internal pressure and six external branch pipe load components involving three orthogonal forces and the respective three orthogonal moments. Xue et al. [119] presented an analytical solution based on thin shell theory of two intersecting cylindrical shells subjected to transverse moment on the nozzle. The presented results were in very good agreement with experimental and numerical results, and with the results obtained by WRC Bulletin 297 when d/D is small. Xue et al. [120,121] provided a thin shell theoretical stress analysis of intersecting cylindrical shells subjected to external loads transmitted through branch pipes. Continuity and uniqueness conditions were applied, and the nozzle was subjected to three kinds of branch pipe forces.

Lotsberg [122] derived analytical expressions based on classical shell theory for stress concentration factors at welds in pipelines and tanks subjected to internal pressure and axial force. Numerical analyses were compared with results from axisymmetric FE analyses for verification of the presented methodology. The derived stress concentration factors can be used together with a hot spot stress *S*–*N* curve for calculation of fatigue damage. Xue et al. [123,124] worked out a universal analytical method for pressurized cylindrical shells with attached nozzles. The solution was based on the improved Morley equations. A sufficient agreement of the results with those obtained from 3D FEM for all cases was noted. Petrovic et al. [125] considered the influence of the torque moment affecting the free end of a slanted branch on the pressure vessel cylindrical shell. The stress in the cylindrical shell was analysed on 245 models with different geometrical characteristics. For every considered model, maximum stress envelope was prepared. Sommerville and Walter [126] investigated the effects of 2D axisymmetric modeling of the cylindrical nozzle to cylindrical vessel intersections on the through-the-wall stress distribution at the nozzle corner. Using 2D finite elements the proposed method corrected inaccuracies following from treating the intersection as an axisymmetric section. Mair [127] summarized some of the published methods of calculating stress intensification factors (SIF) for a limited range of lateral branch connections and maked recommendations based on comparisons with finite element analysis (FEA) studies. It also included recommendations on how such FEA studies should be applied in order to provide suitable SIF values.

In more recent research on normally intersecting cylinders, Khan et al. [128] performed a comparative study of the stress field around a reinforced and an unreinforced normal intersection of two internally pressurized cylindrical shells under in-plane and out-of-plane bending. Kuo and Hsu [129] provided analytical solution for stresses of normally intersected reinforced nozzle-cylindrical shells under internal pressure or nozzle radial thermal expansion. The computer code NUTSHELL was written and the results were compared with experiments and FEM results. Xue et al. [130] carried out the stress analysis based on the theory of thin elastic shells for two normally intersecting cylindrical shells with a large diameter ratio. The modified Morley equations were used for the analysis of the shell with the cut-out. The boundary forces and displacements at the interaction were expanded into Fourier series, and every harmonics of boundary coefficients was obtained by numerical quadrature. The results obtained were in agreement with those from the FEM and experiments. Xue et al. [131] presented the analytical results for stresses in two normally intersecting cylindrical shells with nozzles due to external run pipe moments. The results were in good agreement with the previous test results and with Moffat's results of 3D FE analyses. Xue et al. [132] presented a thin shell theoretical solution for two normally intersecting cylindrical shells due to the external branch pipe moment.

The plastic behaviour of the structure containing junction under internal pressure was discussed by Cloud and Rodabaugh [133], who derived an estimate of the plastic limit pressures of cylindrical nozzles in cylindrical shells based on assumed velocity distribution. Design graphs based on the analysis had been prepared. Biron and Courchesne [134] analysed how to satisfy lower bound theorem of limit analysis in the cylinder-cylinder tee connections subjected to internal pressure. Srinivasaiah & Schroeder [135] provided lower bounds to limit pressures at a tee intersection of cylindrical shells based on a 3D stress field. The results agreed with those from experimental tests. Robinson [136] performed a parametric survey of lower-bound limit pressure for the cylinder-cylinder intersection, with results compared with other known papers and experimental results. In the next paper Robinson [137] gave further comments to that problem and provided explanation of some apparently inconsistent experimental results. Hamilton et al. [138] proposed a simple method to obtain lower bound limit loads of pressurized cylinder-cylinder intersections using generalized yield criteria. Yang et al. [139] developed a modified elastic compensation method for limit analysis of nozzle-to-cylinder junction. Three representative examples were calculated, and the results were compared with other known solutions. Lee et al. [140] established a method for determining the limit pressure of piping branch junctions using FE computations. An approximate analytical formula for calculating the limit pressure was proposed. Lee et al. [141] presented plastic limit loads of piping branch junctions with local wall thinning under in-plane bending, based on 3D FE limit analyses using elastic—perfectly plastic materials. Two locations of wall thinning were considered: one in the run pipe (opposite to the intersection), and the other one in the branch pipe (next to the intersection). Simple approximations of wall thinning on the plastic limit loads were proposed. Lee et al. [142] extended the validity of plastic limit load solution of thin-walled branch junction to thick-walled cases. This was achieved by renormalization of the thin-walled expression. Skopinsky and Berkov [143] presented a new method and numerical procedure for determining the plastic limit load in cylindrical shell intersections using elastic-plastic finite element analysis. The proposed method was based on the maximum criterion of the rate of the change of the relative plastic work. A parametric study of the non-radial cylindrical shell intersections subjected to in-plane moment on the nozzle was performed to examine the influence of an angular parameter on the plastic limit moment on the basis of the proposed criterion.

Kataoka et al. [144] examined the relationship between collapse loads and local primary membrane stress of cylinder-to-cylinder intersections. It was shown that evaluation by stress intensification factor is overly-conservative. The proposed concept of collapse strength reduction factor (CSRF) provided more accurate strength evaluation of this structure. Kataoka et al. [145] extended the method of CSRF for cylinder-to-cylinder intersections with general plastic materials considering material and geometric nonlinearity. The results showed that the collapse pressure is larger than that of limit load analysis. Lee et al. [146] presented an approximate closed-form plastic limit load solutions for piping branch junctions under out-of-plane bending combined with pressure. Skopinsky and Berkov [147] proposed a new criterion for determining the plastic limit load in shell intersections using elastic-plastic FE analysis. Then, a numerical procedure was described to define the plastic pressure. A parametric study of the radial intersections of cylindrical shells under the internal pressure loading was performed to examine the influence of the diameter ratio on the plastic limit pressure on the basis of the proposed criteria.

The plastic behaviour of the junction structure under moment load was discussed by Xuan et al. [148], who evaluated plastic limit load of piping branch junctions subjected to out-of-plane moment loadings, with the failure mode of global collapse. The new approximate formula for the out-of-plane plastic limit moment was presented. Its accuracy was validated by comparison with FE analysis and experimental results. Xuan et al. [149] found the limit load of welded piping branch junctions subjected to combined internal pressure and bending. The non-linear FE analysis was used to propose the closed limit load solution. Xuan et al. [150] reported experimental results for the limit load of forged piping branch junctions with comparison with existing solutions. Some relationships of the limit load with structural dimensions and crack size were proposed. Kim et al. [151] discussed plastic limit loads for thin-walled piping branch junctions under combined internal pressure and in-plane bending. The results were obtained by 3D FE limit analysis. Xuan et al. [152] presented an approximate analysis of plastic limit load of forged piping branch junctions. A closed- form solution under in–plane moments was obtained. Results were compared with six experimental data of real piping branch junctions and with results of FE analysis.

Kim and Lee [153] presented plastic limit load solutions for a thin-walled branch junction under internal pressure and in-plane bending. The 3D FE analysis was applied. Kim et al. [154] proposed the closed-form yield loci in the limit analysis of thin-walled piping branch junctions under combined pressure and in-plane bending. The small-strain 3D FE limit load analyses were used. Ryu et al. [155,156] analysed by 3D FEM the plastic limit pressures and the limit moment of the piping branch junction with local wall thinning under internal pressure. Two locations of the wall thinning were considered. Simple approximate formulae were proposed. Lee et al. [157] presented approximate closed-form plastic limit load solutions for piping branch junctions under out-of-plane bending alone and under combined pressure and out-of-plane bending. 3D FE limit analyses for an elastic-perfectly plastic material were used. Comparison with published experimental plastic limit load data showed that the predictions agreed relatively well with the test data. Lee et al. [158] investigated by the FEM the buckling behavior of girth-welded circular steel tubes subjected to bending. Elastoplastic large-deformation analysis incorporating weld-induced geometric imperfection and residual stress was carried out. Results showed that the flexural behaviour of girth-welded circular steel tubes always involved local buckling near the girth weld on the compression side, which significantly affected the moment versus end-rotation response. Skopinsky et al. [159] used applied methods of nonlinear analysis to determine the plastic limit loads in shell intersection configurations under combined internal pressure, in-plane moment and out-plane moment loadings. The numerical analyses of shell intersections were performed using FEM, geometrically nonlinear shell theory in quadratic approximation and plasticity theory. The graphical results of the cylindrical shell intersection under different two-parameter and three-parameter combined loadings were presented. Skopinsky et al. [160] investigated the pad reinforced nozzle connections of the cylindrical vessel under internal and external loads using the special-purpose computer program SAIS for the elastic–plastic analysis and the stress analysis in intersecting shells. The results of a parametric study of unreinforced and pad reinforced vessel models with a nozzle under internal pressure, in-plane moment, and out-plane moment loadings were discussed.

Nadarajah et al. [161] performed limit and shakedown analysis of nozzle/cylinder interactions under internal pressure and in-plane moment loading. Preiss [162] performed shakedown calculations of two cylinder–cylinder intersections with constant moment load and varying internal pressure load. These were example cases for the application of check against progressive plastic deformation as stated in the new European UFPV standard, Annex 5.B. In this context, the usage of the deviatoric maps of stress states to obtain proper self-equilibrating stress fields was shown. Some problems and corresponding possible solutions for performing the shakedown check using a FE model with shell elements were stated and shown. Abdalla et al. [163] proposed a simplified approach to analysis of shakedown of a cylindrical vessel-nozzle intersection subjected to the steady internal pressure and cyclic out-of-plane bending moments on the nozzle. Appropriate Brie diagram was generated and compared with previous ones. The results were verified by FE simulations of elastic–plastic cyclic loading. Additionally, Abdalla et al. [164] determined the maximum moment carrying capacity and the elastic limit load.

Chen and Schnobrich [165] presented many numerical results based on FEM for the non-linear behaviour of intersecting cylinders. Brown et al. [166] presented a comparison of stress results obtained using 3D FE code with results obtained from experimental testing for the cylinder-to-cylinder structure with a variable shell thickness. It was concluded that the FE code can accurately and economically predict the structural response of complex components. Sabir [167] developed new strain-based finite elements and used them for the analysis of cylinders with holes and normally intersecting cylinders. A method of substructing was introduced to enable a solution to the large number of non-banded set of simultaneous equations encountered. The solutions showed good agreement when compared with experimental results. Kirkwood et al. [168] used the FE code to model an equal-diameter branch pipe intersection subjected to out-of-plane and twisting moments. The predicted stresses were compared with results from tests of a welded branch junction, and also with the values from the current UK power piping code BS 806.

Khan and Shah Syed [169] considered five models of normally intersecting cylindrical shells with radius ratios ranging from 0.528 to 1.000 with out-of-plane moment loading. For each diameter ratio different combinations of thickness ratios and radius-to-thickness ratios of the run shell were considered. The FEM was used to find the magnitude and location of the maximum stress. Natarajan et al. [170] developed a FE model to study the stresses in the neighbourhood of a cylinder-cylinder intersection with internal pressure loading. Various parametric finite element studies were conducted. The selected model is then validated by applying it to various available cylinder intersection models and comparing the results. The finite element results were further compared with a solution obtained using a shell theory. Wu and Lu [171] worked out a quadrilateral cylindrical finite element based on 2D shell theory for elastoplastic analysis of nozzle-vessel intersection. Examples for stress analysis indicated a good agreement with those based on 3D brick elements and experimental data. Shen and Li [172] developed a method based on 3D FE analysis to evaluate stress intensities in the neighborhood of cylinder-cylinder intersection under internal pressure.

Skopinsky [173] proposed a classification of the model joints of intersecting cylindrical shells. The stress analysis was performed with FEM based on modified mixed variational formulation. The computer program SAIS was developed and parametric numerical studies were presented. In the case of non-radial cylindrical shell intersections subjected to external loading the code SAIS was used by Skopinsky [174] to perform additional stress analysis. Skopinsky [175] used thin shell theory and FEM to investigate shell intersections with torus transition under internal pressure loading. The developed special-purpose computer program SAIS was employed for elastic stress analysis of the shell intersections. Comparison of calculated results with experimental data was presented. The results were given in the form which allowed one to analyse the effects of changing the parameters on stress ratios in the shell intersections. Skopinsky [176] presented a comparative study of 3D and

2D FE analyses for normally intersecting cylindrical shells under pressure loading. The comparison of stress results obtained using the 3D computer program with experimental data was given.

Hoseinzadeh et al. [177] developed a simple method, based on a parametric study conducted by ABAQUS FE software, for design and analysis of metallic pipe joints under torsion. Many case studies of typical metallic joints under torsion were considered. A prototypical joint was designed by using the developed design curves, and the stress distributions were verified by the same software. Tang et al. [178] carried out the FE analysis of elastic stresses in the cylinder-nozzle intersection under pressure and in-plane nozzle moment. It was found that the total stress value under the combined loads was not completely equal to the total value gotten by adding the two stress states under separate loads. Ahmad et al. [179] carried out a 3D FE study on stress analysis in pressurized piping tee intersections. Then the stress concentration factor and the plastic collapse load were found for intersections with different depths of wall thinning. Qadir and Redekop [180] carried out the 3D FE analysis of a pressurized vessel-nozzle intersection with wall thinning damage. The effect of stress concentration factor (SCF) of wall thinning on damage was examined. A parametric study was conducted for a wide range of tee joints. Lv and Wang [181] performed FE analysis with ANSYS code of stresses in the nozzle zone at channel of pressure vessel. The results revealed that the stress concentration region lied at the junction of channel and pipe.

Some research was devoted to intersecting cylinders loaded dynamically. Jones [182,183] performed an approximate rigid-plastic analysis of shell intersections loaded dynamically, in which a maximum permanent radial deflection was found. Aslanyan et al. [184] considered an eigenvalue problem of 3D elasticity for a multi-structure consisting of a 3D solid linked with a thin-walled elastic cylinder. An asymptotic method was used to derive the junction conditions and to obtain the skeleton model for the multi-structure. Explicit asymptotic formulae were obtained for the first six eigenfrequencies. Sawa et al. [185] performed the FEM stress analysis for strength of adhesive butt joint of similar hollow cylinders under static and impact tensile loadings. It was found, in particular, that characteristics of the joints subjected to impact loading were opposite to those subjected to static loading. Cheng and Widera [186] investigated the burst pressure of a series of cylinder-cylinder intersections with various diameters and wall thicknesses subjected to short term dynamic loading. The FEM code LS-DYNA was used. A correlation equation for prediction of the dynamic burst pressure was proposed.

Jiang and Yahiaoui [187] used the 3D FEM to analyse complex intersections of multipass welded piping branch junction, with boundary nonlinearities associated with welding. The residual stress distributions on the both run and branch pipes and their cross section were given. Hossain et al. [188] measured experimentally the residual stresses in a nozzle-to-cylinder weld after thermal ageing at 550^{o} C. It was found that the thermal treatment had significantly relaxed the welding residual stresses. Jiang and Yahiaoui [189] analysed by FEM the effect of cooling rate on the final residual stress state in a thick-walled piping intersection. It was found that the magnitudes and overall spatial distributions of residual stresses were very sensitive to the cooling rate. The results and modelling technique presented show that the residual stress profiles in multipass welded complex geometries can be efficiently optimized through the convenient cooling rate control. Petrova and Bouzid [190] outlined an analytical model of a flanged joint with a tube sheet of a multi-pass shell-to-tube heat exchanger subjected to a non-axisymmetric thermal loading. The analytical results were compared with those following from FEM analyses.

Jiang and Yahiaoui [191] developed a 3D thermomechanical model to account the effect of welding sequence on the residual stress distribution in the multipass piping branch junction. Three possible symmetrical welding sequences were evaluated. It was found that the high residual stress was formed in the vicinity of weld region irrespective of the welding sequence. Law et al. [192] measured the significant residual stresses in new connections to an existing pressure gas pipeline. An integrity assessment of the welded branch connection was performed based on the existing analysis codes. It was found that the residual stresses estimated from the codes overestimated the real stresses. Ure et al. [193] implemented a lower bound method based on Melan's theorem into the Linear Matching Method ratchet analysis procedure. Then a ratchet analysis of a pipe intersection subject to cyclic thermo-mechanical loading using the proposed numerical technique was applied. The pipe intersection considered here had multiple materials with temperature-dependent properties. Verification of the results was given via the full elastic-plastic analysis in Abaqus.

Schoessow and Kooistra [194] conducted a strain-gauge test on a cylindrical shell to which two pipes were attached. The stresses were of magnitude that demanded the respect and attention of designers. Calladine and Goodall [195] performed pressure tests on circular vessels with circular cutouts and radial branches. The limit pressures observed for vessels with cutouts agreed well with a lower-bound analysis. Fidler [196] reported stress level of photoelastic analysis of oblique cylinder intersections subjected to internal pressure. Corum and Greenstreet [197] performed experimental tests of elastic stress analysis of cylindrical-to-cylindrical shell models and compared the results with some theoretical predictions. Warren [198] performed photoelastic tests on complex intersections of reinforced thin shell cylinders with longitudinal and circumferential stiffeners. Gwaltney et al. [199] tested four carefully machined cylinder-to-cylinder shell models intersected at the right angle with different diameter-to-thickness ratios, and the results were compared with theoretical predictions obtained from a thin-shell FE analysis. Comparisons of measured and predicted stress distributions were presented for three of these loadings: internal pressure as well as in-plane and out-of-plane moments applied to the nozzle. The agreement was shown to be reasonably good for the four models. Khab and Hsiao [200] presented the results of an experimental stress analysis of the intersection region of two straight cylindrical shells. In-plane and out-plane moments were applied to the attached shell. The measurements demonstrated that the local stress concentration in the intersection region of the main shell increased with the increase of the acute angle between the axes of the two shells. Fang et al. [201] investigated the elastic strength of cylindrical vessels with hillside nozzle under out-of-plane moment on the nozzle. A total of 5 model vessels with different hillside nozzle angles and loading directions were fabricated for the study by experimental and FE numerical simulation methods. The results indicated that the maximum elastic stress occurred at the inside corner of the nozzle-cylinder intersection in the transverse section of the cylinders.

Gao et al. [202] studied elastic stress distribution, deformation characteristics and stress concentration factors of a cylindrical vessels with lateral nozzle under internal pressure. Three full-scale vessels were investigated both experimentally and by the 3D FEM analysis. It was indicated that the maximum stress occurred at the acute side of the intersection. Moffat [203] presented results of experimental stress analyses of four equal-diameter branch junctions of cylindrical pipes of different thickness that had been subjected to six external moment loadings together with internal pressure in one case. The results were presented and discussed in the light of current American and British piping design codes.

Sang et al. [204] provided results of limit and burst pressures for a cylindrical shell intersection with intermediate diameter ratio. The results were found experimentally and compared with those obtained by the nonlinear FEM. Li et al. [205] tested experimentally and calculated by 3D FEM the plastic limit load and deformation characteristics of three pressurized cylinders with lateral nozzle. Distinct characteristics occurred in each of the three vessels. Wu et al. [206] determined the plastic limit moment for cylindrical vessels with a nozzle under in-plane moment on the nozzle. Three full-scale models were tested and the 3D FEM analysis was performed. An empirical formula for the problem was proposed.

Murthy et al. [207] conducted fracture and fatigue tests as well as FE analysis on two shell-nozzle junction specimens until their collapse. The factor of safety for crack initiation, the through-wall crack size at the collapse was obtained. Mangerig and Romen [208] presented the results of seven fatigue tests on welded pipe intersections in steel-concrete composite bridge structures. Wang et al. [209] used experiments and FEM analysis to determine the burst pressure of pressurized cylinders with hillside nozzle. Three full-scale test models with different angles of the hillside nozzle were fabricated. It was found that the elastic-plastic FE analysis was a viable option to estimate the burst pressure in the structure. Xue et al. [210] carried out experimental burst tests by pressurizing test vessels with nozzles to burst. A parametric study was carried out as well. The results showed that the proposed equation resulting from the parametric analysis could be employed to predict the static burst pressure of cylindrical shell intersections for a wide range of geometric ratios. Wang et al. [211] obtained the equations of constraint-dependent J-R curves for the cracks of a dissimilar metal welded joint for connecting pipe-nozzle of nuclear pressure vessel. Different constraint equations derived from the three pairs of crack growth amount all agree with the experimental J-R curves.

Johns et al. [212] presented theoretical and experimental results for discontinuous stresses arising at a change of wall thickness in a cylinder–cylinder, a cylinder–hemisphere and a cone–spherical torus junction in pressure vessels. Many numerical results were given. Abbas et al. [213] analyzed the effects of geometric misalignment in the cylinder–cylinder junction of a pressure vessels. Stresses at the affected area of the cylinders were predicted with analytical analysis and FEM ANSYS code.

Among other problems of intersecting cylinders available in the literature, let us mention Khathlan [214], who developed the computer code to analyse the stress and strain distributions around the junction of two normally intersected circular cylinders. The vessel and nozzle solutions were incorporated with the appropriate continuity conditions between the two cylinders. The code was used to develop tables of stress concentration factor to complement those published by the Welding Research Council. Kim et al. [215] discussed the effect of reinforcement on plastic limit loads of branch junctions of tubes under internal pressure and in-plane / out-of-plane bending. It was found that the reinforcement was most effective when applied to the branch pipe. Laghzale and Bouzid [216] developed a simple analytical model able to predict the change in the residual contact pressure at the tube-to-tubusheet joint interface. The results from the analytical model were checked and compared with those of FE analysis. It was shown that relaxation of the contact stress due to creep can be significant and should be considered in design. Wang and Koizumi [217] analysed experimentally and numerically the buckling of cylindrical shells with longitudinal rigid or flexible joints under external pressure. It was clarified that the buckling depended on the shell dimension and on imperfections along the joint. Honda et al. [218] used the CSRF method to analyse the elastic-plastic collapse characteristics of 45 deg cylinder-to-cylinder intersections. The 45 deg-laterals were weaker than 90 deg-laterals, and inelastic analysis provided greater strength of 45 deg-laterals than the elastic analysis. The results of elastic-plastic analysis showed that the overly large plastic strain occurred on 45 deg-laterals. Mao and Bao [219] comparatively investigated buckling behaviours of T joint and pipe by varying geometric parameters and methods of analysis. The effects of the wall thickness and ellipticity on buckling behaviour were taken into account. Through rigorous FE numerical analysis, the buckling behaviours of the T joint and pipe were discussed in terms of deformation pattern, stress distribution, and critical pressure. Some interesting and useful conclusions were drawn.

5 Cone – cylinder junctions

Many parts of complex shell structures are designed as cylindrical shells joined with conical shells. Some typical parts of such shell structures are given in Fig. 6.

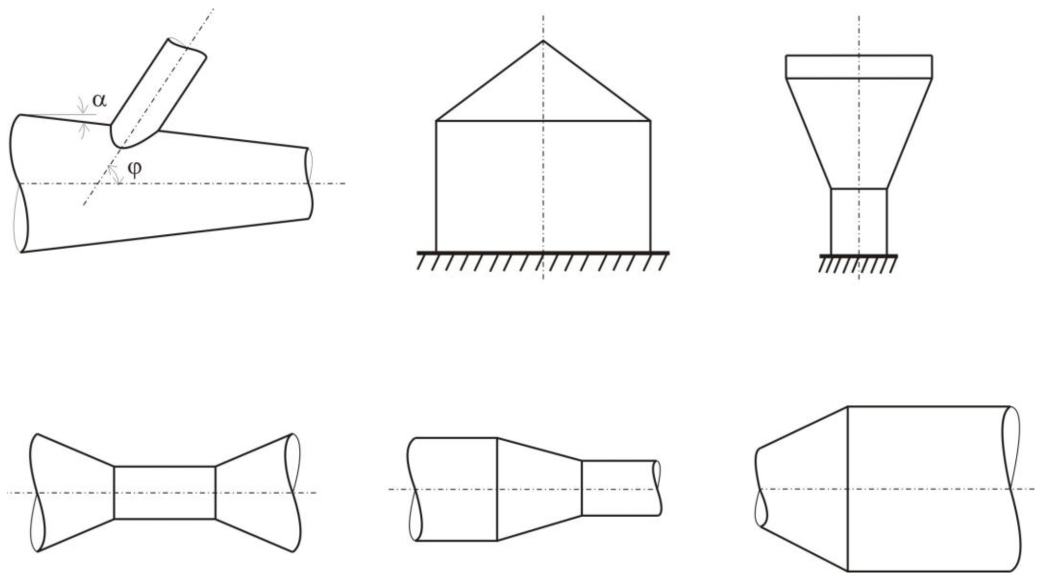

Figure 6. Some typical configurations of cone – cylinder compound shell structures

Within the linear theory of thin elastic shells, Watts and Lang [220] proposed an analytical method for calculating stresses in a pressure vessel with a conical head. The solutions were presented taking care of shear, direct stress and bending for various values of a number of parameters. The abundance of numerical tables and graphs was given. Becker [221] developed the theory for stresses at coaxial intersections of cones and cylinders of uniform thickness. Some adjustments were made to obtain a low stress in the intersection. Alwar and Ramamurt [222] discussed an asymmetric bending problem of cylindrical-conical shell junction. The theoretical model and the numerical method were worked out and some numerical solutions were given. Graff [223] analysed junction stresses for a conical section joining cylinders of different diameter subject to internal pressure. Jerath and Boresi [224] derived boundary conditions at a cone-cylinder shell junction. Skopinsky [174] used the linear elastic thin shell theory and the FE SAIS code to analyse effects of changing geometric and angular parameters on the stress ratios in non-radial cylindrical shell intersections subjected to external loading. Skopinsky [225] employed the shell theory to investigate the cone-cylinder intersection. The FE SAIS code was used for elastic stress analysis. Comparison with experiments was presented. Thin and moderate-thickness shells were analysed. Zamani et al. [226] presented an elastic solution of cylinder-truncated cone shell intersection under internal pressure. The general solution of the cone equations was based on power series method. The effect of cone apex angle on the stress distribution in conical and cylindrical parts of structure was investigated. In addition, the effect of the intersection and boundary locations on the circumferential and longitudinal stresses were evaluated.

In order to describe the behaviour of structures containing cone–cylinder junctions, interesting experiments were conducted. Borg [227] measured stresses and strains near intersections of conical and cylindrical shells. The test models had three different apex angles and three different thickness-to-cylinder diameter ratios. The results were in agreement with the Geckeler theory of cylindrical shells and other linear shell models. Teng et al. [228] described the development of an experimental facility for testing model steel silo transition junctions. Issues covered included the fabrication of quality model junctions using thin steel sheets, the loading method and the precise 3D measurement of geometric imperfections and deformed shapes using a laser-displacement meter. Typical experimental results of a cone–cylinder–skirt–ring junction were presented to demonstrate the capability of the developed facility. Zhao and Teng [229] presented the carefully conducted experimental results on buckling of cone-cylinder intersections under internal pressure. The FEM predictions were also given. Zhao and Teng [230,231] presented the results of a series of tests on cone–cylinder–skirt–ring junctions in steel silos under simulated bulk solid loading. The tests included the geometric imperfections measurements and the failure behaviour. Determination of buckling modes and loads based on displacement measurements was examined in detail. Then the finite element modelling of the experimental results was performed. The comparisons showed that in FE analyses one should take into account the geometric imperfections, effects of welding and the interaction between the junction and the stored solid. The implications of the experimental and FE numerical results for the design of steel silo transition junctions were discussed. Zhao [232] presented the experimental study of buckling behavior of imperfect ring-stiffened cone-cylinder intersections under internal pressure. The results from nonlinear bifurcation analysis using the perfect shape and nonlinear analysis using the measured imperfect shape were presented and compared with the experimental results. El Damatty et al. [233] conducted experiments on elevated water tanks consisting of a truncated cone with a top superimposed cylindrical cap. Fundamental frequencies as well as the frequencies of vibration of the modes during which the cross section of the tank remains circular were analysed. Results of the experiments were used to validate the assumptions employed in a previously developed analytical model for the free surface sloshing motion and a numerical model for the vibration of the liquid-shell system. Khalili and Showkati [234] presented experimental results and FE numerical analyses of buckling of T-ring stiffened cone-cylinder intersection under internal pressure. Two classes of non-linear analyses were compared, and the experimental results were compared with the design proposals.

The effect of material plasticity on the compound shell behaviour was analysed by Jones [235], who discussed influence of internal pressure on the behaviour of a cylindrical nozzle which intersected axisymmetrically a conical pressure vessel for the case when both shells were made of rigid, perfectly plastic materials. It was observed that the lower and upper bounds of the title problem were coincident to three significant figures. Taylor and Polychroni [236] used the limit analysis to investigate minimum weight conditions for concentrated ring reinforcement at the intersections between equal thickness conical and cylindrical parts of vessels subject to internal pressure. Myler and Robinson [237] reported some results on the limit analysis of intersecting conical pressure vessels. Attention was paid to the necessity of reinforcement in the region of the junction. A computer code was written for calculating the lower bound limit pressure. Teng and Rotter [238,239] employed an elasto-plastic axisymmetric large-deflection FE analysis to study the stress distribution, plastic collapse behaviour and classical limit load analysis of steel silo transition junctions. The distribution of stresses, the effect of large deflections, the formation of a plastic collapse mechanism, and the collapse process were investigated. An improved but still simple equation was then proposed for use in design. Then a major parametric study of the plastic collapse behaviour and strength of the complex transition junctions was performed. It was noted that the modified method gave a much closer overall approximation to the finite element results over the wide range of parameters studied. Kalnins and Updike [240] analysed two failure modes for junctions between cylindrical and conical shells: the axisymmetric yielding and the low-cycle fatigue. The effect of reinforcement on the strength was discussed in detail.

Various buckling problems of cylinder–cone structures were discussed by Greiner and Ofner [241] who performed an elastic-plastic buckling analysis at cone-cylinder junctions of silos. Knoedel [242] discussed cylinder-cone-cylinder intersection under axial compression. Teng [243] presented a detailed finite-element investigation into the axisymmetric failure behaviour and strength of intersections of the large end of a cone and a cylinder subjected to uniform internal pressure. A rationally based simple equation for the plastic limit loads from a small-deflection elastic-plastic analysis was established. The beneficial effect of large deflections on failure strength was also developed. In the accompanying paper Teng [244] extended the previous results into the nonsymmetric bifurcation buckling behaviour and strength of these intersections and established the accurate and simple buckling strength equations. The range of shell geometries whose strength was governed by nonsymmetric buckling was established. An approximate equation to account for the effect of large deflections on failure strength was developed. Teng [245] generalized the Rotter effective area method for the plastic limit loads of cone-cylinder intersections in silos to include also the focal pressure effects. A simple formula was proposed for the collapse strength of different types of metal shell intersections. The formula was suitable for codification purposes. Teng and Gabriel [246] investigated the collapse behavior and strength of the cone-cylinder intersections under internal pressure with locally increased wall thickness. Simple equations for the plastic limit loads were developed and the effect of geometrical change was briefly discussed. Sweedan and El Damatty [247] developed a simplified procedure for design of hydrostatically loaded combined steel conical tanks against buckling. The numerical results obtained with FEM together with a non-linear regression analysis were used to develop magnification functions that relate the overall shell stresses to the membrane stresses which can be evaluated analytically.

Several dynamic problems of cylinder-cone shell structures were discussed. Lashkari and Weingarten [248] performed an analytical and experimental investigation to determine natural frequencies and mode shapes of a cone-cylinder segmented shell. The FE technique and holographic interferometry were used. Liu and Liu [249] extended the transfer matrix method to free vibrations of joined conical-cylindrical shells made of composite materials. Numerical results for natural frequencies and mode shapes were given. Kamat et al. [250] investigated by the FE method the dynamic instability of parametrically excited laminated composite joined conical-cylindrical shells. The influence of various parameters on the dynamic stability regions of cross-ply laminates was brought out. Dynamic characteristics of the segmented cylindrical-conical shell were analysed by FEM in the book by Chróścielewski et al. [4]. Two mode shapes are given in Fig. 7 as examples.

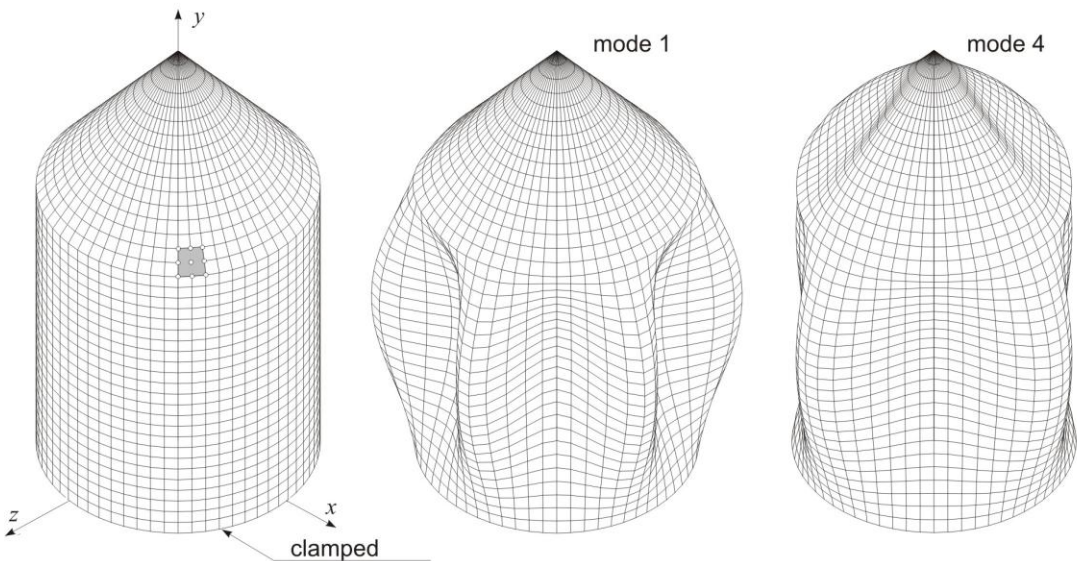

Figure 7. Two mode shapes of cone – cylinder segmental shell

El Damatty et al. [251] analysed dynamic characteristics of the truncated conical vessels having a superimposed top cylindrical cap. Experimental testing of small models were first conducted and the tested specimen were then simulated numerically by a 3D FEM. An extensive parametric study is then conducted to determine the dynamic characteristics of full-scale combined conical vessels. Equations and charts describing the natural frequencies, the mode shapes and the generalized and effective masses of the vessels were obtained. Qu et al. [252] proposed a variational method to study free vibrations of joined cylindrical-conical shells subjected to classical and non-classical boundary conditions. The results validated from FEM ANSYS code were found to be in a good agreement with previously published results. Kouchakzadeh and Shakouri [253] used continuity conditions at the joining section of the cones to formulate and solve the free vibrations problem of joined cross-ply laminated conical shells. All combinations of boundary conditions could be assumed. The effects of semi-vertex angle, the meridional length and the shell thickness on the natural frequencies were investigated. Ma et al. [254] presented a free and forced vibration analysis of coupled conical–cylindrical shells with arbitrary boundary condition. New numerical examples were also conducted to illustrate the forced vibration behaviour of the coupled conical–cylindrical shells subjected to the excitation forces in different directions. Shakouri and Kouchakzadeh [255] investigated natural frequencies and mode shapes of two joined isotropic conical shells. The equations of Donnell type were solved assuming trigonometric response in circumferential and series solution in meridional directions, and all combinations of boundary conditions could be assumed. The results were compared and validated with the available results in other investigations and also with model testing. The effects of semi-vertex angles and meridional lengths on the natural frequency and circumferential wave number of joined shells were analysed.

Additional elastic buckling problems were investigated by Teng and his associates. In particular, Teng [244] analysed by the FEM the cone-cylinder intersection under nonsymmetric buckling behavior and internal pressure. Accurate and simplified buckling strength equations were derived. The range of shell geometries was defined whose strength was governed by non-symmetric buckling. Teng [256] investigated the elastic buckling and strength of cone-cylinder intersection with a ring, under localized circumferential compression. Approximate strength equations were developed. Teng and Barbagallo [257] analysed the out-of-plane buckling strength of rings attached to cone-cylinder intersections under an elastic restrain of the neighbor shells. A simple closed-form design formula was proposed and the results were compared with the FE shell buckling analysis. Teng and Chan [258] developed a simple method for assessing the out-of-plane buckling strength of ringbeams provided at the cone-cylinder intersection under internal pressure of an elevated storage silo and tank. Teng and Ma [259] described the elastic buckling strength of ring-stiffened cone-cylinder intersections under internal pressure. Two main buckling modes were identified. Simple expressions were proposed which relate the number of circumferential buckling waves to the geometrical parameters of the intersection. Teng and Zhao [260] deployed the research on the buckling of internally-pressurized cone–cylinder intersections and state-of-the-art finite element analyses to provide another description of their failure due to misoperation overpressure. The validity of the existing formulae for real vessels with geometric imperfections was next examined through a comparison of theoretical predictions with experimental results. The calculations demonstrated that the cone–cylinder intersection buckled first, followed by the buckling of the spherical partition. Zhao and Teng [261] worked out a rational FE model based on experimental and numerical results for postbuckling behavior and strength of the cone-cylinder intersections subjected to internal pressure. A design proposal was established in the Eurocode format for practical use.

Among other problems discussed in the literature, let us mention Mahdi et al. [262] who analysed the effect of cylindrical part length on the crushing behaviour, failure mechanisms and modes of cone-cylinder-cone intersection of the composite shells. Theoretical model and experimental results were compared. Patel et al. [263] discussed nonlinear thermoelastic buckling/post-buckling characteristics of cross-ply laminated joined conical-cylindrical and conical-cylindrical-conical panels subjected to the uniform temperature rise. The influence of semi-cone angle, material properties, and the number of circumferential waves on the response was studied. Gorla and Haddad [264] used the FE analysis to find stresses and heat transfer in a cone-cylinder pressure vessel. Heat transfer rate was computed from structural and thermodynamic random variables. Bardia et al. [265] compared the cone-to-cylinder junction reinforcement requirements provided in two ASME codes based on different criteria.

6 Sphere – cylinder junctions

Many pressure vessels, oil, gas or water containers and variety of specialised fittings used in refineries are constructed by joining spherical and cylindrical shell parts. Some examples of such junctions are sketched in Fig. 8.

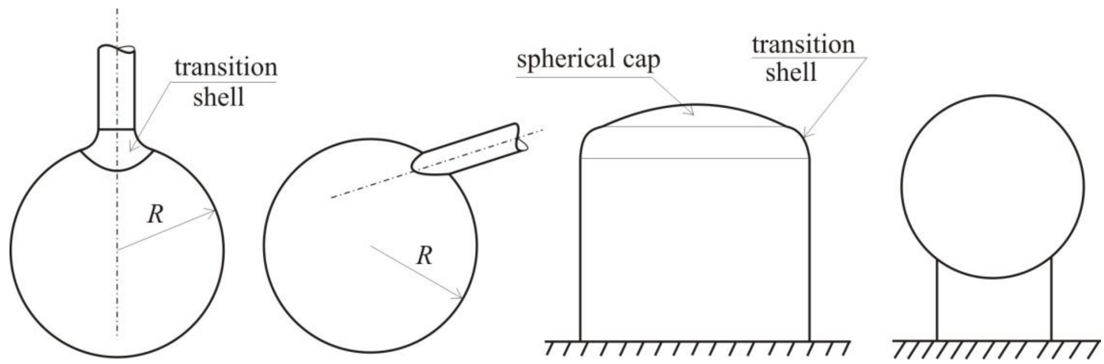

Figure 8. Examples of shell junctions of sphere – cylinder type

Various theoretical and numerical methods were applied to this problem within the linear theory of thin elastic shells. In particular, Watts and Lang [266] described a computational procedure to obtain the stresses in a pressure vessel with a hemispherical head. Tabulated results were given at the junction for various ratios of parameters. Penny [267] analysed stress concentrations at the junction of a spherical pressure vessel and cylindrical duct caused by certain axisymmetric loadings. Varga [268] discussed elastic displacements and stresses in the region of intersection of the cylinder with a spherical shell subjected to internal pressure. The numerical results were compared with experimental ones. Johnson [269] analysed stress distribution in a spherical shell with a non-radial nozzle. The classical linear K-L theory of shells was applied, and the cylinder radius was much smaller than the sphere radius. Four displacement continuity conditions along the junction were proposed. Ellyin [270] presented experimental results for seven cylinder-sphere intersecting shells for wide range of parameters. The experimental results were compared with theoretical predictions with good agreement. Bryson et al. [271] performed a parametric study for stress distribution in the reinforced cylinder-to-sphere intersection. Jayaraman and Rao [272] presented an analytical solution to the problem of a reinforced junction of a pressure spherical shell with a conical nozzle. The reinforcement and the conical nozzle were considered as an elastic discontinuity in the spherical shell causing perturbations in the undisturbed membrane state of stress in the spherical shell. The computer software was developed and applied to a wide range of discontinuity parameters to assess their influence on the stress concentrations. Brooks [273] developed a computer code based on thin shell theory for reducing the stress level at the reinforced cylinder-to-sphere pressure vessel intersection. All three forces and moments on each nozzle, the internal pressure and thermal loading were considered. Chen and Zhou [274] used simplified methods to calculate the redundant moments and forces at the junction of the hemisphere and cylindrical shells of different thicknesses**.**

In more recent papers, the FEM was utilised. Murthy et al. [275] investigated by FEM the spherical shell-cylindrical nozzle multipass weld junction of steam generator. The obtained residual stresses were at the extent of heat affected the region from the weld. Luo [276] analysed by FEM the sharp-angle and round-angle transitions between the spherical shell and the nozzle on the stress distribution in the junction. The round transition could insignificantly reduce the maximum bending and the peak stresses as compared with the sharp-angle transition, but not the membrane stresses. It was recommended to adopt the sharp-angle transitions in those connections. Luo and Guo [277] used FEM to analyse stresses in the spherical shell with opening nozzle. Al-Gahtani et al. [278] presented a parametric FE study of the junction of spherical vessel with the cylindrical nozzle under internal pressure. Wide range of dimensions was studied. It was found that the minimum required cap size was linearly related to the nozzle size.

Elasto-plastic approach was applied by Leckie [279] who analyzed shakedown pressures for flush cylinder-sphere intersections. Ellyin and Sherbourne [280] presented a general method for finding lower and upper bounds on the collapse pressure for a rigid-plastic vessel composed of intersecting cylindrical and spherical shells. The Tresca yield condition and the associated flow rule were obeyed. The solutions were presented in terms of generalized stresses and may be used in conjunction with any choice of the yield surface. Ellyin [281] analysed the effect of yield surfaces on the limit pressure of intersection of the cylindrical and spherical shell. The limit analysis was performed with the Tresca yield surface for the rigid–perfectly-plastic material with associated flow rule.

Robinson et al. [282] calculated lower bounds to the limit pressure for a spherical vessel with a circumferential slot at its junction with a radial cylindrical protruding branch. Nadarajah et al. [283] worked out a simple method of estimating limit loads in the nozzle-sphere intersections with internal pressure and radial load. A sequence of elastic FE analyses and the lower bound theorem was applied. The results were compared with detailed FE elastoplastic analysis and good coincidence of them were obtained. Weiss and Rudolph [284] performed analyses by the FE concerning the fatigue strength of nozzle-to-spherical shell intersections subjected to internal pressure, axial force and torsional moments. Stress concentration factors established represented the maximum von Mises stress to an approximate normal stress.

Some design solutions of sphere – cylinder junction were proposed by Leckie et al. [285] who applied theoretical studies to a complex component of a power plant. A rational and simple design procedure emerged which was presented in terms of the representative deformation and rupture stresses. The theoretical predictions were compared with the results of a series of tests performed on the component. Zheng et al. [286] worked out a unique design of the junction between a thick cylindrical pressure vessel shell and a thinner hemispherical head. Its basic structure, stress characteristics, and the engineering design method were described. Jiang et al. [287] studied the residual stresses in the welding connections between the hydrogen nozzle and a sphere head in a hydrofining reactor. The results showed that large axial stresses were generated on the internal surface of the hydrogen nozzle due to angular deformation. A new connection structure between the nozzle and the head was developed.

Among other interesting dynamical solution of sphere – cylinder shell intersections, let us mention Jones [182,183] who proposed an approximate rigid-plastic analysis of such shell intersections loaded dynamically. The two common cases of cylindrical nozzles intersecting either spherical or cylindrical shells were considered in some detail for various geometrical parameters. Summers and Jones [288] provided experimental results on dynamic plastic behavior of the cylindrical nozzle intersecting a hemisphere shell under impulsive velocity field. Attwater et al. [289] performed the 3D FE analysis of spherical pressure vessels with radial disposed cylindrical nozzle intersections under the axisymmetrical loading. Stress data were obtained and the maximum principal stresses were plotted. The results were related to those following from British Standards. Xue and Zhang [290] studied the failure mechanism and impact factor of load carrying capacity of welded hollow spherical joints connected with circular steel tubes. The experimental research on compressive bearing capacity of such junctions was also given. Wu et al. [291,292] utilized the domain decomposition method to investigate free vibration characteristics of the combined cylindrical – spherical shells with different boundary conditions. Exact solutions were found and compared with those obtained by the FE ANSYS software to compare reliability and accuracy.

7 Junctions of shells of revolution

Shells of revolution are among the most popular shell structures used in modern technology. Various structures of this type are constructed by joining two or more regular axisymmetric shell parts at their circular edges thus leading to variety of useful shell shapes. Hyperboloidal cooling towers, oil and water tanks, pressure vessels, rockets, and silos sketched in Fig. 9 are only a few examples of such structures.

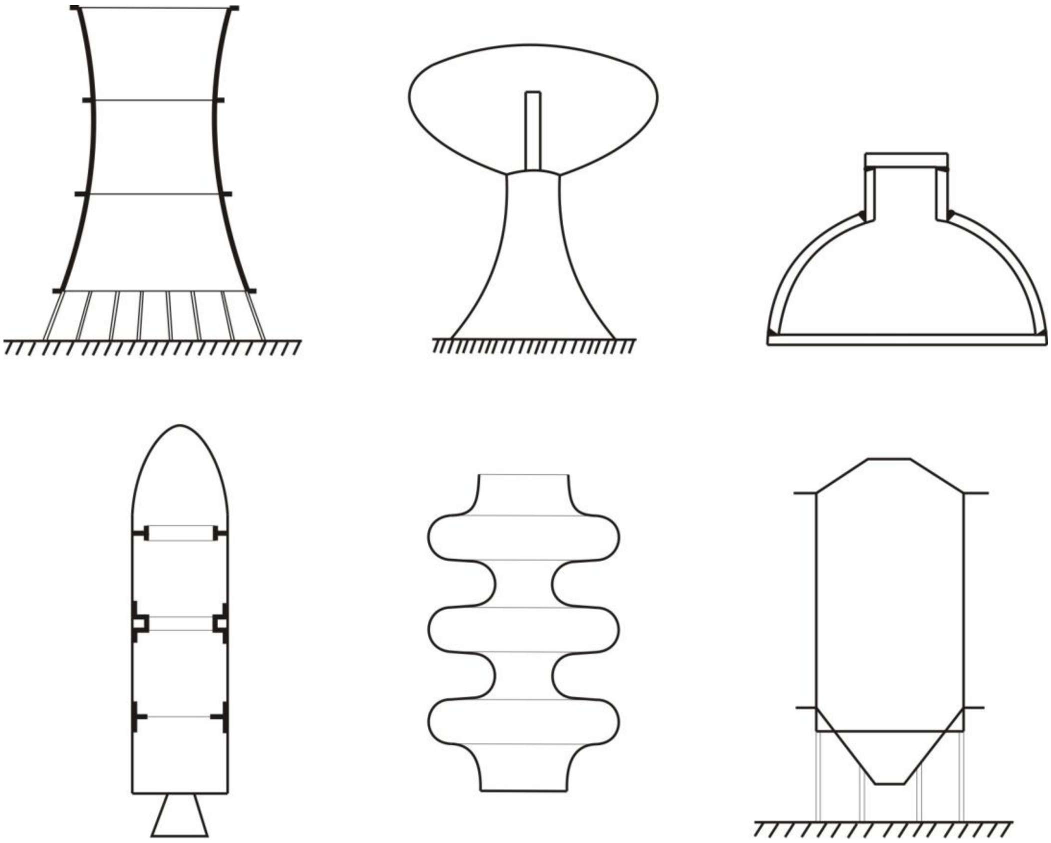

Figure 9. Examples of compound shells of revolution

The linear theory of thin elastic shells was used by Chernykh [293] who presented explicit analytical and numerical results for compound shells of revolution. Among the results were a) the junction of two shells directly and through the reinforcing ring, b) the shell junction with different temperatures, c) axisymmetric and anti-symmetric external loadings, d) junctions of toroidal shell parts, e) the spherical shell supported on the cylinder, f) junction of cylinders with defects in the joined middle surfaces. References to several earlier Russian books reporting the shell junction problems were given. Zingoni [294] developed a simplified but reasonably accurate approach for calculating junction stresses in general shells of revolution with certain variation of junction thickness along the shell meridian. The method was applied to the 3-region egg-shaped digestor shell, to yield explicit closed-form analytic results for stresses throughout the digestor. Any desired parametric studies may also be conducted on the basis of these analytical results.

Plastic behaviour of the material was used by Ellyin and Sherbourne [280], who discussed the general theory of the plastic analysis of axisymmetric shells of revolution. Conditions for the construction of a complete solution and bounds to it were defined. Approximate solutions, in conjunction with yield surfaces for the cylindrical shells, were outlined. Galishin [295] performed an axisymmetric thermoelastoplastic stress-strain analysis of isotropic shells of revolution with a branched meridian. The results were extended by Galishin [296] for laminated transversally isotropic shells. Gnit'ko and Merzlyakov [297] performed non-axisymmetric thermoelastoplastic analysis of branched shells of revolution by the semi-analytic finite element method. The method was based on the principle of virtual displacements of the linear Kirchhoff-Love type shells undergoing small elastoplastic strains.

Bushnell [298] developed a comprehensive computer programme BOSOR4, based on the finite difference numerical technique, for analyses of stress, stability, and vibration of segmented, ring-stiffened branched shells of revolution. The programme performed large deflection axisymmetric stress analysis, small-deflection nonsymmetric stress analysis, modal vibration analysis with axisymmetric nonlinear prestress included, and buckling analysis with axisymmetric or nonsymmetric prestress. A large number of problems were solved to demonstrate the scope and practicality of the program. Then Bushnell et al. [299] used the refined code BOSOR6 to analyse the failure of axially compressed cylindrical shells containing the frangible joints. In the analysis plasticity, moderately large deflection shell theory and various boundary conditions were discussed. The numerical results agreed to within 2% with the experimental joint failure. Nemeth and Anderson [300] performed the axisymmetric shell analysis that parametrically accessed the structural behaviour of the space-shuttle solid rocket booster field joint. The rocket was subjected to quasi-steady state internal pressure loading for both the original joint and the redesigned joint. A method for simulating contact between adjacent shells of revolution was proposed and discussed.

The FE displacement method was applied by Lakshminarayana [301] to analyse the axisymmetric bending of laminated composite shell junctions using laminated anisotropic shell theory. Numerical results were presented for cylinder-cone shell junction and cylinder-geodesic-isotensoid dome junction. These results were compared with results for isotropic case. Pei and Harik [302] applied an iterative method for analysis of axisymmetrical general isotropic shells and joined shells of revolution with varying rigidity. Solution was obtained by the stiffness approach. Special geometric shapes – cylindrical, spherical, conical – were used in the elements.

Flores and Godoy [303] analyzed the postbuckling behavior of elastic cone-cylinder and sphere-cylinder shells of revolution. Asymptotic approximation to the secondary path was used. It was found that the bifurcation loads of the compound shell were lower than those of the individual components.

Intersection of a cylindrical shell with a plate appears in many compound shell structures. Linear strength analysis was applied by Sokolov [304], who calculated moments and deformations in a plate flange welded to a tube and loaded with tensioning load. The numerical results were compared with experimental data. Sis'mekov and Kagan [305] investigated the junction of the plate ring with a spherical shell. The unknown boundary loads in the junction were found from two equations of the force method. Complete formulas and graphs were obtained for practical use. Redekop and Schroeder [306] generalized the theory presented previously for the 3D stress analysis of intersection of a cylindrical shell with a plate to include non-axisymmetric loading cases. The geometry of the intersection was partitioned into three parts: cylindrical shell, plate, and ring. A comparison was given of theoretical results with experimental data. Flavin [307] analysed a right elastic cylinder with plane ends perpendicular to the generators, to which plates were bonded to the plane ends. One plate was given a general displacement in its own plane, while the other was fixed. It was proved that the strain field converged in energy norm to that of a simple shear state. Various applications were discussed. Laberge and Baldur [308] presented curves for the stress concentration factors due to inside corner radii between cylinders and flat-head closures. The analysis was based on a FE study. The paper indicates the degree of conservatism of the ASME Boiler and Pressure Vessel Code. Abrosimov [309] analysed numerically the dynamic nonlinear deformation, buckling and postbuckling behaviour of composite plate-shell structures made of rigidly joined plates and shells of revolution. For solving the IBVP an explicit 2^{nd}-order time integration scheme was used. Szybiński et al. [310] showed numerically the influence of parameters defining a circular stress relief groove, position of the groove and its radius on stresses and strains in flat ends of high-pressure vessels. The results were verified in experimental investigations.

Several other problems of cylinder–plate junction were also discussed. Lee et al. [311] carried out the thermal ratcheting test with the cylinder-to-plate junction. The ratched deformations were analyzed with the constitutive equations of the non-linear combined hardening model. The analysed results were in good agreement with those of the structural tests. Baldur et al. [312] discussed FE analysis of thermal load on the plate–cylinder intersection. The numerical results were condensed into a rapid method for the determination of peak stresses needed for performing the fatigue analysis in pressure vessels subjected to a significant variable thermal load. A set of coefficients provides a convenient method of stress evaluation suitable for design purposes. Lambert and Bell [313] developed a pipe-plate joint as a simplified model of tubular joint geometries for fatigue studies. Two specimens were tested and estimates of stress intensity factors were made. Separately, FE analyses for various discrete crack configurations were performed. The results were discussed, with particular emphasis on the accuracy and the implications for fracture mechanics modelling. Lee et al. [314] carried out the progressive inelastic deformation behaviour under thermal loads of the cylinder-plate junction with various plate thickness and travelling lengths. The thermal ratchet deformations were analysed with constitutive equations of the non-linear combined hardening model. The results were compared with those of the structural tests and a reasonably good agreement was found.

Tso and Hansen [315] analysed wave propagation through cylinder-plate junctions. Lee and Choi [316] analyzed the free vibrations of simply supported cylindrical shells with an interior rectangular plate by using the receptance method. The frequency equation of the combined system was obtained by considering the continuity conditions at the joint between the plate and the shell. The numerical results were compared with those given in published works and to experimental results with good agreement. Filippov and Haseganu [317] analysed low frequency vibrations of a thin cylindrical shell joined with an annular thin plate. Filippov [318] calculated the critical external pressure and the buckling modes of a thin circular cylindrical shell joined with an annular thin plate. Wang et al. [319] proposed simplified strength checking formulae for a semi-cylindrical header-nozzle intersection under combined piping loads. Three methods were presented and compared. The plastic work curvature method was found to be the most reliable one among the three methods.

Zeman [320] analysed the ratcheting limit of flat end cylindrical shell connections under internal pressure. The ratcheting limits for two different models were given for an ideal-elastic ideal-plastic constitutive law, the Mises yield condition and the associated flow law. The results were compared with each other and with the limits for global plastic deformation given in the literature. Preiss [321] calculated equivalent stresses according to Tresca's yield criterion and principal stresses for the flat end-to-cylindrical shell connection subjected to internal pressure. A stress concentration factor was defined. For different values of the ratio of fillet/groove-radius to plate thickness, approximation functions for those stress concentration factors were obtained. Preiss et al. [322] performed the limit load and creep design of highly non-linear cylinder-plate intersection. Within the small-strain theory, the influence of shear force acting at the intersection on the limit load pressure was shown.

8 Tubular joints

Tubes of small diameter can be connected directly by welding, through flanges, using adhesive or bolts and by other means. To design and analyse more complex 3D tubular joints advanced FEM computer codes are required. Some examples of tubular joints are sketched in Fig. 10.

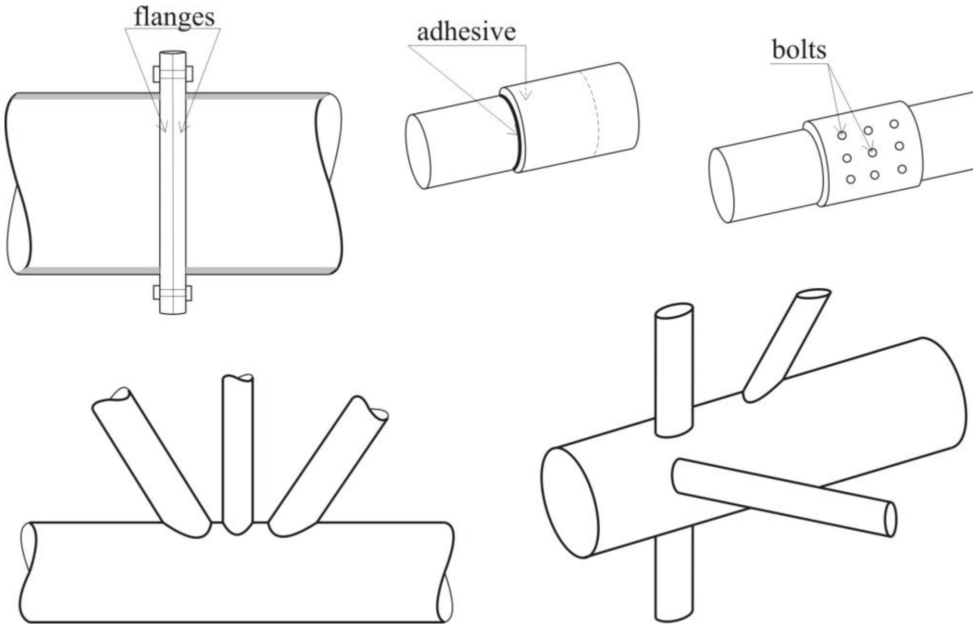

Figure 10. Some examples of tubular joints

Within the linear theory of shells, Esslinger [323] proposed simple and useful formulas for calculation of the pipe connections. It was assumed that the socket entirely took up the membrane stress. Fessler and Levin [324] determined experimentally the stress distribution on the inner surfaces near tee junction of thick pipes. An approximate method was developed, and the calculated and measured results were compared. The method took account of the spatial shape and variable rigidity of the flange. Rodabaugh and Alterbury [325] developed formulas which provide stresses in and near a tapered transition joint in pipelines and pressure vessels. Many useful graphs were presented. Comparison with test results was given with a good agreement. Scordelis and Bouwkamp [326] presented a method, based on the Donnell-Jenkins shell equations, for analysis of tubular tee-joints under any input loading or displacement pattern. The analytical results obtained by the computer were compared to those obtained previously experimentally. Applying the FE numerical analysis, Haswell [327] presented a study of a cracked tubular joint using several different structural models and consistent input parameters. A comparison of the full-scale analysis and the simple representations allowed conclusions to be drawn on the applicability of simplifications to the problem and also allowed errors to be quantified. Golovanov [328] described a scheme of joining isoparametric shell elements with a bend in the middle surface. Tubular joint loaded by internal pressure was analysed, where stiffness of the welded joint was taken into account. Bowness and Lee [329] performed the FE study to find stress fields and stress intensity factors in tubular joints. Various modeling assumptions were critically discussed. Nwosu et al. [330] performed the FEM numerical analysis for stress distribution along the intersection of internally ring-stiffened tubular T-joints under the action of axial and in-plane/out-of-plane loads. Stress concentration factors were computed. Parametric analyses and comparison with known analytical and experimental results revealed that stiffening could considerably reduce the stress concentrations. Chen et al. [331] proposed a new analytical method for load distribution on the thread teeth in a cylindrical pipe threaded connection. The results showed that the load was concentrated mainly on the last 4-5 threads. The FEM analysis validated the new method. Stael et al. [332] proposed an alternative method for determining the stress concentration factors of complex tubular bridge joints. This method was applicable to any tubular joint and accurately determined all SCFs of the joint. These SCFs allowed fast computation of the hot spot stresses of the tubular joint loaded with any realistic load condition.

Morgan and Lee [333] presented an extensive FE parametric study for distributions of stress concentration factors in tubular K-joints under the unbalanced out-of plane moment loading. The results could be utilized to obtain the accurate and safe fatigue life estimates. Chang and Dover [334] carried out comprehensive thin shell FE analyses for 330 different tubular Y and T-joints. A new set of equations was derived from the joint geometric parameters. These equations were used to predict the stress distributions along the intersection and the hot spot stress concentration factor. Mao and Bao [219] investigated buckling behaviours of T joint and pipe by varying geometric parameters and analysis methods. The equations of critical pressure and buckling wave number in the elasto-plastic range were established. The FEM was adopted to explore the effects of nonlinearities on buckling. Through FEM numerical analysis, the buckling behaviours of the T joint and pipe were discussed in terms of deformation pattern, stress distribution, and critical pressure. Pavlović et al. [335] compared performance of the ring flange connection and the novel friction considering connection of a real tubular tower segment. The FEM is used to investigate possible failure modes of the connections. Recommendations for the design are provided together with a numerical example validated by experimental tests.

Design procedures for joining tubes were also discussed. Hoseinzadeh et al. [177] developed a simple engineering approach for design and analysis of adhesively bonded metallic pipe joints under torsion. Many parameters were taken into account. A prototypical joint was designed and verified with FEM. Zhao [336] surveyed the latest design procedures for welded steel tubular joints. Both static design and fatigue design were covered. Zhao et al. [337] summarized the third edition of IIW static design procedures for welded hollow-section joints. Research background on yield stresses, moment resistance of multi-planar joints, chord stress functions, overlap joints, and plate-to-tube joints were discussed. Holderness and Bruno [338] demonstrated in-plant repair strategy of seal welded and hydraulic expended tube-to-tubesheet joints in a high pressure feedwater heater. Ahmadi et al. [339] applied a parametric FE analyses for development of fatigue design formulas for outer brace SCFs in offshore three-planar tubular DKT-joints. Effects of geometrical characteristics and loading conditions on the values of SCFs were studied. Lukankin et al. [340] considered two coaxial cylindrical shells with different radii joined by a stiffening ring either rigidly or by hinges. A numerical algorithm for constructing solutions of the resulting equations is proposed, in which the conditions for the joining of the shells with the stiffening ring were satisfied exactly. Various cases of stability of the structure were investigated.

Su et al. [341] investigated by the 3D FEM the fatigue strength of the welded tube-to-tubesheet joint under a periodic axial load along the tube. It was shown that there existed many fatigue striations in the fracture face. Yang and Guan [342] developed a one-dimensional analytical model for stress analysis of composite pipe joints under combined torsional and tensile loads. Good correlation with FEM results was found. Ahmadi and Zavvar [343] used the geometrically parametric study followed by the nonlinear regression analysis to develop a new set of formulas for the stress concentration factors (SCFs) for the fatigue design of ring-stiffened KT-joints under the out-of-plane bending loadings. The numerical data was extracted from the FE analysis of 118 models, which were validated against the test results obtained from an experimental study.

9 Some other types of shell junctions

Complex shell structures are designed by joining regular shell parts at their common boundaries. Due to variety of geometries of shell base surfaces this leads to a great number of complex shell structures bounded only by the designer imagination and technological requirements. As a rule, such complex shell structures can only be analysed by direct numerical codes. Only for some simple shell problems with junctions special techniques of analysis have been developed. A few cylindrical shells with misalignment, joined with a plate or joined to the supporting structure are given in Fig. 11 as examples of such simple cases.

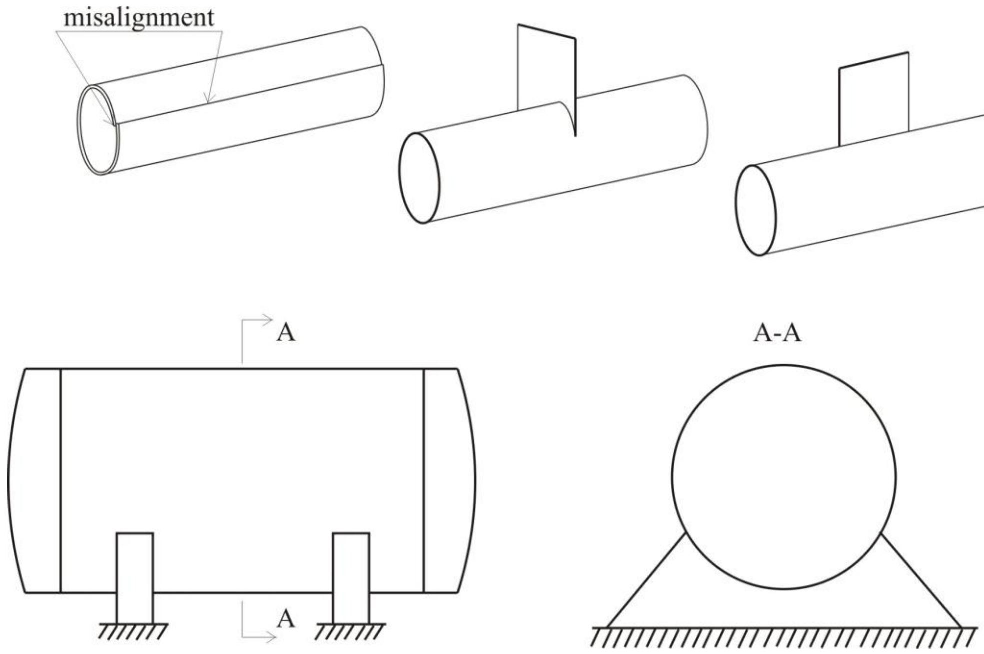

Figure 11. Examples of junction of the cylindrical shell with the plate and supporting structure

Influence of misalignment at the shell junction may be of importance for engineering design. Zeman [344] treated the problem of angular misalignment at longitudinal weld joints of cylindrical shells, following from a residual moment and a residual force. Stress concentration factor was derived, and the measurement procedure proposed. Bock and Zeman [345] proposed a power-series approximation for the local type of peaking in welded longitudinal joints of cylindrical shells with angular misalignment of the cylinders. Good agreement with the FE analysis was noted. Ong and Hoon [346] worked out an improved theory for prediction the peak bending stresses at the longitudinal welded joints of pressurized cylindrical shells due to angular distortion. The results were validated against FEM results. Zingoni [347] analysed discontinuity effects at cone-cone axisymmetric shell junction. The results were based on the one-term asymptotic-series solution for the axisymmetric bending of a non-shallow conical shell. As an example, the stress distribution in a large liquid-filled elevated rhombic tank was evaluated. The stresses obtained on the basis of the closed-form analytical approach developed in the paper were shown to be in good agreement with those obtained from a FE analysis. Brabin et al. [348] carried out the FE analysis for elastic stress distribution at pressurized cylindrical pressure vessels axial junction having a misalignment in the circumferential joint.

Various closing heads of cylindrical shells are used in compound shell structures. Galletly [349] derived the influence coefficients for edge displacements at a junction of a cylinder closed by a torispherical head and a cylinder joined to a smaller cylinder by an ellipsoidal-toroidal shell. Accurate stress distributions can be obtained quickly without a specialized knowledge of shell theory. Kraus et al. [350] solved the Love-Meissner equations by the finite differences with computer. A wide range of shell parameters of the ellipsoidal heads were treated, and the numerical results were compared with experimental ones. Hoffman [351] derived minimum weight proportions for variable thickness domes of long cylindrical pressure vessels, under restriction that the membrane stress satisfied the

Mises-Hencky yield conditions. Ellipsoidal and torispherical heads were considered. Skopinsky and Berkov [352] used a numerical procedure based on FEM and the SAIS code to formulate a new criterion for the definition of plastic limit load in nozzle connections of ellipsoidal pressure vessels. The parametric study of the joints was given. Błachut and Magnucki [353] discussed some aspects of strength, static stability, and structural optimization of horizontal cylindrical pressure vessels. Stress concentrations at the junction of cylinder-ellipsoidal end closures, choices for wall thicknesses in the vessel, stresses for flexible supports and their dimensions as well as the ultimate load carrying capacity of internally pressurized heads were examined. Aboveground and underground cases were analysed and optimal sizing of whole vessels was discussed for slender and compact geometries.

Vilhelmsen [354] presented reference stress solutions for endcaps derived from collapse loads using the FE method. The computed reference stress could be used for design and remaining life predictions. The reference stresses were compared to the predictions by some codes, and it was demonstrated which combinations of geometries had the potential for premature failure. Skopinsky and Smetankin [355] carried out a parametric study for stress analysis of reinforcement of ellipsoidal pressure vessel head with the offset nozzle. Various reinforcement configurations were considered, and their effects on the maximum stresses in the head-nozzle intersection was performed. Prinz and Nussbaumer [356,357] experimentally investigated the low cycle fatigue capacity of the liquid storage tank shell-to-base connections under multi-axial loading. Twenty-seven tank connections were fatigue tested. The fatigue fracture originated in the base material away from the weld heat affected zone.

In the book by Chróścielewski et al. [4] the non-linear post-buckling behaviour of the clamped wavy cylindrical shell reinforced by the orthogonal plate rib was analysed by FEM. Under the concentrated force applied at the end, the primary equilibrium path and two secondary post-buckling equilibrium paths were calculated. Fig. 12 shows that for some force level five different equilibrium states are possible, but only the equilibrium state A could be reached along the primary equilibrium path.

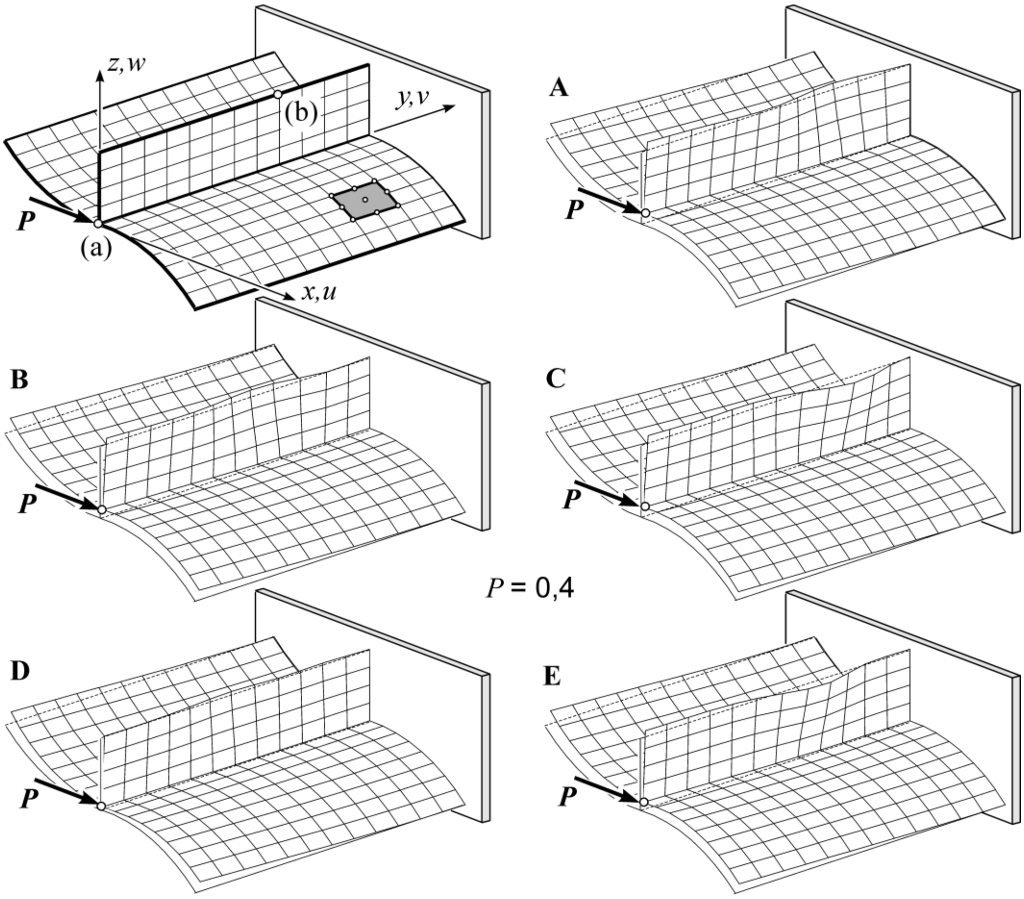

Figure 12. The clamped wavy cylindrical shell reinforced by the rib; five different post-buckling equilibrium states A – E for the same value of P

Some dynamical problems of shells with junctions were also analysed. Langley [358] discussed elastic wave transmission properties of a structural junction of an arbitrary number of curved panels. The junction transmission coefficients and coupling loss factors were found. Redekop [359] performed the vibration analysis of a torus-cylinder shell assembly based on the linear Sanders–Budiansky theory. Solutions were set up using the differential quadrature method. Results were given for some standard-size 90° pipe bends, and these results were compared with FE results. Ozerciyes and Yuceoglu [360] analyzed the free asymmetric vibrations of composite full circular cylindrical shells stiffened by an adhesively bonded non-circular shell segment. The final system of ODE was integrated numerically. Typical mode shapes and natural frequencies were given for several sets of support conditions.

Thermal problems of shells with junctions were posed by Cheng and Weil [361], who discussed the junction between a cylindrical tower and the skirt made by continuous fillet weld. Reduction of the stresses in the weld was achieved by lowering the thermal gradients and slotting the skirt, thereby reducing the stiffness. Nishi et al. [362] evaluated the strength and thermal fatigue life on dissimilar metal joints for ITER reactor first wall components. The results revealed existence of two singular stress solutions. Oka et al. [363,364] discussed the effect of hot feed feel injection time on thermal fatigue life of the shells-to-skirt junction area of coke drums in the refinery. Four identical coke drums were analyzed. Temperature and strain histories were measured and FEM analyses were performed, which were used to calculate the thermal fatigue life. It was found that the injection time strongly affected the number of cycles to fracture. Jiang and Guan [365] applied FEM to find the residual stress and deformation induced by the welding sequence of half-pipes into the shell. The effect of heat input and cooling was analysed. Filling the shell with cooling water could considerably decrease the residual stress.

Khan [366] performed the flexibility tests on 32 test samples of shell-to-shell intersections. The flexibility factors of these connections were experimentally determined. The experimental values were compared to predicted values given in ASME code, Section III, Subsection NB. Recommended equations for flexibility factors were given for the cases where experimental values did not agree with the predictions using the code equations. Then Khan [367] proposed a new equation for stress intensification factors for use in the case of integrally reinforced weld-on fittings. It was found that the pad-reinforced connections had a much higher fatigue life than the unreinforced shell-to-shell intersections. This fatigue life was comparable to or better than that of tested weld-on fittings.

Among other types of analyses of compound shell structures, let us mention Leckie and Penny [368] who calculated stress concentration factors for stresses at nozzle intersections in pressure vessels. Kozlov et al. [369] posed a mixed BVP for an elastic multi-structure consisting of a union of a 3D domain and thin cylinders. A rigorous algorithm was presented which yielded a multi-scaled asymptotic expansion for the displacement and stress fields in the multi-structure. Wood [370] reported experimental results on elastic stresses, deflections and rotations in the vicinity of an unreinforced intersection of 90 deg mitered bend subjected to in-plane bending moment. The results were compared with p-adaptive thin shell FE calculations. Moazed et al. [371] examined numerically with FEM and experimentally a welded T-joint connection of square thin-walled section tube under a multi-axial state of stress. It was shown that the membrane stress in the tube remained similar regardless of the weld geometry, which affected only the bending stress. The stress concentrations were highly localized at the vicinity of the weld toe. Voth and Packer [372] used the FEM to perform the parametric study on the behaviour and design of T-type plate-to-circular hollow section connections. 120 connections were analysed. It was established that the chord length should be at least eight time of the chord diameter. Partial design strength functions were proposed. Mani et al. [373] presented results of parametric study of pull-out radius for steam generator shell nozzle junction for the fast breeder reactor. A FE modelling for the shell nozzle junction was used. The optimized stress values for the five models were presented. Zingoni et al. [374] presented a linear-elastic theoretical formulation for determination of stresses in large thin-walled liquid-filled vessels in the form of multi-segmented spherical shells. The edge effect in the vicinity of the shell junctions was modelled by the bending theory for spherical shells. The developed formulation was in excellent agreement with the FEM modelling, showing that the presented theoretical formulation is a reliable, computationally efficient and accurate means of obtaining stresses in large multi-segmented spherical vessels.

10 Conclusions

We have reviewed the research on the compound thin-walled shell structures composed of regular shell elements joined together along their common boundaries. First, books and some review papers on junctions in shell structures have been characterised in which various approaches to modelling and analysis of compound shells are presented. Then general results on several forms of boundary, continuity, and/or jump conditions at the singular surface curves dividing regular parts of the shell base surface have been reviewed. Alternative forms of mechanical models of shell junctions have been reviewed, and a number of problems treated analytically, numerically and experimentally have been recalled. Finally, we have carefully reviewed results obtained for special shell structures containing the cylinder-cylinder intersection, cone-, sphere-, and plate-cylinder junctions, tubular joints, junctions of shells of revolution as well as some other special types of junctions of multi-shells commonly used in modern technology.

The limited size of this paper has not allowed us to review some other problems associated with shell junctions. As an example let us mention the junctions of plate structures, which should be reviewed separately or may be treated to some extend as special cases of junctions in shell structures. One should be aware that many interesting problems associated with shell junctions may not be available in printed publications, for they are prepared by some technical organisations and companies only as internal reports with limited accessibility in openly published media.

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